Electricity, Magnetism & Optics

Gauss's Law

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Chapter Description

- Aims

  Students will understand the concept of electric flux, and how it relates to the calculation of electric charge within a closed surface .

- Expected Outcomes
  - ➢ Able to describe the concept of electric flux
  - ➢ Able to relate the electric flux to the electric charge within a closed surface
  - ➢ Able to apply Gauss's law to analyse electric field due to symmetric distribution of charge

- Mathematical concepts
  - ❖ Dot product
  - ❖ Surface integral

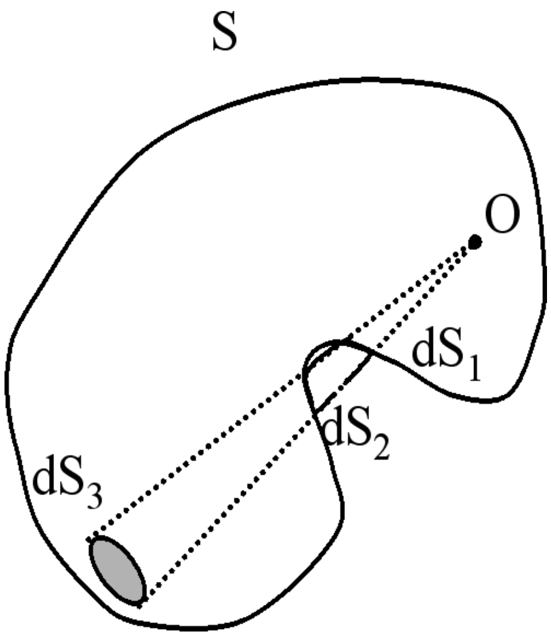
Content

2.1 Electric Flux

2.2 Gauss's Law

2.3 Applications of Gauss's law

2.4 Charges on conductors



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2.1 Electric Flux

- The electric field vector can be determined using Coulomb's law if the charge if given

- But there's an alternative to the relation between charge and electric field using the concept of **electric flux**

- The concept flux can be described as how much of something passing through a particular area.

- In this case, an electric flux can be defined as the number of electric field lines passing through an area

Defining electric flux: Uniform electric field over a flat surface

- A more accurate definition of electric flux is the **dot product between the electric field and the area** it passes through

- For a flat surface with uniformly distributed electric field, the electric flux, Φ_E is written in vector form and scalar form as

$$\Phi_E = \vec{E} \cdot \vec{A} \quad \text{(uniform electric field, flat surface)}$$

$$\Phi_E = EA\cos\phi \quad \text{(uniform electric field, flat surface)}$$

where ϕ is the angle between the electric field and and the area.

Defining electric flux: Non-uniform electric field

- Unfortunately, a flat surface situation is too specific, and thus, a generalized definition of electric flux to cover all types of situation is needed.

- For a non-uniform electric field, the concept of **surface integral** is very helpful to calculate the electric flux through an area. The surface area is defined first, and is then divided into many infinitesimally small area, $d\vec{\mathbf{A}}$. Adding all the electric flux passing through each of the small area leads to

$$\Phi_E = \int \vec{\boldsymbol{E}} \cdot d\vec{\mathbf{A}} = \int E \cos\phi \, dA$$

(general definition of electric flux)

2.2 Gauss's Law

- The relation between charge, electric field and electric flux can further be explained using Gauss's law.

- Gauss's law specifies that all the electric flux through any closed surface is proportional to the total electric charge inside the aforementioned closed surface

- Gauss's law can be derived from Coulomb's law (and vice versa).

Deriving Gauss's law: Simple case

- A simple case of Gauss's law can be derived from a single charged object.
- Imagine a perfect sphere with radius *R* surrounding a single charged object, with the object exactly in the middle of the sphere. The electric field on the surface of the imagined sphere will be $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$
- While the total electric flux will be

$$\Phi_E = EA = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\left(4\pi R^2\right)$$

$$= \frac{q}{\varepsilon_0}$$

- This shows that the electric flux in only dependent on the charge inside the imaginary sphere!

Deriving Gauss's law: Generalized

- The same discussion can also be used for an arbitrary non-spherical closed surface, giving the same result, which is that the electric flux through the non-spherical closed surface is only dependent on the total charge enclosed within it.
- Thus, Gauss's law can be generalized into

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0}$$

(General form of Gauss's law)

- The symbol Q_{encl} refers to the TOTAL or NET charge inside the closed surface. It doesn't tell, for example, the number of charged objects within it.

$\oint$

Discussion

What is the electric flux inside a sphere with exactly ONE electron and ONE proton?

