Universiti Malaysia PAHANG

Engineering • Technology • Creativity

Electricity, Magnetism & Optics

Current, Resistance and Electromotive Forces

by

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Chapter Description

- Aims

  Students will understand the concept of electric current and relate it with material's resistivity

- Expected Outcomes
  - Able to describe electric current and how it relates to charge flow
  - Able to differentiate resistance and resistivity
  - Able to analyze resistance of a conductor based on its geometry

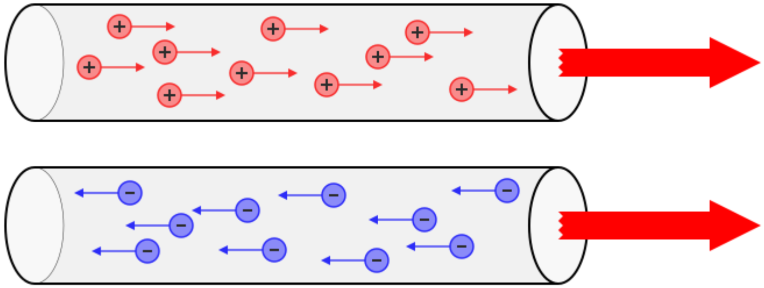
Content

5.1 Current

5.2 Resistivity and Resistance

5.3 Electromotive Force

5.4 Energy and Power in Circuits



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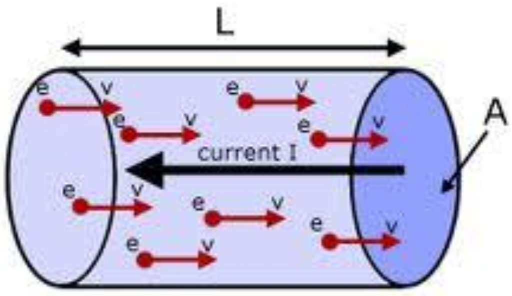
5.1 Current

- We've only discussed with charged objects at rest. But what about moving charges??
- Electrons are distributed evenly on the outer surface of conductors, so the electric field is zero inside it. However, electrons in the conductor is still moving freely!
- This movement of charge from one region to the other is called an **electric current**

Drift velocity

- Imagine if there’s an electric field inside the conductor from a source. The charged particles in the conductor (usually electrons) will be subjected to electric force $\vec{F} = q\vec{E}$



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- In vacuum, the charged particles will accelerate indefinitely

- However, in conductors, the electrons undergoes <u>collision</u> with atoms’ nuclei, or other electrons etc
- <u>Electrons can be said to be moving very slow</u>, or drifting.

- **The drift velocity** is written as $\vec{v}_d$

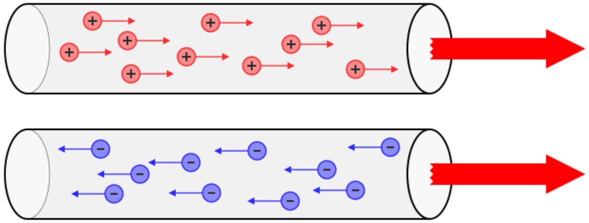
Defining Current

- Current is the movement of charge, or in other words, it is defined as the **rate at which charge flows through a given surface**.

$$I = \frac{dQ}{dt} \quad \text{(Definition of current)}$$

- The **unit for current is Ampere (A)** where 1 A = 1 C/s

- In a conductor like metals, the moving charges are always negative (electrons). However, it is different in other materials. Example, in ionic solution the moving charge could be both positive (cation) AND negative (anion).



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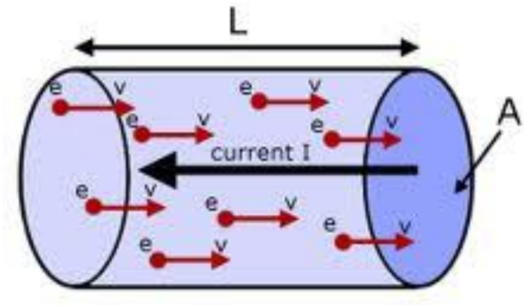
- The flow of current is defined to be in the direction of flow of positive charges!

