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Electricity, Magnetism & Optics

Capacitance and Dielectrics

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Chapter Description

- Aims

  Students will understand the nature of capacitors and how they are used to store electric energy

- Expected Outcomes
  - ➢ Able to describe capacitors and capacitance
  - ➢ Able to calculate the amount of energy stored in a capacitor
  - ➢ Able to analyze capacitors connected in series and parallel in a circuit

- Mathematical concepts
  - ❖ Integral

Content

4.1 Capacitors and Capacitance

4.2 Capacitors in Series and Parallel

4.3 Energy Storage in Capacitors

4.4 Dielectrics

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4.1 Capacitors and Capacitance

- Other than batteries, there is another device that could store electric energy which is called capacitors.

- A capacitor consists of at least two conductors, separated by an insulator (including vacuum)

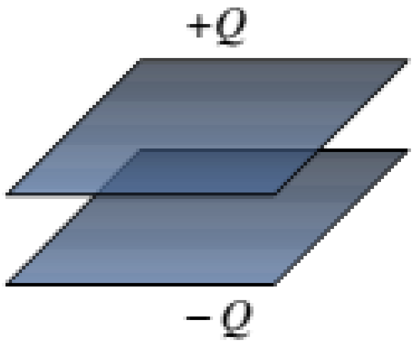
- While the amount of energy stored in the capacitor is governed by a property called the capacitance

Capacitors

- Consider a capacitor with two conductors separated by an insulator. The net charge of both conductors is initially zero. **The capacitor is charged by transferring electrons from one conductor to the other** (how it is transferred wili be covered in next chapter).

- After a while, both conductors will have charge of equal magnitude but opposing sign. This still gives net zero charge for both conductors and not violating the principle of charge conservation.

- In this chapter, by definition a capacitor with charge Q stored in it means one conductor at a higher potential stores charge $+Q$, and the other one stores charge $-Q$



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Defining Capacitance

- Now let's consider the electric field between the two conductors. According to Gauss's law, it will be proportional to the charge *Q* on each conductor. Therefore, the potential difference or voltage will also depend on *Q*.

- If the charge on each conductor is increased to 2*Q*, both electric field and voltage will also increase to 2*E* and 2*V*, respectively. But notice that the RATIO of charge to voltage doesn't change!

- This **ratio is defined as capacitance**, and the unit is farad (F)

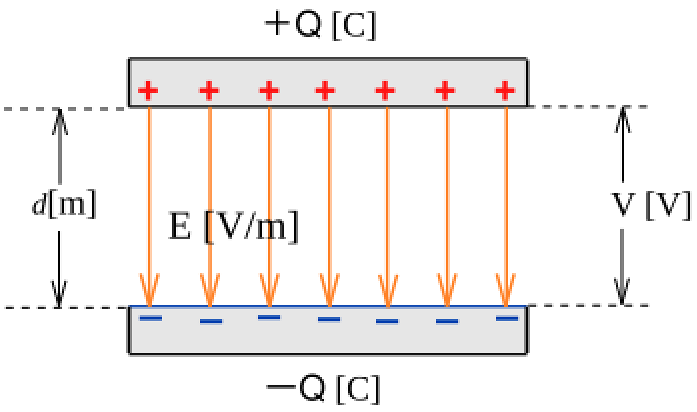
$$C = \frac{Q}{V_{\mathrm{ab}}} \quad \text{(definition of capacitance)}$$

Calculating Capacitance I

- Now let's continue the discussion considering electric field.
- To make it easier, consider a uniform electric field between two parallel conducting plates, each with area *A* and separated by distance *d*
- This type of capacitor is also known as the **parallel-plate capacitor**
- The electric field can be calculated using Gauss's law, giving the result of

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{Q}{A}$$

where σ is the surface charge density.



