Electricity, Magnetism & Optics

Electric Potential

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Chapter Description

- Aims

  Students will understand the concept of electrostatic potential and relate it to electric energy .

- Expected Outcomes
  - ➢ Able to describe the meaning of electric potential
  - ➢ Able to calculate the electric potential energy from a collection of charges
  - ➢ Able to visualize electric potential in space using equipotential surfaces

- Mathematical concepts
  - ❖ Summation
  - ❖ Definite integral

Content

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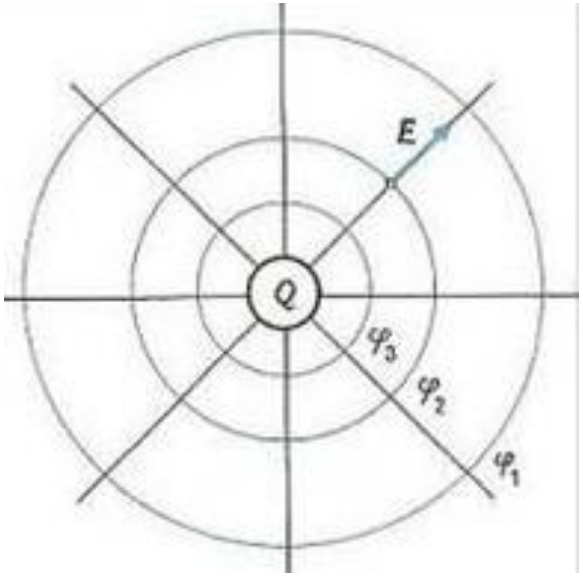

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3.1 Electric Potential Energy

- When we talk about electricity, the two most common concepts that one usually thinks about are voltage and energy

- In daily lives, battery is often associated with electricity and the most important parameter of a battery is the voltage it provides. Other than that, we are usually concerned with the voltage rating of our power supply. This voltage is closely related to electric energy as we will see in this chapter.

- But first, let us recap the concept of work, potential energy and the principle of conservation of energy in mechanics

Potential Energy in Mechanics I

- In mechanics, work done by any physical forces is defined as

$$W_{a\to b} = \int_a^b \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = \int_a^b F\cos\phi \cdot dl$$

- Also, *potential energy* can be used to express work done by the force

- As an example, the work done to an object moving from point A to point B can be written as

$$W_{a\to b} = U_a - U_b = -\left(U_b - U_a\right) = -\Delta U$$

where the work is actually the <u>*change of energy* of the object between the points</u>.

Potential Energy in Mechanics II

- Consider a falling object due to force of gravity. The starting potential energy at point A is higher than at point B, meaning the work done is positive.
- This can be said that the force of gravity does positive work and the gravitational potential energy decreases.
- However, due to the principle of conservation of energy, the decrease of potential energy is offset by the increase of kinetic energy of the falling object.