Current in term of drift velocity



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- Current can also be expressed by drift velocity
- Consider positively charged particles that are moving through a conductor with cross-sectional area *A* and an electric field *E*
- Assume there are *n* moving particles *per unit volume. n* is called concentration of particles.
- All particles moves at v_d. In a time interval *dt*, each particle moves a distance $v_d\,dt$
- So, the amount of particles moving is $n A v_d dt$, (since $A v_d dt$ is the volume of the cylinder)
- Therefore, charge moving = $dQ = q(n A v_d dt)$

$$I = \frac{dQ}{dt} = n|q|v_d A$$ (General expression of current)

Current density

- Current is a scalar quantity, but a positive or negative sign can be attached to it to show the direction. However, it is better if a vector is used instead
- Current density is a vector quantity expressing the flow of charge divided by area.
- In our situation before where the electric field and drift velocity is uniform in the conductor, current density can be written as

$$J = \frac{I}{A} = n|q|v_d$$ (Definition of current density)

or in vector form

$$\vec{\boldsymbol{J}} = nq\vec{\boldsymbol{v}}_d$$ (Vector of current density)

5.2 Resistivity and Resistance

- The current density $\vec{J}$ in a conductor depends on the electric field in the conductor, $\vec{E}$ and on properties of the conductor material

- Usually, this relationship is linear $\vec{J} \propto \vec{E}$
- This is called the **Ohm's law**

- Sometimes, the relationship between current density and electric field is not linear but it won't be discussed in this course.

Resistivity

- The proportionality constant for Ohm's law is also known as the ***resistivity***
- Resistivity of a material, ρ is defined as the **ratio of *E* and *J***

$$\rho = \frac{E}{J} \quad \text{(Definition of resistivity)}$$

- Unit for resistivity is **V · m/A** or **Ω · m**
- A perfect conductor would have *zero resistivity*, while a perfect insulator would have *infinite resistivity*.

- **Metals have much lower resistivity than ceramics or polymers**. (Silver and copper has the lowest resistivity among metals)
- ρ usually is constant at a certain temperature and does not depend on electric field. Materials with constant ρ is called *ohmic* conductor

Resistivity of different materials at room temperature

Material	ρ at 20 °C
Metals	
Silver	1.59×10^{-8}
Copper	1.68×10^{-8}
Gold	2.44×10^{-8}
Aluminium	2.82×10^{-8}
Tungsten	5.60×10^{-8}
Steel	20×10^{-8}
Alloys	
Nichrome	100×10^{-8}
Constantan	49×10^{-8}

Material	ρ at 20 °C
Semiconductors	
Carbon (Graphite)	3.5×10^{-5}
Silicon	2300
Insulators	
Glass	$10^{10} - 10^{14}$
Mica	$10^{11} - 10^{15}$
Sulfur	10^{15}
Wood	$10^{8} - 10^{11}$
PET	10^{21}
Teflon	10^{23}

Temperature dependence of resistivity

- The resistivity of a *metallic* conductor nearly always increases with increasing temperature

- As *T* increase, the ions of the conductor vibrates with greater amplitude, making it easier for electrons to collide with an ion
- This will decrease v_d and reduce the current

$$\rho(T) = \rho_0\left[1+\alpha\left(T-T_0\right)\right]$$

(Temperature dependence of resistivity)

ρ_0 : resistivity at reference temperature, T_0,
T_0 : reference temperature (0 °C or 20 °C)

- α is called the ***temperature coefficient of resistivity***, and varies according to materials

Resistance

- According to Ohm's law, $\vec{E} = \rho \vec{J}$ and *ρ* is constant.
- However, usually current and potential difference is more widely used. Also, it's much easier to measure *I* and *V*
- Thus, Ohm's law equation needs to be rewritten using *I* and *V*

- Imagine a wire with resistivity *ρ*, cross-sectional area *A* and length *L*.
- The *direction* of the current is from higher potential to lower potential
- *V* = *EL* and *I* = *JA*
- Therefore, $\frac{V}{L} = \rho \frac{I}{A}$ or $\frac{V}{I} = \frac{\rho L}{A}$

$$R = \frac{\rho L}{A}$$ (relationship between resistance and resistivity)

$$V = IR$$ (relationship between voltage, current and resistance)

5.3 Electromotive Force

Consider the two statements below.

1 A conductor needs to be a part of a *closed loop* or *closed circuit* to have a steady current.

Imagine what happens if it isn't part of a closed circuit

2 If a charge q goes around a complete circuit and returns to its starting point, the potential energy must be the same at the end of the round trip.

But, there is always a decrease in potential energy when charges move through a conductor due to collision of the charges with ions or other charges in the conductor.

Therefore, there must be some part in the circuit in which the potential energy increase!

