Process Monitoring

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Chapter 3a
Principal Component Analysis

Process Monitoring

Chapter Description

- Aims
  - Understand the basic principles of multivariate techniques.

- Expected Outcomes

  - Comprehensively explain in writing as well as solve mathematically the principles of multivariate analysis based on complex monitoring problem of MSPM framework.

- Other related Information

Subtopics

3.6 The Basic Structure of Matrix

3.7 Eigenstructure

3.6 The Basic Structure of Matrix

- Multiple sets of vector transformations (matrix):
  - $\mathbf{X}^* = \mathbf{XA}$

  where, $\mathbf{X}$ = original

  $\mathbf{A}$ = matrix transformation

- Composite transformations:
  - Successive matrix transformations, eg $\mathbf{A} = \mathbf{TS}$.
  - Main assumption: $\mathbf{T}$ and $\mathbf{S}$ are post-multiplying $\mathbf{X}$
  - Main mechanism: if $\mathbf{T}$ is the matrix of one linear transformation, then $\mathbf{S}$ is the pre-image matrix.

3.6 The Basic Structure of Matrix

According to Green and Carroll, (1976):

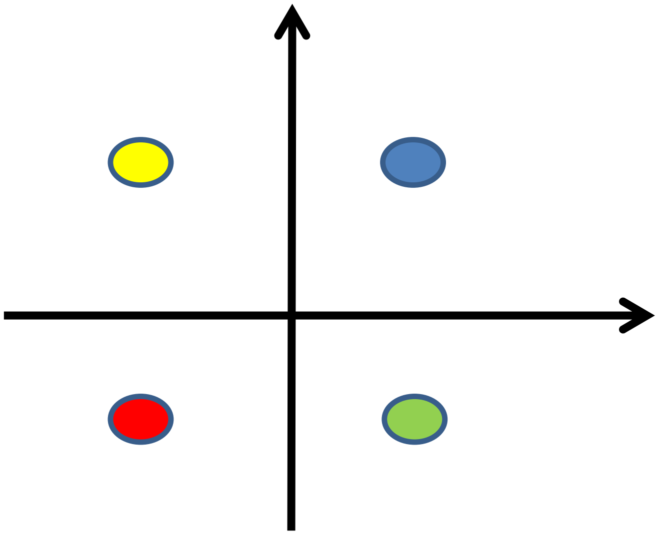
"Any non-singular matrix (determinant $\neq 0$) transformation with real-valued entries can be uniquely decomposed into the triplet matrix product of either:

a) A rotation, followed by a stretch, followed by another rotation

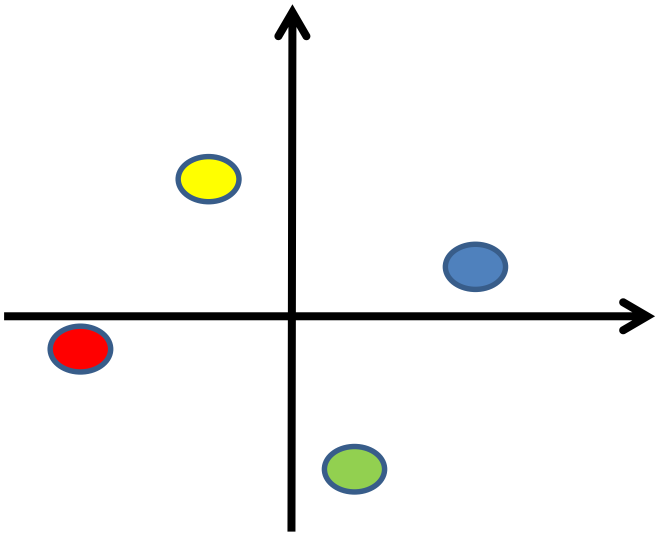
b) A rotation, followed by a reflection, followed by a stretch, followed by another rotation".