Gaussian surface

- The symbol $\oint$, or the circle on the integral sign denotes that **the integration must be performed over a closed surface**

- This closed surface is usually referred to as a Gaussian surface
- Gaussian surface is only an IMAGINARY surface. IT IS NOT a physical object around the charge.

- Usually, a simple Gaussian surface such as any high-symmetry geometrical shapes is chosen to help make calculation easier

Discussion

Name any geometrical shape with a high degree of symmetry. Why does this shape makes calculation of electric flux easier?

2.3 Applications of Gauss's Law

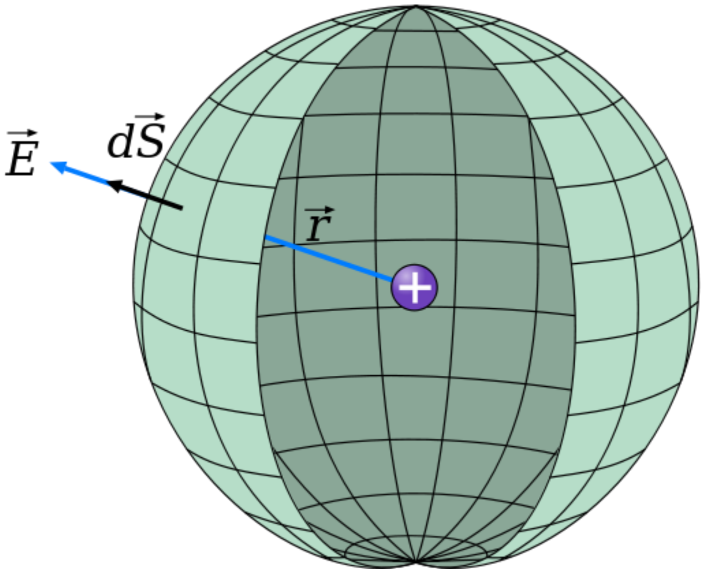
- Gauss's law can be used to calculate the electric field of a known charge distribution, or to calculate the charge distribution if the electric field is given.

- As mentioned, geometrical shapes with a high degree of symmetry is usually chosen to make the calculation of electric field easier

- Some examples are single charged particles, uniformly charged sphere, uniform line charge and uniform plane charge

Single charged particle: Deriving Coulomb's law

- Gauss's law can be used to calculate the electric field due to a single charged particle. A sphere is chosen as the Gaussian surface.



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Left side:
$$\oint \vec{E}\cdot d\vec{A} = E\int d\vec{A} = EA = E\left(4\pi r^2\right)$$

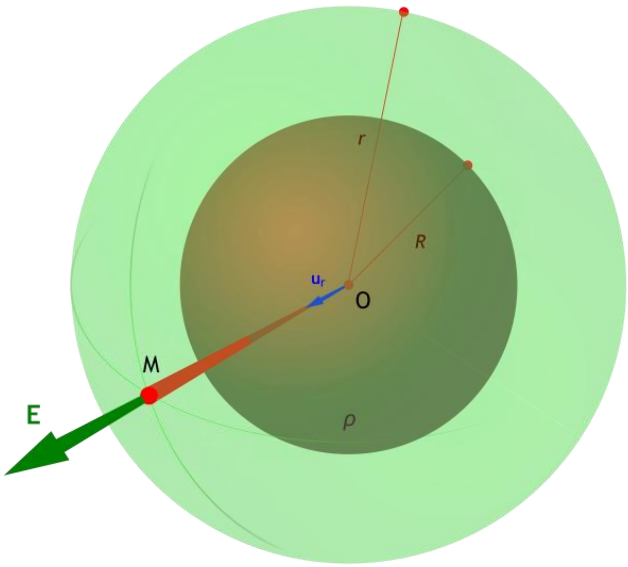
Right side:
$$\frac{Q_{encl}}{\varepsilon_o} = \frac{q}{\varepsilon_o}$$

$$\therefore\ E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

Coulomb's Law!

Uniformly charged sphere I

- Consider a charged insulating sphere of radius *R* with total charge *Q* distributed evenly on it.
- Electric field outside the sphere at distance of *r* from the midpoint ($r > R$), can easily be calculated using Gauss's law



Source: Sharayanan, Wikimedia Commons

Outside sphere ($r > R$)

Left side: $\oint \vec{E} \cdot d\vec{A} = E \int d\vec{A} = EA = E\left(4\pi r^2\right)$

Right side: $\dfrac{Q_{encl}}{\varepsilon_o} = \dfrac{Q}{\varepsilon_o}$

$$\therefore\ E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad (\text{for } r > R)$$

Uniformly charged sphere II

- While the calculation of electric field inside the sphere is a little bit more complicated due to the fact that the amount of enclosed charge is different depending on where *r* is.
- Volume charge density, ρ can be used to help with calculation where $\rho = Q/V$.