Electromotive Force

- In an electric circuit there must be a device that could increase potential energy.
- In this device, a charge travels “uphill”, i.e. from lower to higher potential energy, even though electrostatic force is trying to push it “downhill”

- The influence that makes current flow from lower to higher potential is called **electromotive force (emf)**
- *emf is not a force, but rather, an energy-per-unit-charge quantity*

- The symbol is $\mathcal{E}$
- The SI unit for emf is **volt** (1 V = 1 J/C)
- Battery is an example of *source of emf*. Can you think of others?
- All these devices convert energy of some form (mechanical, chemical, thermal etc) into electric potential energy.

Internal Resistance

- For an *ideal* source of emf, the potential difference across the source is equal to the emf ($V_{ab} = \mathcal{E} = IR$)
- However, real sources of emf don't behave ideally. Charge moving through the material of any sources of emf *encounters resistance* too.
- This is called the **internal resistance**, denoted by *r*

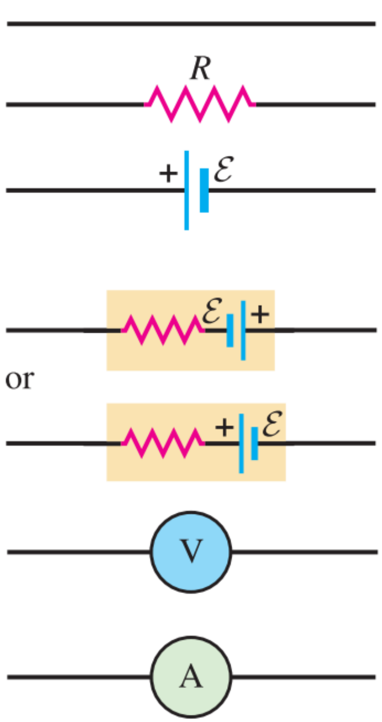
- As the current moves through *r*, it experiences a drop in potential equal to *Ir*
- Thus, when a current flowing through a source from the –ve terminal, *b* to +ve terminal, *a*, the potential difference between *a* and *b* will be

$$V_{ab} = \mathcal{E} - Ir$$

(terminal voltage, source with internal resistance)

- This voltage is called terminal voltage, and is less than emf due to internal resistance. Example, a 1.5 V battery have 1.5 V of emf but lower V_{ab}

Symbols for Circuit Diagram



Conductor with negligible resistance

Resistor

Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)

Source of emf with internal resistance *r* (*r* can be placed on either side)

Voltmeter (measures potential difference between its terminals)

Ammeter (measures current through it)

Voltmeter: measure potential difference *between its terminals*. Ideal voltmeters have infinitely large resistance and measures potential difference without any current diverting through it

Ammeter: measure current *passing through* it. Ideal ammeters have zero resistance and no potential difference between its terminals

5.4 Energy and Power in Circuits

- In electric circuits, we are most often interested in the *rate* at which energy is either delivered to or extracted from a circuit element.
- If the current through the element is I, then in time interval dt an amount of charge $dQ = I\,dt$ passes through an element.
- The potential energy change for this amount of charge is $V_{ab}\,dQ = V_{ab}\,I\,dt$
- Dividing this expression by dt, we obtain the *rate* at which energy is transferred either into or out of the circuit element
- The time rate of energy transfer is **power**, denoted by P

$$P = V_{ab} I$$ (power, or rate at which energy is transferred)

- Unit for power is **W** or watt (1 W = 1 J/C · C/s = 1 J/s)

Power Input to a Pure Resistance

$$P = V_{ab} I = I^2 R = \frac{V_{ab}^{\ 2}}{R}$$ (power delivered to a resistor)

Power Output of a Source

$$P = V_{ab} I = \mathcal{E} I - I^2 r$$

The term $\mathcal{E}I$ is the rate at which work is done on the moving charges.

The term I^2R is the rate at which electrical energy is *dissipated* in the internal resistance of the source.

The difference $\mathcal{E}I - I^2R$ is the *net* electrical power output of the source, i.e. the rate at which the source delivers electrical energy to the circuit

Conclusion

- Current
  - Current is defined as the rate at which charge flows
  - The direction is the flow of positive charge
- Resistivity and resistance
  - Resistivity is the proportionality constant between electric field and current density.
  - Resistance depends on the geometrical shape of the material
- Electromotive Force
  - In an electric circuit there must be a device that could increase potential energy.
  - The influence that makes current flow from lower to higher potential is called electromotive force (emf)

References

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Thank you!

Next chapter:
Direct Current Circuits

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