$$K_a + U_a = K_b + U_b$$

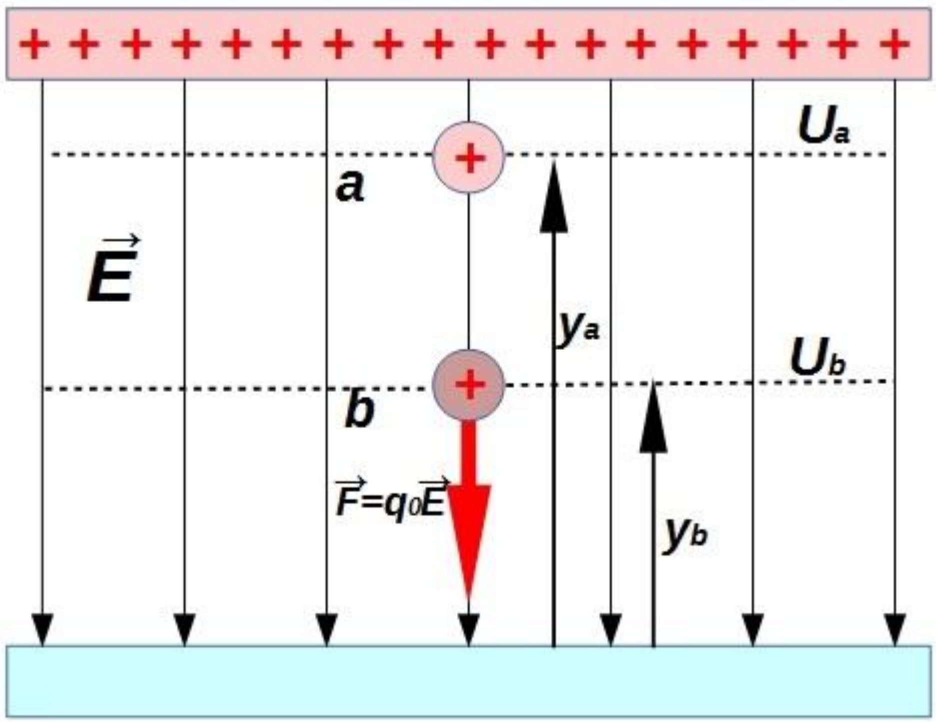
$$-\Delta U = -\left(U_b - U_a\right) = K_b - K_a = \Delta K$$

- As conclusion,

  Positive work → potential decrease + kinetic increase

Potential Energy in a Uniform Electric Field

- Now let's continue the discussion with electric field.
- To make it easier, consider a uniform electric field between two plates with opposite charge sign



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Defining Electrical Potential Energy

- Let's place a positively charged object between the plates. According to coulomb's law, the field exerts a force on the object towards the negative plate with magnitude of $F = q_0 E$

- Since the force is uniform, the work done can be expressed as

$$W_{a \to b} = \int_a^b \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = Fd = q_0 Ed$$

$$W_{a \to b} = -\Delta U = -\left(U_b - U_a\right) = -\left(q_0 E y_b - q_0 E y_a\right) = q_0 E\left(y_a - y_b\right)$$

- The work done is this scenario is also positive! Therefore, the electric potential energy is decreases, and it can be defined as

$$U = q_0 E y$$

Electrical Potential Energy of two charged objects

- But what about the electric potential energy of a charged object moving due to other situation of electric field?
- Imagine two charged object, one is non-moving and the other is moving due to the electric field of the non-moving object
- From Coulomb's law, the force exerted on the moving object is $F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$
- The electric field at the second object is $E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
- And using the definition of electric potential,

$$U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

(electric potential energy of two point charges, q and q_0)

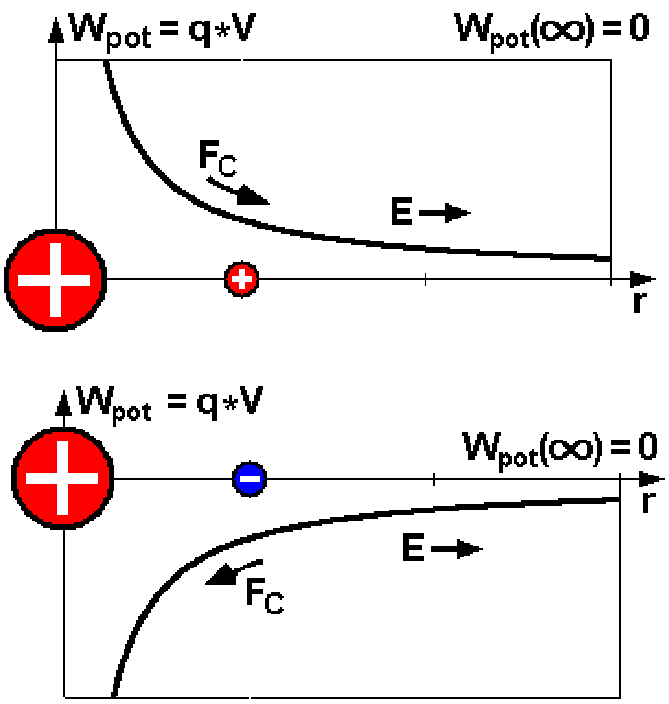
Reference point

- Potential energy is hard to visualize without one reference point where the potential energy is defined as zero ($U = 0$)

- In electricity, the reference is taken at a point far enough from the non-moving charged object that the potential is zero ($r = \infty$)

- If the two charged objects have the same sign, the charged object repels each other and thus, the work is positive.

