BSP1153
Mechanics & Thermodynamics
Vector

by
Dr. Farah Hanani bt Zulkifli

Faculty of Industrial Sciences & Technology
farahhanani@ump.edu.my

Chapter Description

- Expected Outcomes
  - To understand the concept of vectors.
  - To resolve the vectors into their components.
  - To solve problems in vectors.

- References

  - Young, H.D. & Freeman, R.A. University Physics with Modern Physics (14th Ed.) Pearson, 2015
  - University physics with modern physics / Wolfgang Bauer, Gary D. Westfall, Mc Graw Hill, 2014
  - Paul E. Tippens, Physics 7th Edition. Mc Graw Hill, 2013
  - Physics for scientists and engineers : a strategic approach / Randall D. Knight, Boston, MA : Pearson, 2013
  - Giancoli, D.C. Physics for Scientists and Engineers: with Modern Physics (4th Edition). Pearson Prentice Hall, 2013

Chapter's Outline:

Vector & Scalar Quantities

VECTORS

Vector Multiplication

i. Scalar Product

ii. Vector Product

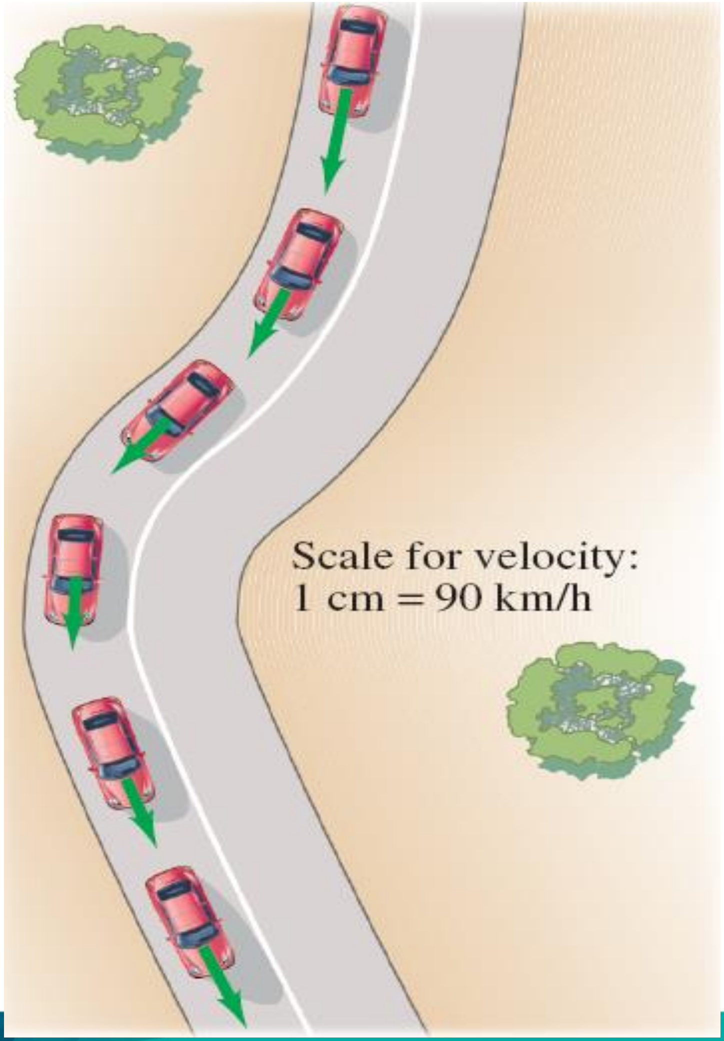
Addition and Subtraction of Vectors

i. Graphical Method

ii. Component Method

Unit Vectors

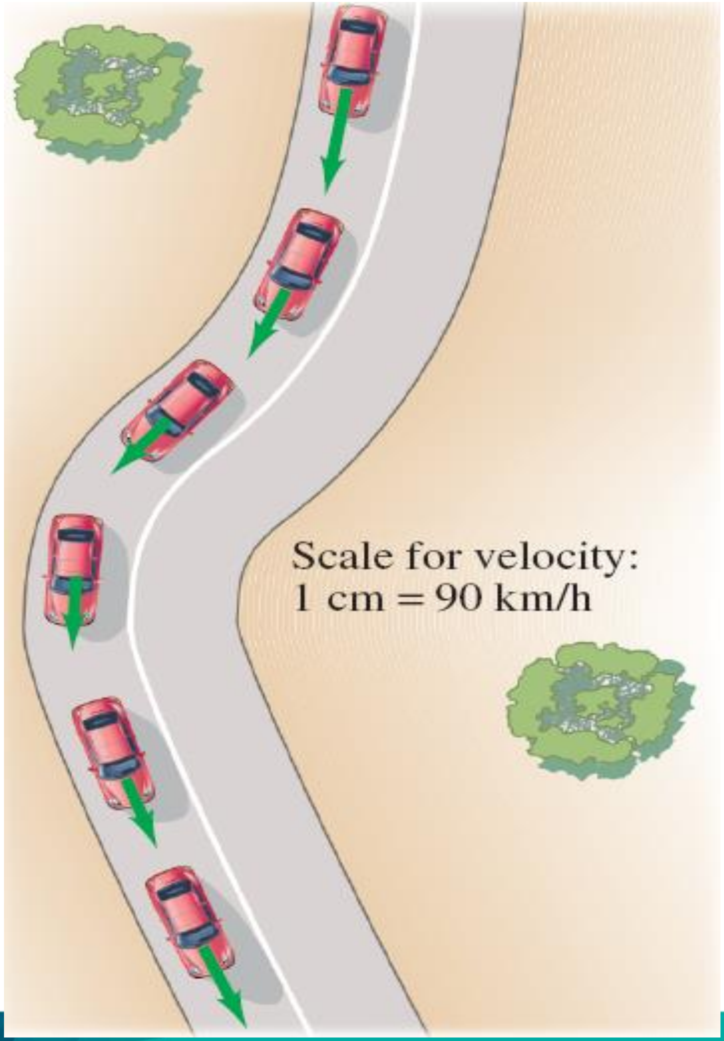
Vectors and Scalars



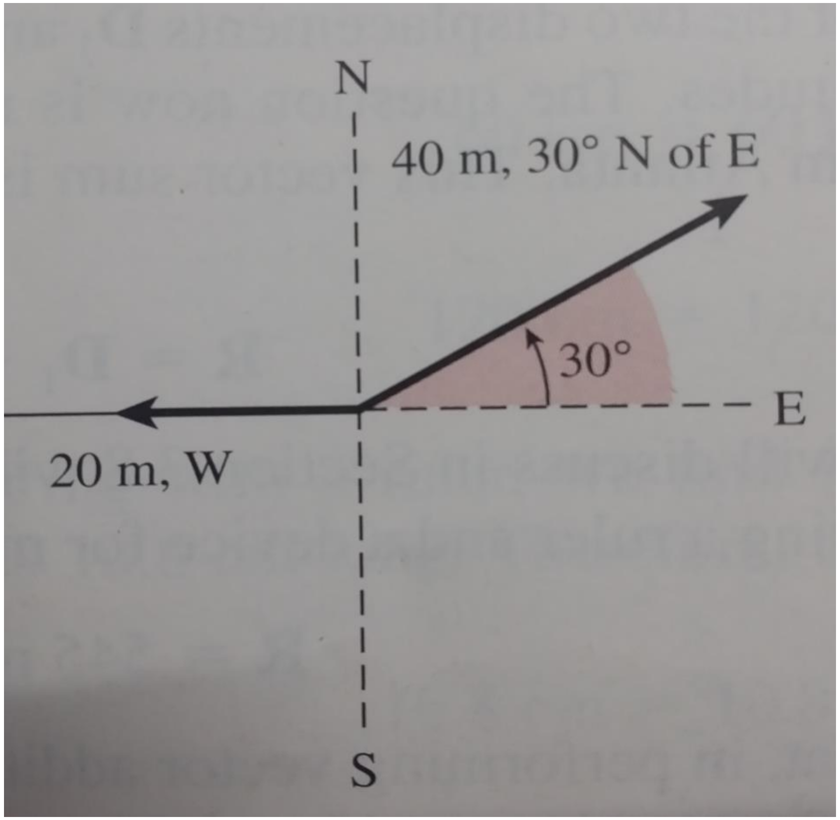
- **Scalar quantity** is consist of magnitude which includes number and unit.
- E.g.: volume = 150 m^3, velocity = 110 km/h.
- It can be added or subtracted.

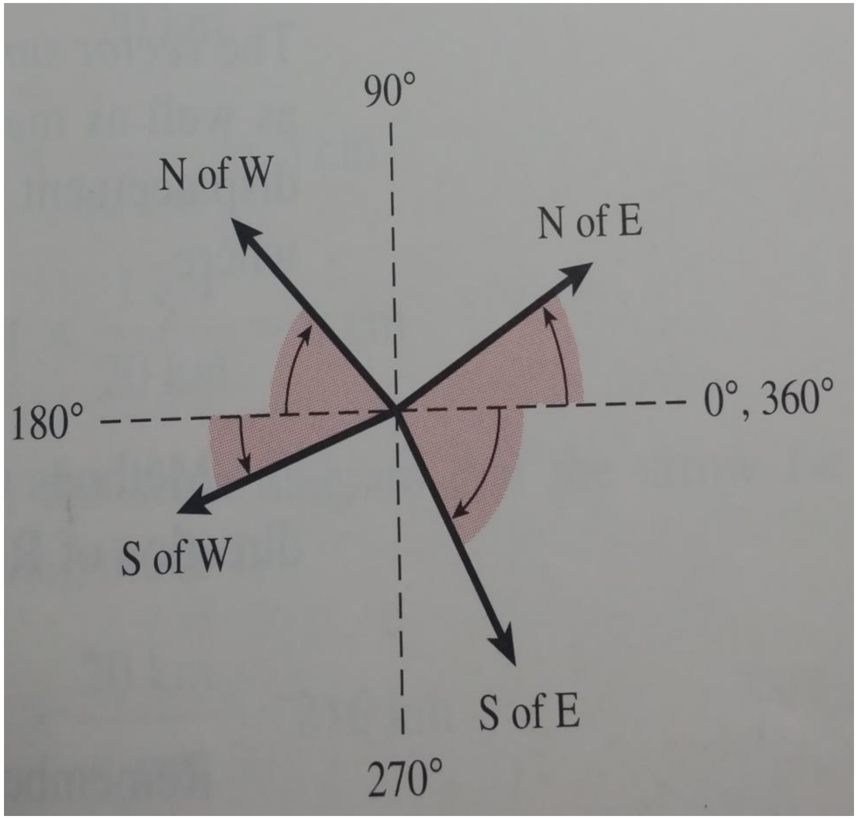
50 km + 150 km = 200 km

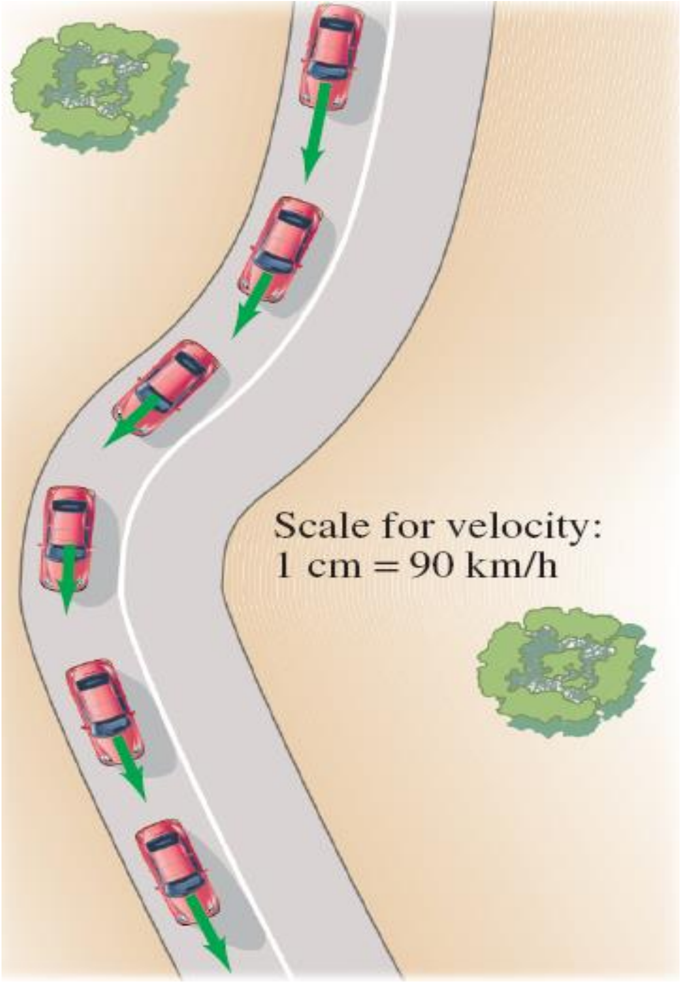
60 min – 15 min = 45 min



- Vector quantity is consist of magnitude **and** direction.
- This should includes number, unit, and direction.
- E.g.:
  - Velocity 50 km/h, 45° N of W.
  - Displacement 20 m, N.