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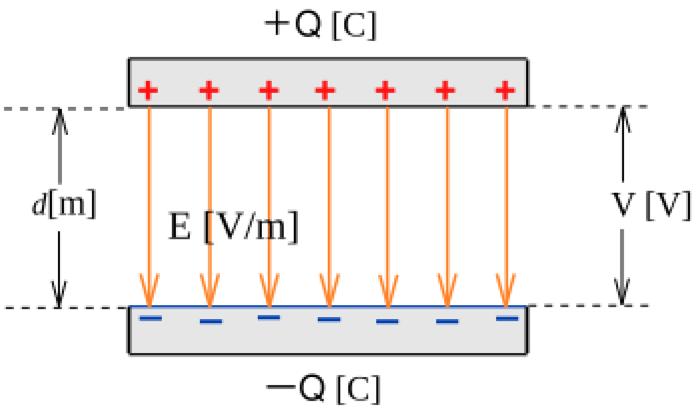
Calculating Capacitance II

- The potential difference can also be easily determined due to the uniform electric field. The potential difference will be

$$V_{ab} = Ed = \frac{1}{\varepsilon_0}\frac{Qd}{A}$$



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- Finally, the capacitance can be written as

$$C = \frac{Q}{V_{ab}} = \varepsilon_0 \frac{A}{d}$$ (capacitance of a parallel-plate capacitor in vacuum)

- Thus, for a parallel-plate capacitor, the capacitance only depends on the geometrical shape of the plates!

4.2 Capacitors in Series and Parallel

- Capacitors are manufactured in certain standard capacitances.
- In order to obtain the capacitance needed, combination of two or more capacitors can be used
- Capacitors can be connected either in series or in parallel (like resistors)
- The **equivalent capacitance** (nett capacitance or effective capacitance), C_{eq} can easily be calculated by looking at the charge stored on each conductor for all capacitors.

Capacitors in Series

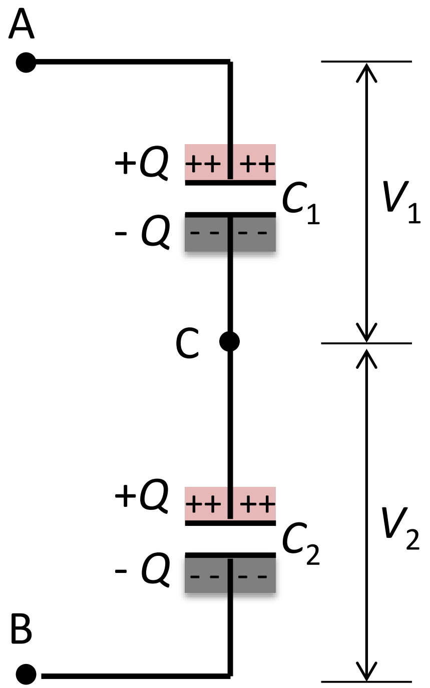
- Consider two capacitors that are connected in series between points A and B. Point C is taken as the point between the two capacitors
- **Both the capacitors must have the same charge**
- The potential difference between all points are

$$V_{ac} = V_1 = \frac{Q}{C_1} \qquad V_{cb} = V_2 = \frac{Q}{C_2}$$

$$V_{ab} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

- Therefore, $\dfrac{V}{Q} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{C_{eq}}$
- Generalising the equation yields

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \quad \text{(capacitors in series)}$$



Capacitors in Parallel

- Now consider another situation where the two capacitors are connected in parallel between points A and B.

- **The potential difference between A and B must be the same.**
- However, the charge on each capacitors will be different.

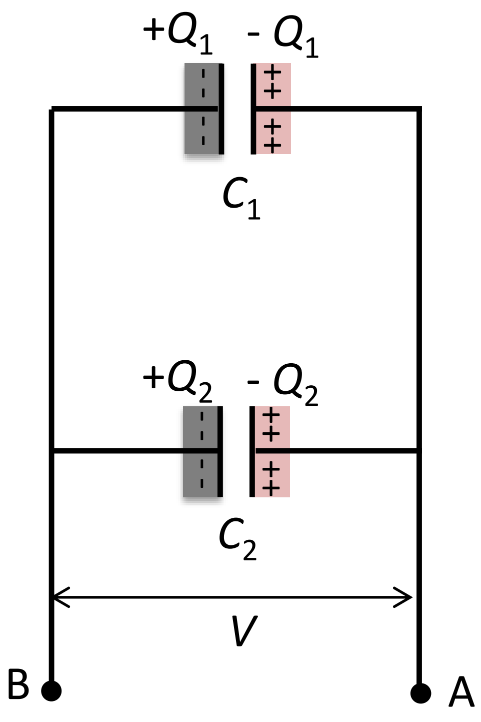
$$Q_1 = C_1 V \qquad Q_2 = C_2 V$$

$$Q = Q_1 + Q_2 = C_1 V + C_2 V$$

- Therefore, $Q / V = C_1 + C_2$

- Generalising the equation yields

$$C_{eq} = C_1 + C_2 + C_3 + .. \qquad \text{(capacitors in parallel)}$$



4.3 Energy Storage in Capacitors

- Capacitors are most commonly used as an electronic device due their ability to store electric energy.