- If they have opposite sign, they attract each other while the work is negative



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Discussion

What is the reference point for gravitational and elastic potential energy?

Multiple Charged Objects

- Now we know that the electric potential energy can be easily calculated if the electric field is known.

- Since the electric field from more than two charged objects at a particular point can be calculated, the potential energy can be written as

$$U = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\right)$$

- Therefore, generalizing the equation using summation will give

$$U = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

(electric potential energy of a collection of charges)

3.2 Electric Potential

- All the discussion of electrical potential energy before depended on knowing the charge of both charged objects. The moving charged objects is actually test charge that we included to make calculations easier.

- Let’s think of a physical quantity that is independent of this test charge.

- Let’s refer to this quantity as **electric potential**.

Discussion

What’s the difference between electric potential and potential energy?

Defining Electric Potential

- Electric potential is defined as the potential energy per unit charge

$$V = \frac{U}{q_0}$$

- This is a scalar quantity since both potential energy and charge are scalar quantity. Thus, potential energy has no direction.

- The SI unit for potential is volt (V) where $1 \text{ V} = 1 \text{ J/C}$

Potential difference

- Work is the change of potential energy and can be written as

$$\frac{W_{a\to b}}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -\left(V_b - V_a\right) = V_a - V_b$$

- The difference $V_a - V_b$ is called the potential of a with respect to b
- This is often called ***potential difference***. In electric circuits, the potential difference is also called ***voltage***.

$$V_{ab} = V_a - V_b = \frac{W_{a\to b}}{q_0}$$ (Potential difference)

Calculating Electric Potential: Charged particles

- Electric potential can be calculated by dividing potential energy by the test charge

$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

(potential due to point charge q at distance r)

- The sign of the potential depends on the sign of the charge, q

- Since this is a scalar quantity, the electric potential due to multiple charged particles is

$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

(potential due to multiple charges)

Calculating Electric Potential: Electric Field

- However, if the electric field is known, it is often easier to calculate electric potential from electric field.
- Inserting the definition of electric field into the work equation

$$W_{a\to b} = \int_a^b \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = \int_a^b q_0 \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$$

- Dividing this by the test charge will give a result of

$$V_a - V_b = \int_a^b \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = \int_a^b E\cos\phi \cdot dl$$

(potential difference as an integral of $\vec{E}$)

Electron Volts

It is useful sometimes to define a unit of energy using the magnitude of electron charge e. The electron volt is defined as the work done on one electron in accelerating it through potential difference of 1 V

$$U = qV$$

$$1\ \text{eV} = 1.602\times 10^{-19}\ \text{J}$$

Calculating Electric Potential: Charge Distribution

- Finally, the electric potential can also be calculated for any type of charge distribution using the two equations below

(i) $$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$ (potential due to a continuous distribution of charges)