N
40 m, 30° N of E
30°
E
20 m, W
S


90°
N of W
N of E
180°
0°, 360°
S of W
S of E
270°



- Green arrow indicated the vector.
- The direction of vector quantity is drawn by the arrow.
- The magnitude of the vector quantity is proportional to the arrow's length.

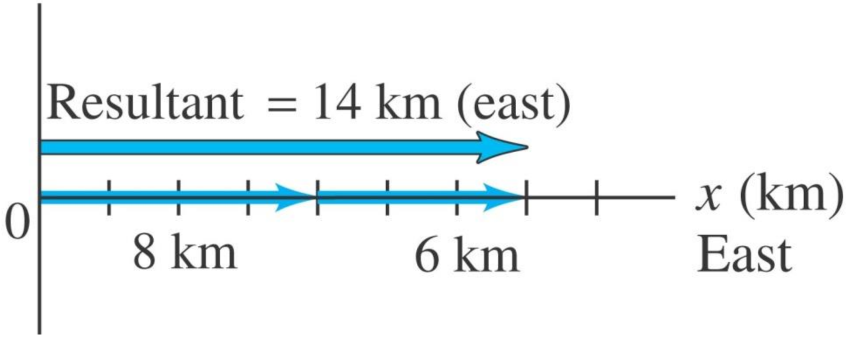
- Usually, vector is write in a bold-type font, with arrow above the symbol. E.g. : $\vec{\mathbf{X}}$

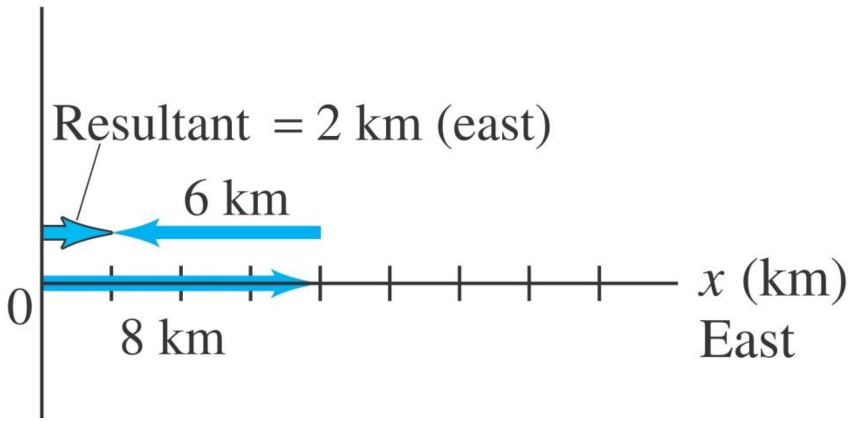
- If we are concern only with the magnitude, an italic-type font or absolute will be used.

$$X \text{ or } \left|\vec{\mathbf{X}}\right|$$

Addition of Vectors – Graphical method

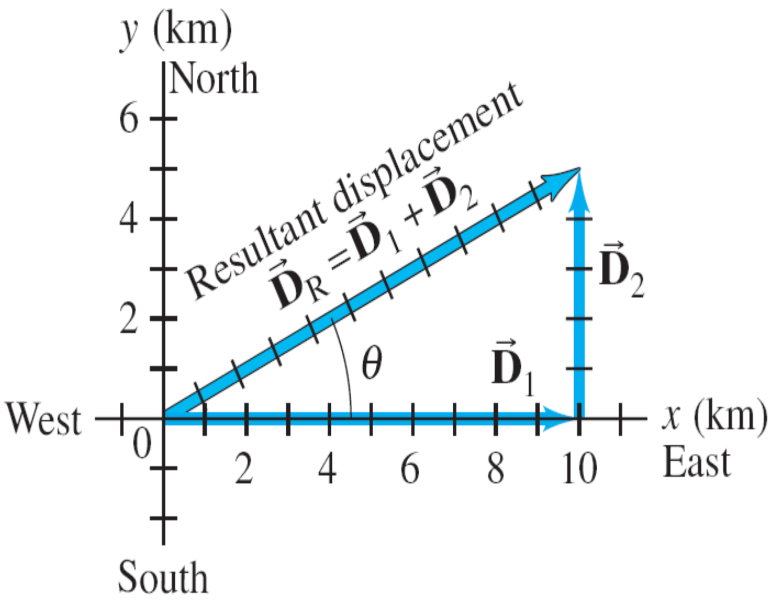
- Vectors can be functioned as algebraic quantities.
- Whenever you are adding, subtracting, multiplying or dividing, directions must be considered into account.
- 2 method of vector addition:
  i. **Graphical Method**
  ii. **Component's Method** ~ more convenient





- For vectors in ONE (1) dimension:
  - Only simples *addition and subtraction* are necessary.
- Please beware on the signs as per shown in the diagram.

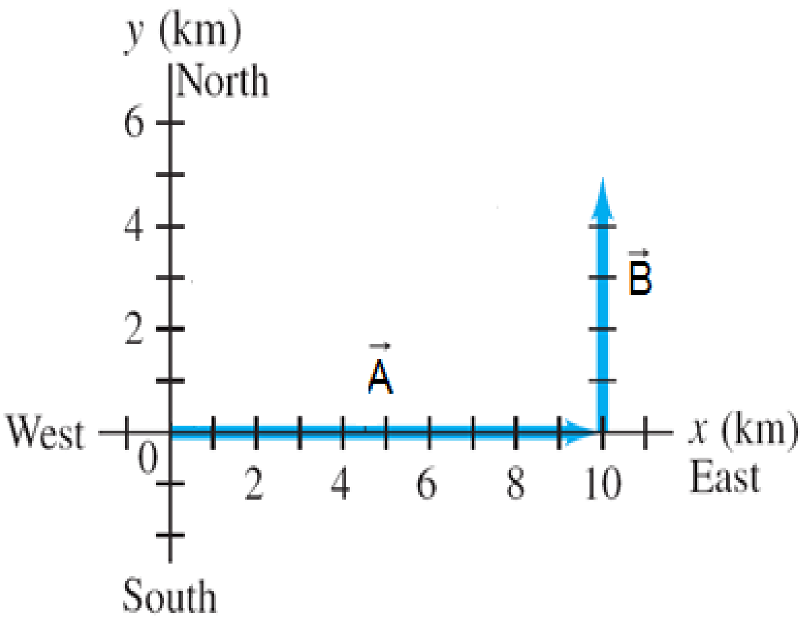
- For vectors in two dimensions, the resultant displacement is represented by arrow labeled $\vec{D}_R$.
- E.g.: Hakim walks 5.0 km north after a 10.km east journey.
- The magnitude is presented by the length of the $\vec{D}_R$