- A **charged capacitors stores electric potential energy, *U***

- Electric potential energy stored in capacitors is equal to the mount of work required to charge it, or equal to the amount of work done to separate opposite charges and place them on two separate conductors.

- This stored energy will be released as work done by electrical forces when the capacitor is discharged

Calculating Energy stored in Capacitor

- The electric potential energy, *U* of a charged capacitor can be determined by calculating the work *W* required to charge it.

- In the middle of charging process, let $v = q / C$, where *v* and *q* is the voltage and charge of the capacitor. During this time, additional work *dW* is required to transfer a small charge of *dq* where $dW = v\,dq = \frac{q}{C}dq$

- Therefore, the total work *W* needed to increase the capacitor charge from zero to final value (or total value) of *Q* is

$$W = \int_0^W dW = \int_0^Q v\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

- Hence, the total potential energy in a fully charged capacitor is

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

(potential energy stored in a capacitor)

4.4 Dielectrics

- Most capacitors have some form of insulating material, called ***dielectrics*** between the conductors

- Rubber and glass are some examples of dielectrics, but most commonly used dielectric material in capacitors is plastic.

- Three reasons dielectrics are placed in between conductors:
  a) It separates both conductors in capacitors
  b) Using dielectrics increase the maximum possible potential difference between the conductors, allowing capacitors to store more energy
  c) The capacitance is ***higher*** with dielectrics instead of vacuum. The capacitance increase by a dimensionless factor K which is known as ***dielectric constant***

Dielectric constant

- The dielectric constant is the measure of how much the capacitance of a capacitor can increase when the dielectric is used as the separator between the conductors.

$$K = \frac{C}{C_0} \quad \text{(Dielectric constant)}$$

C : Capacitance with dielectrics material

C_0: Capacitance in vacuum

Material	*K*	Material	*K*
Vacuum	1	Glass	5-10
Air (1 Atm)	1.00059	Polyvinyl Chloride	3.18
Air (100 Atm)	1.0548	Germanium	16
Water	80.4	Glycerin	16
Benzene	2.28	Strontium titanate	310
Mica	3.1	Barium titanate	1200 - 10000

Induced Charge

- When a dielectric material is inserted between the plates while the charge is kept the same, the potential difference between the plates decreases by *K*
- Therefore, the electric field between the plates will also decrease by *K.*

$$K = \frac{V_0}{V} = \frac{E_0}{E} \quad \text{when } Q \text{ is constant}$$

- Thus, surface charge density also decreased. The surface charge on the conductor doesn't change, but an **induced charge** of the opposite sign appear on each surface of the dielectrics.
- The dielectric is still neutral, but the charge is *redistributed* on the surfaces.

Polarization

- This redistribution of charge on the surface of the dielectric is called **polarization**

- Deriving the relationship between surface charge and charge on the plates gives us the product $K\varepsilon_0$ called *permittivity of the dielectric*

$$\varepsilon = K\varepsilon_0$$ (Definition of dielectric permittivity)

$$C = KC_0 = K\varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d}$$ (Capacitance of parallel plate capacitor with dielectric material)

Conclusion

- Capacitor and capacitance
  - Capacitor are electronic devices which are used to store electric charge and energy, made up from two or more conductors separated by an insulator
  - The capacitance is the ratio of charge stored in capacitors to the voltage between the conductors

- Stored energy
  - The stored energy in capacitors is equal to the amount of energy required to separate the charge from both conductors, and is released as electric force when the capacitor is discharged

References

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- **Physics for Scientists & Engineers 4th Edition**, Douglas C. Giancoli, Pearson, 2008

- **Physics for Scientists & Engineers 9th Edition**, Raymond A. Serway & John W. Jewett, Cengage Learning, 2014

Thank you!

Next chapter:
Current, Resistance and Electromotive Forces