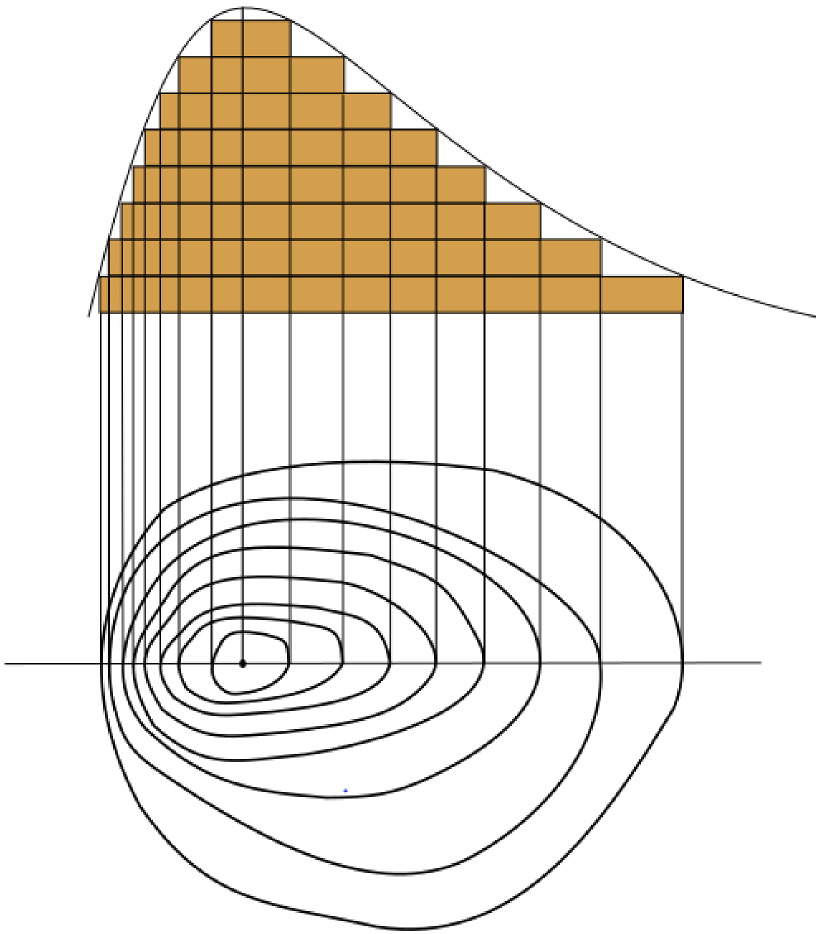
(ii) $$V_a - V_b = \int_a^b \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = \int_a^b E\cos\phi \cdot dl$$ (potential difference as an integral of $\vec{E}$)

- The second equation is especially useful if the electric field can be calculated using Gauss's law

3.3 Equipotential Surfaces

- Electric field line is used to help us visualize electric field vector. Therefore, is there any way to easily visualize electric potential?

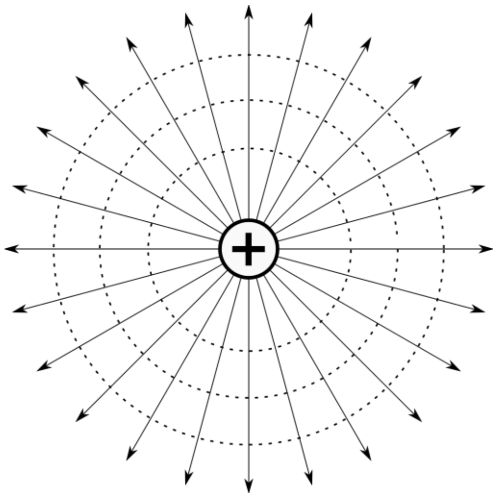
- We can translate the concept of *contour* in geography to help us visualize electric potential



Equipotential Surfaces

- In geography, *contour lines* are used to denote places with the same height from sea level

- In electricity, an **equipotential surface** is used to show regions with the same electric potential, *V*



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- The figure to the left demonstrates both electric field lines (arrow line) and the equipotential surfaces (circle) surrounding a positively charged object

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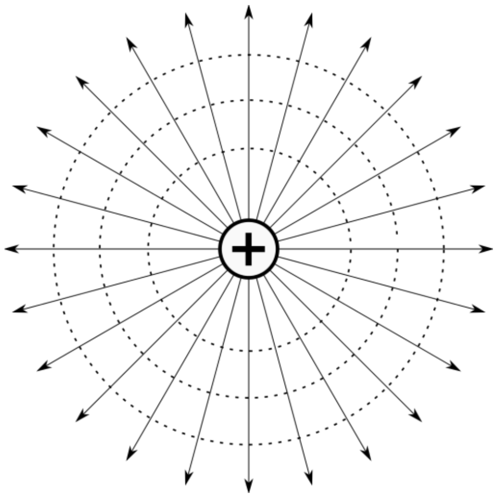