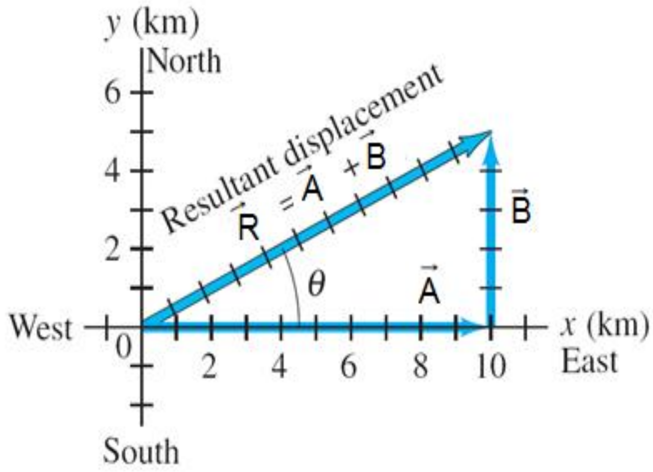
- **The rules are as follow:**

By refer to the coordinate system:

1. The first vector is drawn by using the specific length and direction given.
2. The second vector is drawn by placing its tail at the tip of the first vector.



3. Draw the resultant vector from the origin/ tail of vector $\vec{A}$ to the end/tip of the vector $\vec{B}$.



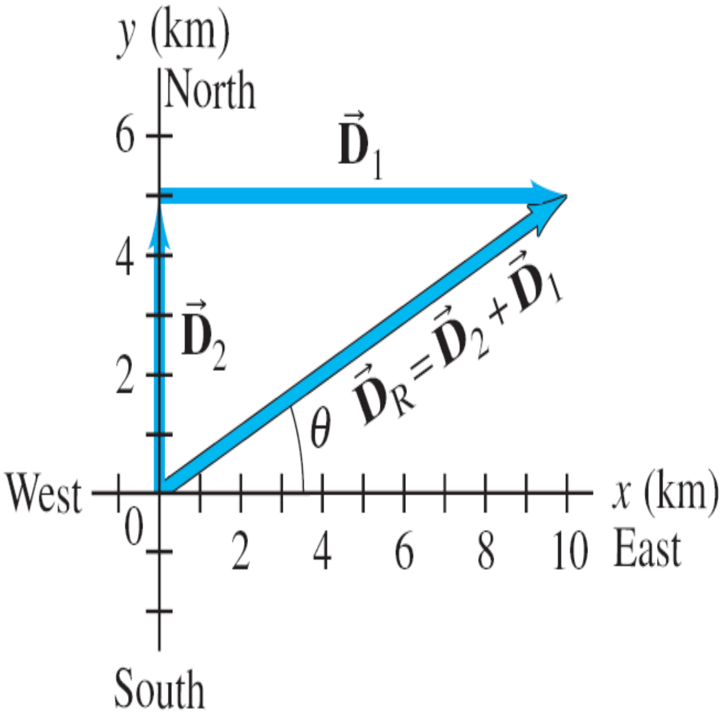
4. Both length and angle of resultant vector could be measured by:
   - Using the scale factor to convert length to actual magnitude
   - Obtained using the theorem of Pythagoras : $D_R = \sqrt{D_1^2 + D_2^2}$

(Tail-to-tip method)

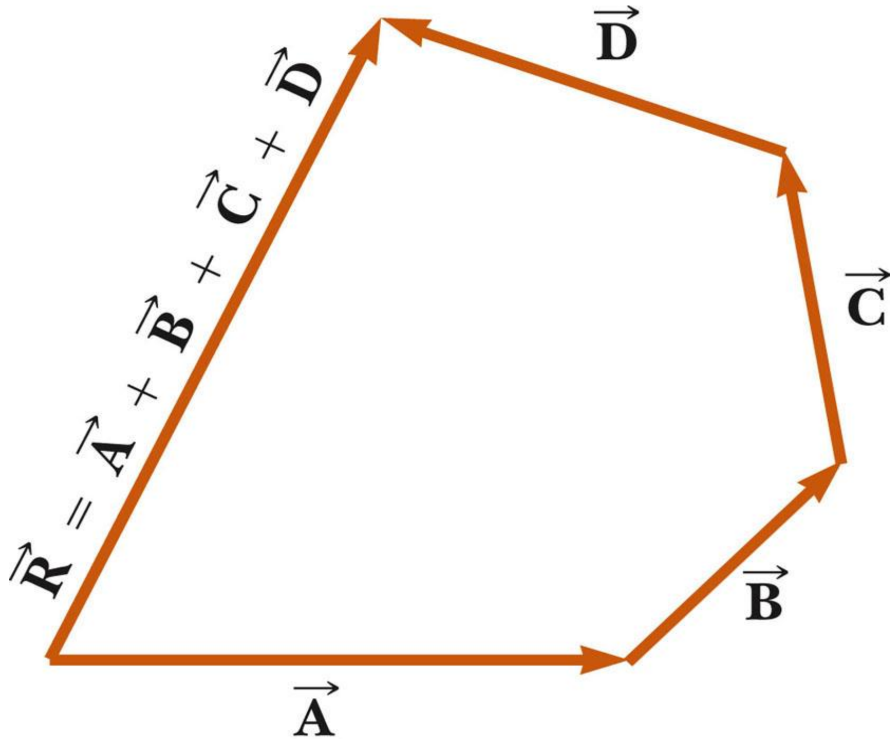
- The resultant vector is not affected by the order of the vectors added.

- E.g., Displacement of 5.0 km N was added to a displacement of 10.0 km east will yield resultant vector of 11.2 km and angle θ=27°

(+*x* axis to resultant), the same when they reverse order (e.g. before).

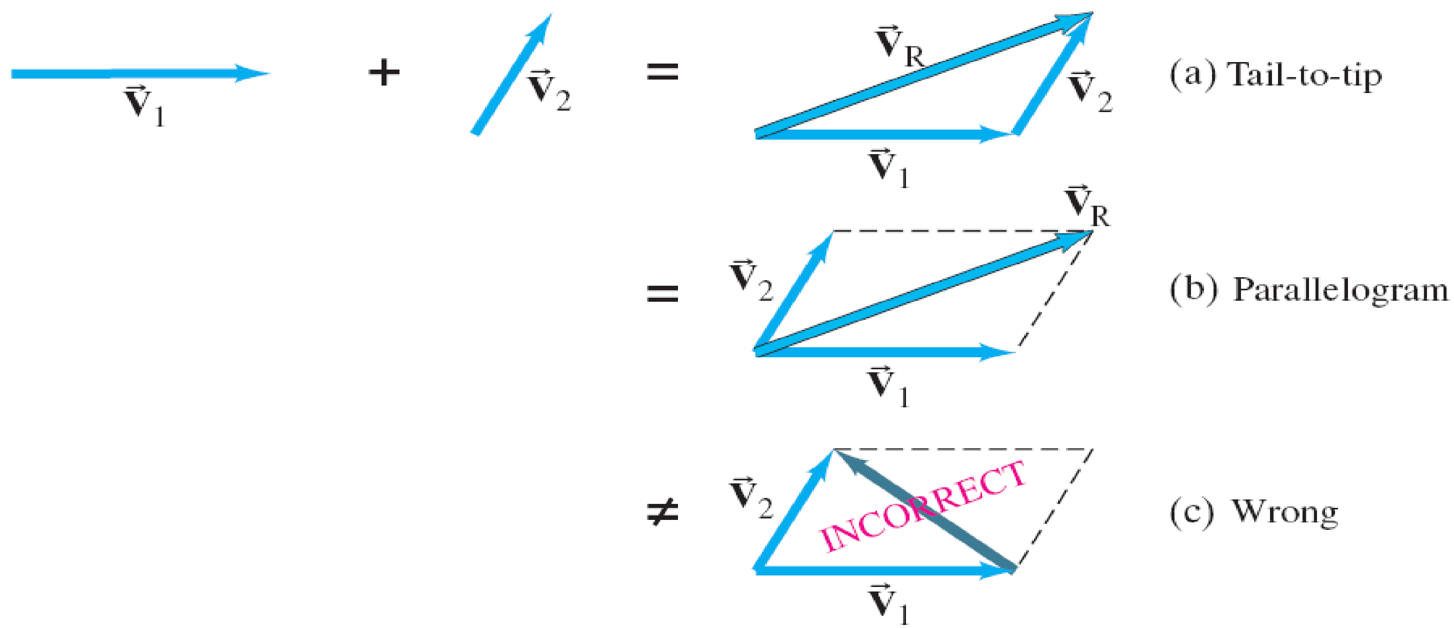


- If there are more than two vectors, you have to keep added the vectors until completed.
- The resultant vector will be drawn from the tail of first vector to the tip of last vector.

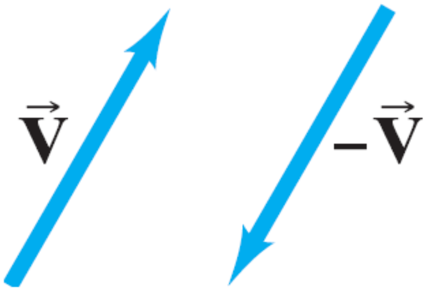


Cont. Vector and Scalar: Addition of Vectors – Graphical method

- A second method for vector addition graphically is the parallelogram method.
- How? Both vectors are drawn from the same origin.



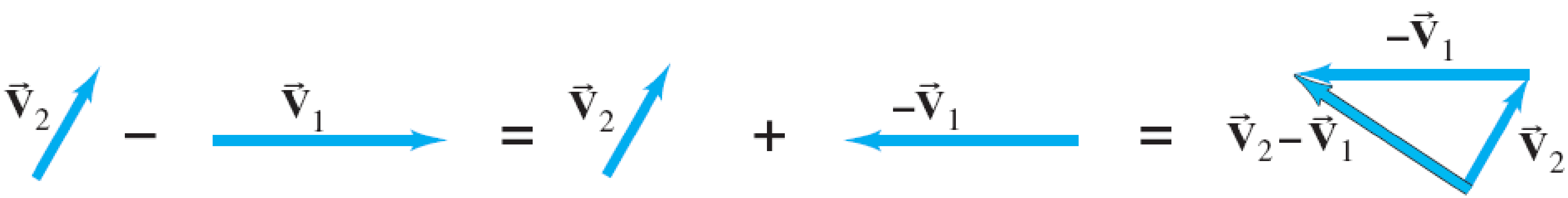
Subtraction of Vectors



For vectors subtraction:

- the negative of a vector has same magnitude but opposite direction.

Then we add the negative vector.

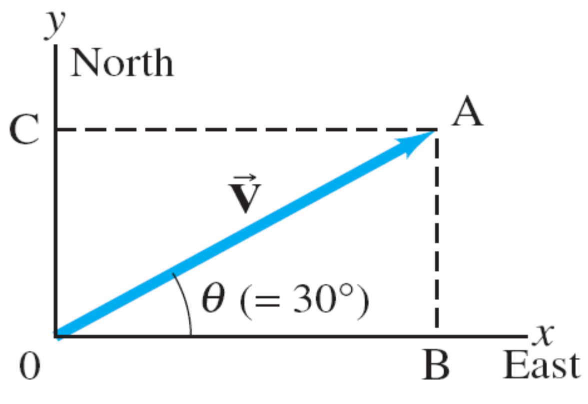


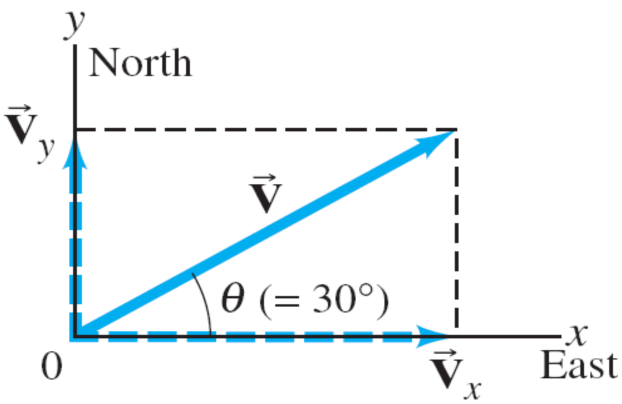
- Thus, the subtraction of vectors could be defined as:

$$\vec{V}_2 - \vec{V}_1 = \vec{V}_2 + (-\vec{V}_1)$$

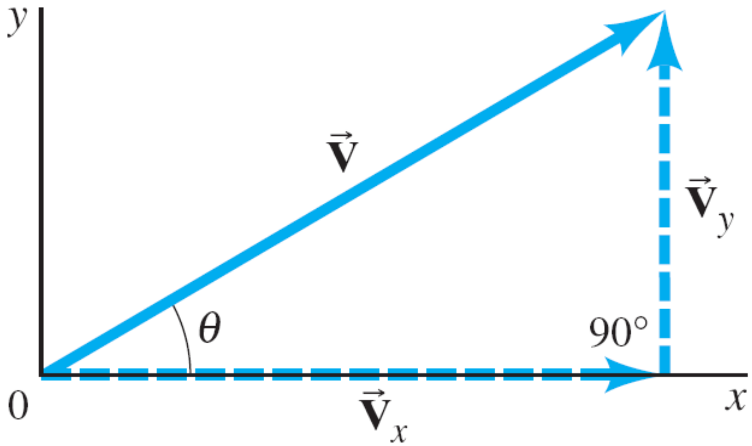
- So, the method of tip-to-tail could be applied.

Addition of Vectors – Component method





- Usually, vector component can be presented by dashed-arrow.
- v_x and v_y, are the magnitude of vector component
- As can be seen, $\vec{V}_x + \vec{V}_y = \vec{V}$ by the parallelogram method of adding vector.



$$\sin\theta = \frac{V_y}{V}$$

$$\cos\theta = \frac{V_x}{V}$$

$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

Trigonometric functions can be used to found the resultant and angle of the vector if the components are perpendicular.

- From trigonometric function; we found that:
- The vector projected along x-axis is called x-component:

$$v_x = v\cos\theta$$

- The vector projected along y-axis is called y-component:

$$v_y = v\sin\theta$$

- The magnitude and direction of vector *v* can be found using

$$v = \sqrt{v_x^2 + v_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{v_y}{v_x}$$

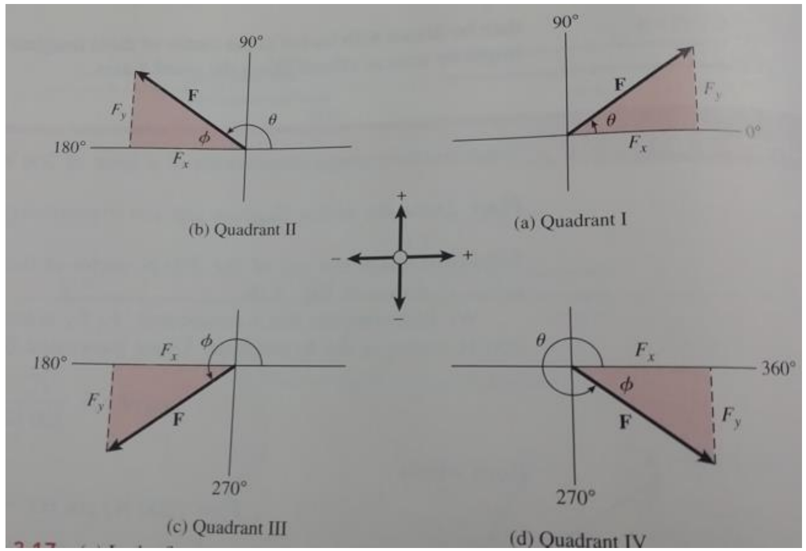
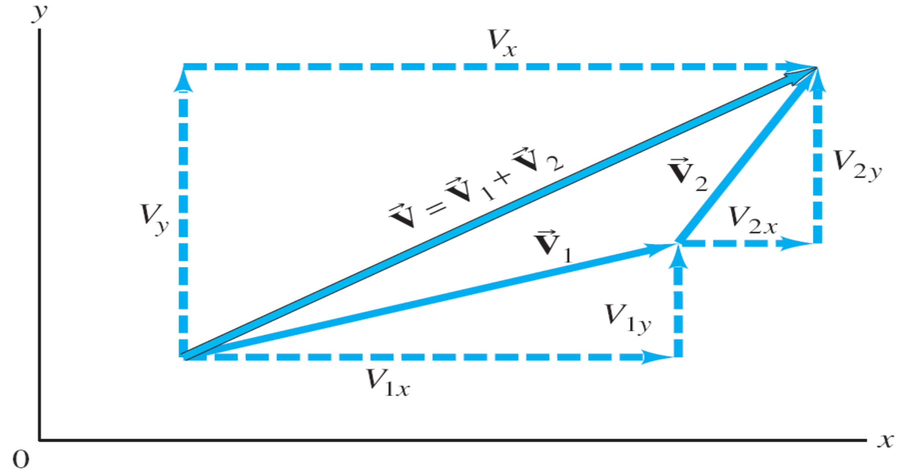


Figure 3.17 (a) In the first quadrant, angle θ is between 0° and 90°; both F_x and F_y are positive. (b) In the second quadrant, angle θ is between 90° and 180°; F_x is negative and F_y is positive. (c) In the third quadrant, angle θ is between 180° and 270°; F_x and F_y are negative. (d) In the fourth quadrant, angle θ is between 270° and 360°; F_x is positive and F_y is negative.

- The addition of any two vector, (in component) $\vec{V} = \vec{V}_1 + \vec{V}_2$ implies that:

$$V_x = V_{1x} + V_{2x}$$

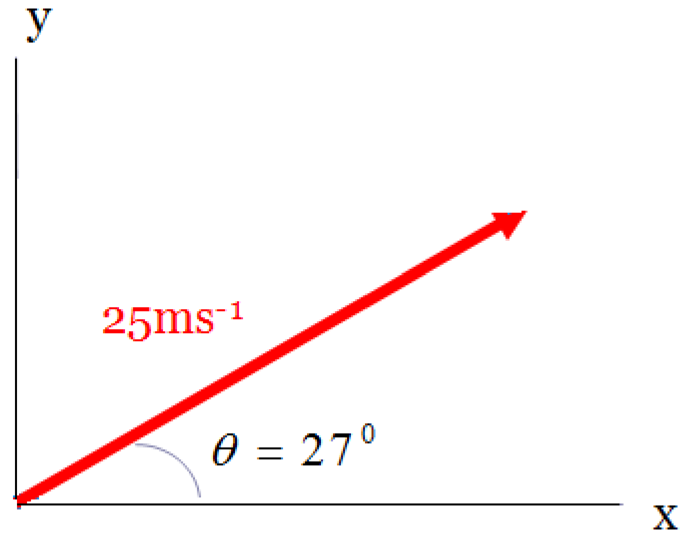
$$V_y = V_{1y} + V_{2y}$$



Addition of Vectors – Component method example

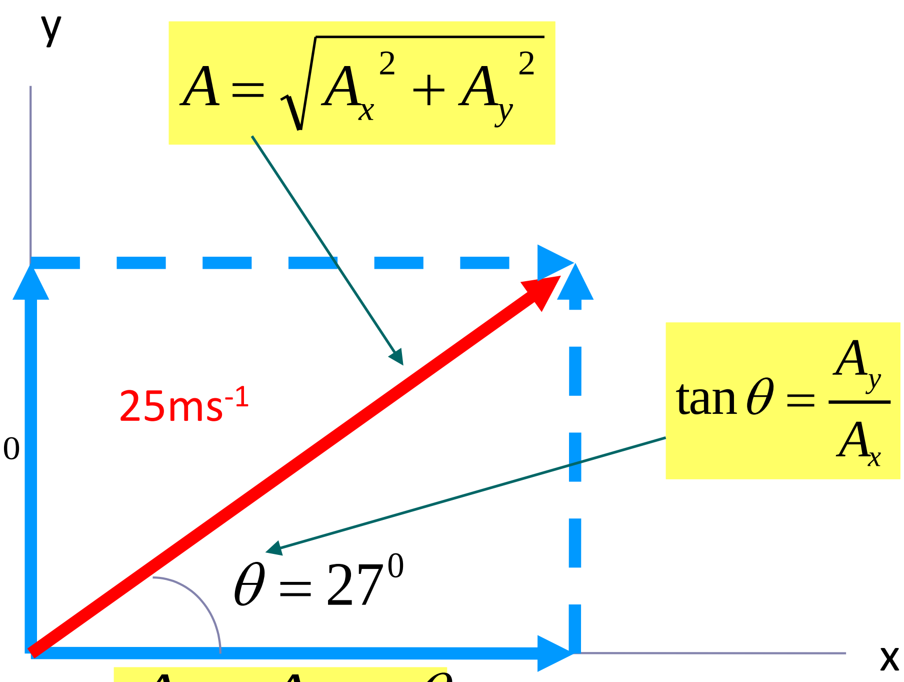
Example:

Calculate the x and y component of vector $\vec{A}$



Addition of Vectors – Component method example

Solution:



$$A = \sqrt{A_x^{\,2} + A_y^{\,2}}$$

$$\tan\theta = \frac{A_y}{A_x}$$

$$A_y = A\sin\theta$$
$$= 25\sin 27^0$$
$$= 11 \text{ ms}^{-1}$$

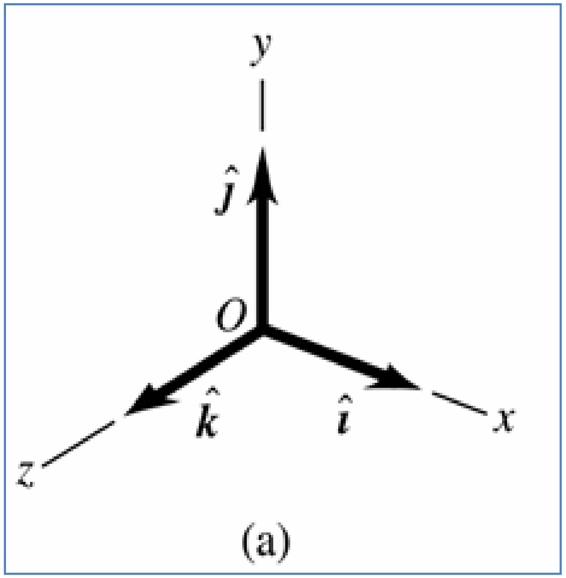
$$A_x = A\cos\theta$$
$$= 25\cos 27^0$$
$$= 22 \text{ ms}^{-1}$$

Unit Vector

- What? A no units vector with a magnitude of 1.

- Purpose? To explain the direction of (e.g., x, y and z) in coordinate system.

- The unit vectors i, j, k are labeled with “hat” (^) symbol.



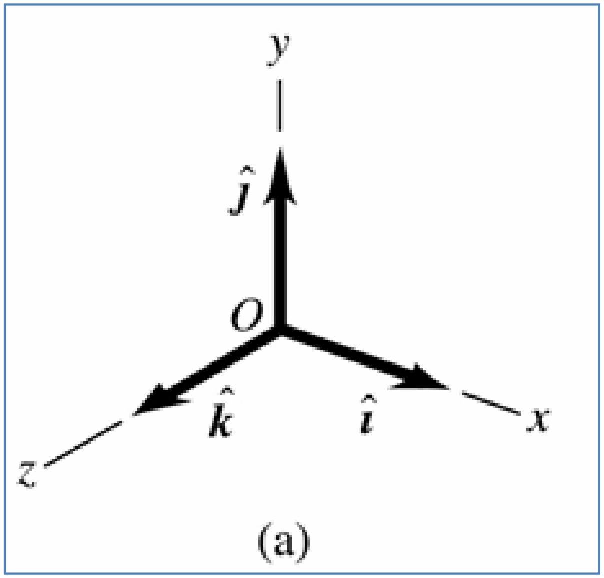
Cont. Vector and Scalar: Unit Vector

- This relationship between component vectors $i,\ j,\ k$ and components (x, y, z) is:

$$\vec{\mathbf{A}}_{\mathbf{x}} = \mathbf{A}_{\mathbf{x}}\,\hat{\mathbf{i}}$$

$$\vec{\mathbf{A}}_{\mathbf{y}} = \mathbf{A}_{\mathbf{y}}\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{A}}_{\mathbf{z}} = \mathbf{A}_{\mathbf{z}}\hat{\mathbf{k}}$$



- A vector can also be written in terms of its components as:

$$\vec{A} = A_x\hat{i} + A_y\,\hat{j} + A_z\hat{k}$$

If there are two vectors $\vec{A}$ and $\vec{B}$, expressed in their components; their vector sum in terms of unit vectors can be expressed as:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{R} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j} + \left(A_z + B_z\right)\hat{k}$$

$$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$$

Example 4

Given the two displacements

$$\vec{D} = \left(6\hat{i} + 3\hat{j} - \hat{k}\right)\text{m} \quad \text{and} \quad \vec{E} = \left(4\hat{i} - 5\hat{j} + 8\hat{k}\right)\text{m}$$

Find the magnitude of the displacement $2\vec{D} - \vec{E}$.

Solution:

Identify, Set Up and Execute:

Letting $\vec{F} = 2\vec{D} - \vec{E}$, we have

$$\vec{F} = 2\left(6\hat{i} + 3\hat{j} - \hat{k}\right)\text{m} - \left(4\hat{i} - 5\hat{j} + 8\hat{k}\right)$$

$$= \left[(12-4)\hat{i} + (6+5)\hat{j} + (-2-8)\hat{k}\right]\text{m}$$

$$\vec{F} = \left(8\hat{i} + 11\hat{j} - 10\hat{k}\right)\text{m}$$

The magnitude of $\vec{F}$,

$$\mathbf{F} = \sqrt{\mathbf{F_x}^2 + \mathbf{F_y}^2 + \mathbf{F_z}^2}$$

$$\mathbf{F} = \sqrt{(8\text{ m})^2 + (11\text{ m})^2 + (-10\text{ m})^2} = 17\text{ m}$$

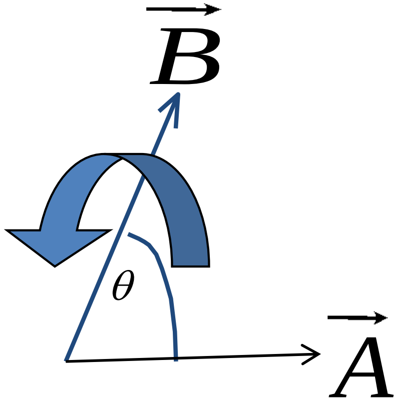
Product of Unit Vectors

1. **Scalar/dot** product
   - useful where a scalar result is wanted from the product of two vectors.

2. **Vector/cross** product
   - useful where a vector result is wanted from the product of two vectors.

Product of Unit Vectors (Scalar Product)

- The scalar or **dot product** is denoted by $\vec{A} \bullet \vec{B}$
- Even if $\vec{A}$ and $\vec{B}$ are both vectors, the quantity $\vec{A} \bullet \vec{B}$ are scalar.



Angle θ measured counterclockwise wrt positive x-axis

Product of Unit Vectors (Scalar Product)

$\vec{A} \bullet \vec{B}$ - What? Magnitude of $\vec{A}$ multiplied with the magnitude of $\vec{B}$ in the direction of $\vec{A}$:

$$\vec{A} \bullet \vec{B} = AB\cos\theta = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$$

- The scalar product may be positive (when $0<\theta<90^o$), negative ($90^o < \theta < 180^o$) and zero ($\theta = 90^o$).

Product of Unit Vectors (Scalar Product)

- Because of these properties of scalar product:

$$\hat{i}\cdot\hat{i} = j\cdot j = k\cdot k = (1)(1)\cos 0^\circ = 1$$

$$\hat{i}\cdot j = \hat{i}\cdot k = j\cdot k = (1)(1)\cos 90^\circ = 0$$

- The scalar product in term of component can be expressed as:

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

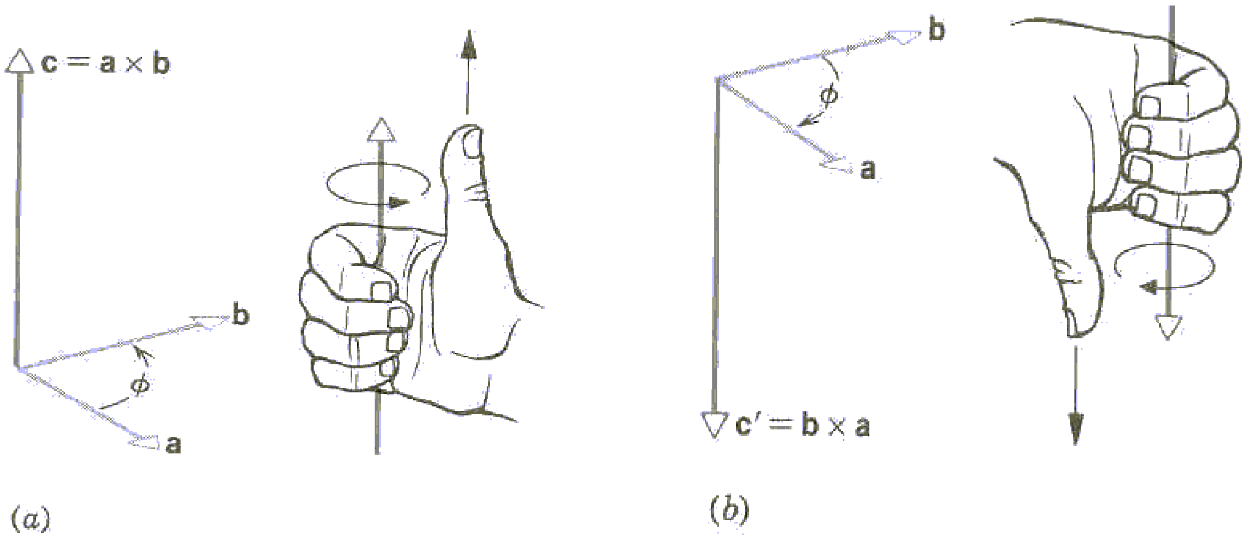
Product of Unit Vectors (Vector Product)

- The vector product, also called as cross product is denoted by $\vec{A}\times\vec{B}$
- Definition: vector quantity with a direction perpendicular *to both* $\vec{A}$ *and* $\vec{B}$ *)*
- The **magnitude** can be measured as followed:

$$\vec{A}\times\vec{B} = AB\sin\theta$$

Cont. Vector and Scalar: Product of Unit Vectors (Vector Product)

- Direction? Use Right Hand Rule



See you in Chapter 3!