Inside sphere (*r* < *R*)

$$Q_{encl} = q_{in} = \rho V' = \rho \cdot \frac{4}{3}\pi r^3$$

Right side: $$\frac{Q_{encl}}{\varepsilon_o} = \frac{q_{in}}{\varepsilon_o} = \frac{4\pi r^3 \rho}{3\varepsilon_o}$$ where $$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$\therefore\ E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R^3}r \quad (\text{for } r < R)$$

Uniformly charged line

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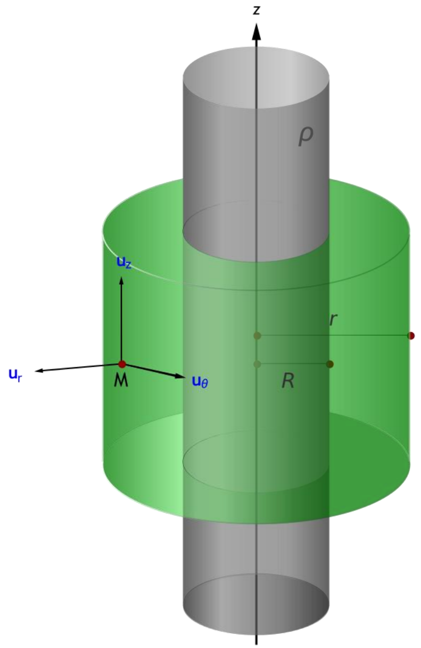
- The concept of using density to represent charge distribution is very helpful in other cases as well.
- Consider an infinitely long line of wire where the charge is distributed evenly with line charge density of λ charge per unit length.



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- The Gaussian surface is taken as a cylinder around the wire.

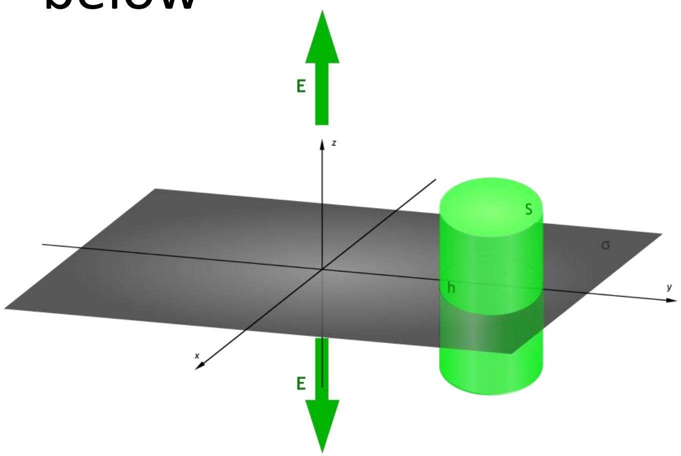
Left side: $\oint \vec{E} \cdot d\vec{A} = E\int d\vec{A} = EA = E(2\pi rl)$

Right side: $\dfrac{Q_{encl}}{\varepsilon_o} = \dfrac{q_{in}}{\varepsilon_o} = \dfrac{\lambda l}{\varepsilon_o}$

$$\therefore\ E = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{r}$$

Uniformly charged plane sheet

- Let's consider another type of density distribution, which is the uniform distribution of charge over an infinite plane sheet, with surface charge density of σ charge per area.
- The Gaussian surface is again taken as a cylinder as shown below



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Left side: $\oint \vec{E} \cdot d\vec{A} = E \int d\vec{A} = EA$

Right side: $\dfrac{Q_{encl}}{\varepsilon_o} = \dfrac{\sigma A}{\varepsilon_o}$

$$\therefore\ E = \frac{\sigma}{2\varepsilon_o}$$

Discussion

Calculate the electric field of TWO infinite planes of charge, where one if positive, and the other is negative

Conclusion

- Electric flux
  - Electric flux is the measure of the amount of electric field lines passing through an area

- Gauss's law
  - Gauss's law relates the electric flux from a closed surface to the charge enclosed in the surface

- Gaussian surface
  - A proper Gaussian surface can be chosen to easily calculate the electric field using Gauss's law

References

- **University Physics 14th Edition**, Hugh D. Young, Roger A. Freedman, IOP Publishing Ltd, 2015
- **Physics for Scientists & Engineers 4th Edition**, Douglas C. Giancoli, Pearson, 2008
- **Physics for Scientists & Engineers 9th Edition**, Raymond A. Serway & John W. Jewett, Cengage Learning, 2014

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Thank you!

Next chapter:
Electric Potential