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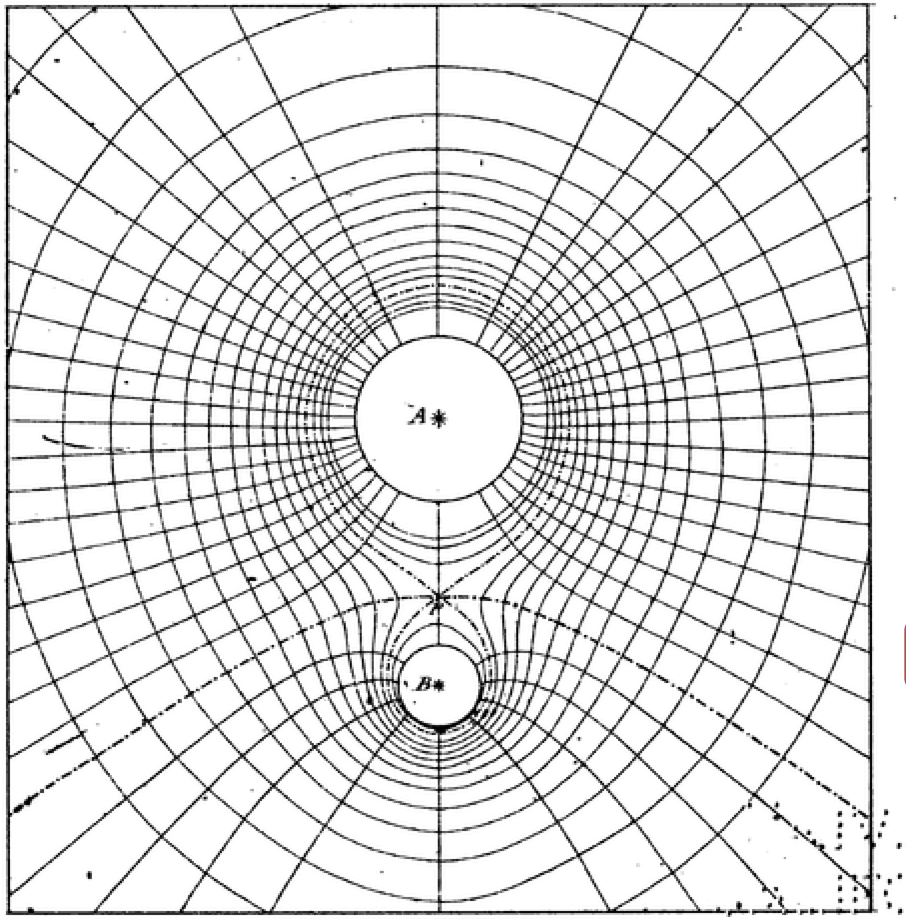
- The figure to the left demonstrates both electric field lines (arrow line) and the equipotential surfaces (circle) surrounding a positively charged object
- Notice that **the equipotential surface is always perpendicular to the electric field line**



Lines of Force and Equipotential Surfaces.

A = 20. B = 5. P, Point of Equilibrium. AP = $\frac{2}{3}$ AB.

Source: Wikimedia Commons

An example of electric field lines and equipotential surface from two charged objects with different charge amount.

Discussion

1) Which line is the electric field line, and which is the equipotential surface?
2) Do the two charges have same or opposite sign charge?

Conclusion

- Electric Potential
  - Electric potential is defined as the potential energy per unit charge

- Electric Potential Difference
  - The change of potential for a charge moving between two points is referred to as electric potential difference

References

- **University Physics 14th Edition**, Hugh D. Young, Roger A. Freedman, IOP Publishing Ltd, 2015

- **Physics for Scientists & Engineers 4th Edition**, Douglas C. Giancoli, Pearson, 2008

- **Physics for Scientists & Engineers 9th Edition**, Raymond A. Serway & John W. Jewett, Cengage Learning, 2014

Thank you!

Next chapter:
Capacitance and Dielectrics