## UNIVERSITY PHYSICS

## Chapter 4 MOTION IN TWO AND THREE DIMENSIONS

PowerPoint Image Slideshow


## FIGURE 4.1



The Red Arrows is the aerobatics display team of Britain's Royal Air Force. Based in Lincolnshire, England, they perform precision flying shows at high speeds, which requires accurate measurement of position, velocity, and acceleration in three dimensions. (credit: modification of work by Phil Long)

## FIGURE 4.2



A three-dimensional coordinate system with a particle at position $P(x(t), y(t), z(t))$.

## FIGURE 4.3



The displacement $\Delta \vec{r}=\vec{r}\left(t_{2}\right)-\vec{r}\left(t_{1}\right)$ is the vector from $P_{1}$ to $P_{2}$.

## FIGURE 4.4



Two position vectors are drawn from the center of Earth, which is the origin of the coordinate system, with the $y$-axis as north and the $x$-axis as east. The vector between them is the displacement of the satellite.

## FIGURE 4.5



Displacement vector with components, angle, and magnitude.

## FIGURE 4.6



Trajectory of a particle undergoing random displacements of Brownian motion. The total displacement is shown in red.

## FIGURE 4.7



A particle moves along a path given by the gray line. In the limit as $\Delta t$ approaches zero, the velocity vector becomes tangent to the path of the particle.


## Horizontal motion,

 constant velocity

A diagram of the motions of two identical balls: one falls from rest and the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent the horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity whereas the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls, which shows the vertical and horizontal motions are independent.

Vertical motion, constant acceleration

The particle starts at point $(x, y, z)=(0$, 0,0 ) with position vector $\vec{r}=0$. The projection of the trajectory onto the $x y$ plane is shown. The values of $y$ and $z$ increase linearly as a function of time, whereas $x$ has a turning point at $t=5 \mathrm{~s}$ and 25 m , when it reverses direction. At this point, the $x$ component of the velocity becomes negative. At $t=10 \mathrm{~s}$, the particle is back to 0 m in the $x$ direction.


## FIGURE 4.10



A skier has an acceleration of $2.1 \mathrm{~m} / \mathrm{s}^{2}$ down a slope of $15^{\circ}$. The origin of the coordinate system is at the ski lodge.

## FIGURE 4.11



The total displacement $s$ of a soccer ball at a point along its path. The vector $\overrightarrow{\boldsymbol{s}}$ has components $\overrightarrow{\boldsymbol{x}}$ and $\overrightarrow{\boldsymbol{y}}$ along the horizontal and vertical axes. Its magnitude is s and it makes an angle $\theta$ with the horizontal.

## FIGURE 4.12


(a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes.
(b) The horizontal motion is simple, because $a_{x}=0$ and $v_{x}$ is a constant.
(c) The velocity in the vertical direction begins to decrease as the object rises. At its highest point, the vertical velocity is zero. As the object falls toward Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity.
(d) The $x$ and $y$ motions are recombined to give the total velocity at any given point on the trajectory.

## FIGURE 4.13



The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

## FIGURE 4.14



The trajectory of a tennis ball hit into the stands.

Trajectories of projectiles on level ground.
(a) The greater the initial speed $v_{0}$, the greater the range for a given initial angle.
(b) The effect of initial angle $\theta_{0}$ on the range of a projectile with a given initial speed. Note that the range is the same for initial angles of $15^{\circ}$ and $75^{\circ}$, although the maximum heights

(a)

(b)

## FIGURE 4.16

## Golf Shot



Two trajectories of a golf ball with a range of 90 m . The impact points of both are at the same level as the launch point.

## FIGURE 4.17

Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because Earth curves away beneath its path. With a speed of $8000 \mathrm{~m} / \mathrm{s}$, orbit is achieved.

## FIGURE 4.18


(a)

(b)
(a) A particle is moving in a circle at a constant speed, with position and velocity vectors at times $t$ and $t+\Delta t$.
(b) Velocity vectors forming a triangle. The two triangles in the figure are similar. The vector $\Delta \overrightarrow{\boldsymbol{v}}$ points toward the center of the circle in the limit $\Delta t \rightarrow 0$.

The centripetal acceleration vector points toward the center of the circular path of motion and is an acceleration in the radial direction. The velocity vector is also shown and is tangent to the circle.


## FIGURE 4.20



The position vector for a particle in circular motion with its components along the $x$ - and $y$-axes. The particle moves counterclockwise. Angle $\theta$ is the angular frequency $\omega$ in radians per second multiplied by $t$.

Position Vector at $\boldsymbol{t}=\mathbf{2 0 0} \mathbf{n s}$


Position vector of the proton at $t=2.0 \times 10^{-7} \mathrm{~s}=200 \mathrm{~ns}$. The trajectory of the proton is shown. The angle through which the proton travels along the circle is 5.712 rad , which a little less than one complete revolution.

## FIGURE 4.22

The centripetal acceleration points
 toward the center of the circle. The tangential acceleration is tangential to the circle at the particle's position. The total acceleration is the vector sum of the tangential and centripetal accelerations, which are perpendicular.

## FIGURE 4.23



The tangential and centripetal acceleration vectors. The net acceleration $\overrightarrow{\mathbf{a}}$ is the vector sum of the two accelerations.

## FIGURE 4.24

## $\overrightarrow{\mathbf{v}}_{\text {PE }}$ $=\overrightarrow{\mathbf{v}}_{\mathrm{PT}}+\overrightarrow{\mathbf{v}}_{\mathrm{TE}}$

When constructing the vector equation, the subscripts for the coupling reference frame appear consecutively on the inside. The subscripts on the left-hand side of the equation are the same as the two outside subscripts on the right-hand side of the equation.

## FIGURE 4.25

10 m/s $\longrightarrow \quad \overrightarrow{\mathbf{v}}_{\mathrm{TE}}$ Velocity of train with respect to Earth
$-2 \mathrm{~m} / \mathrm{s}$
$8 \mathrm{~m} / \mathrm{s}$

Velocity vectors of the train with respect to Earth, person with respect to the train, and person with respect to Earth.
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The positions of particle $P$ relative to frames $S$ and $S^{\prime}$ are $\overrightarrow{\mathbf{r}}_{P S}$ and $\overrightarrow{\mathbf{r}}_{P S^{\prime}}$, respectively.


## FIGURE 4.27



A car travels east toward an intersection while a truck travels south toward the same intersection.

## FIGURE 4.28



Vector diagram of the vector equation $\overrightarrow{\mathbf{v}}_{\mathrm{CT}}=\overrightarrow{\mathbf{v}}_{\mathrm{CE}}+\overrightarrow{\mathbf{v}}_{\mathrm{ET}}$.
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Vector diagram for Equation 4.34 showing the vectors $\overrightarrow{\mathbf{v}}_{\mathrm{PA}}, \overrightarrow{\mathbf{v}}_{\mathrm{AG}}, \overrightarrow{\mathbf{v}}_{\mathrm{PG}}$.


## FIGURE 4.29



## EXERCISE 22



## EXERCISE 36



## EXERCISE 46



## EXERCISE 48

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## EXERCISE 83



## EXERCISE 100

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