$$\mathbf{A} = \mathbf{P}\Delta\mathbf{Q}'$$

3.6 The Basic Structure of Matrix

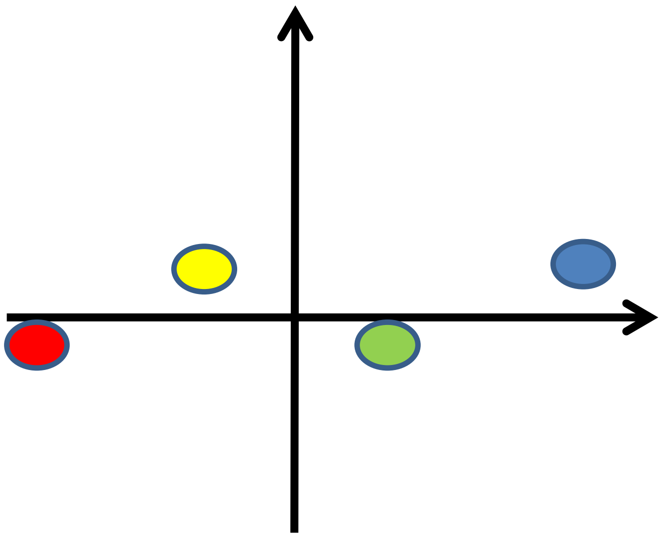


Original Coordinate, **X**

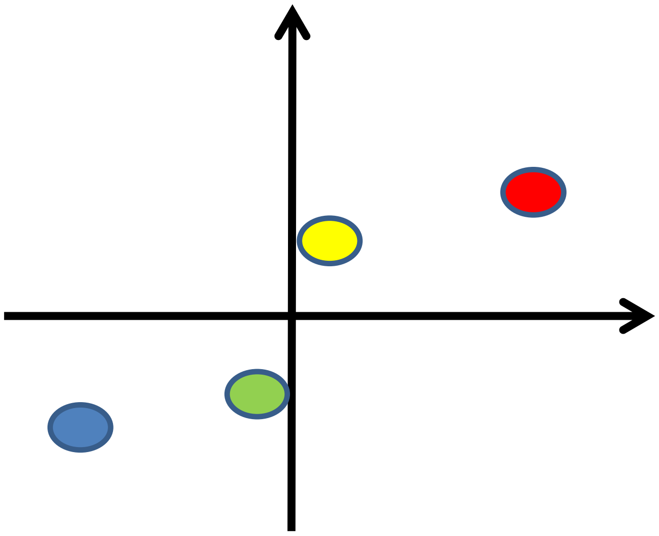


Rotation, **XP**

3.6 The Basic Structure of Matrix



Stretch, **XPΔ**



Rotation (improper), **XPΔV**

3.7 Eigenstructure

- Eigenvectors = invariant vectors, in which, those vectors that map into themselves or multiples of themselves under a given transformation map, $\mathbf{A}$.

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$$

- Eigenvalues (characteristic roots) = a scalar (scale) corresponds to that particular eigenvectors.

3.7 Eigenstructure

- In order to find λ and $\mathbf{x}$, it is necessary to solve the **characteristic equation**:

$$|\mathbf{A} - \lambda\mathbf{I}| = 0$$

- Let **A** = $\begin{bmatrix} -3 & 5 \\ 4 & -2 \end{bmatrix}$, determine the corresponding eigenvectors as well as eigenvalues for **A**!

3.7 Eigenstructure

How would **A** behave if one chose the two eigenvectors as the new basis vectors?

– If $\mathbf{U}$ is a set of basis vectors of the transformation matrix $\mathbf{A}$, then the original transformation $\mathbf{A}$ behaves as a stretch (or a stretch and reflection) relative to this special basis of eigenvectors:

$$\mathbf{D} = \mathbf{U}^{-1}\mathbf{A}\mathbf{U}$$

where, $\mathbf{D}$ = diagonal matrix whose entries are the eigenvalues of $\mathbf{A}$.

$\mathbf{U}$ = eigenvector matrix of $\mathbf{A}$.

References

- Green, P.E., and Carroll, J.D., (1976). *Mathematical Tools for Applied Multivariate Analysis.* New York, USA: Academic Press.

- Jackson, J.E., (1991). *A User's Guide To Principal Components.* John Wiley and Sons. USA.

Authors Information

Credit to the authors: