UMP OPEN COURSEWARE

Universiti Malaysia PAHANG
Engineering • Technology • Creativity

CHAPTER 8

# BEE3143:POWER SYSTEM ANALYSIS- Symmetrical Components

**Expected Outcomes**

Able to solve unbalanced fault analysis

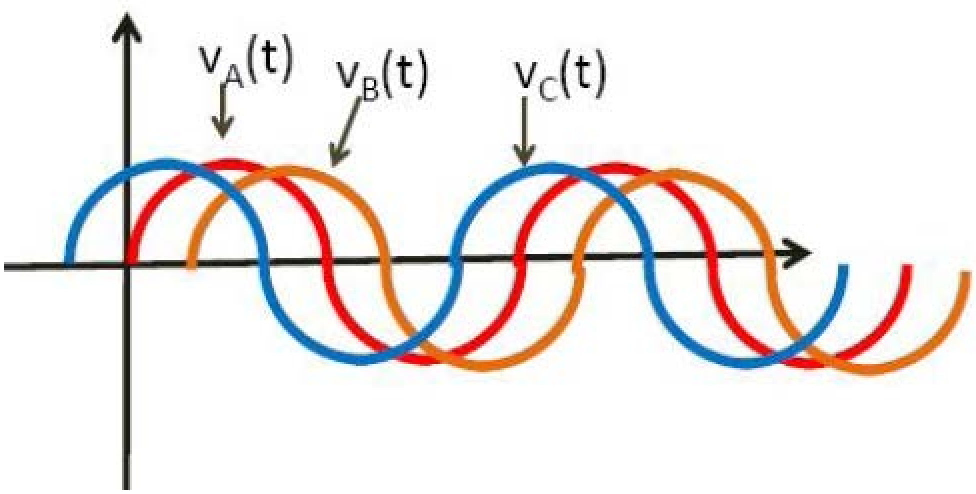
# Introduction

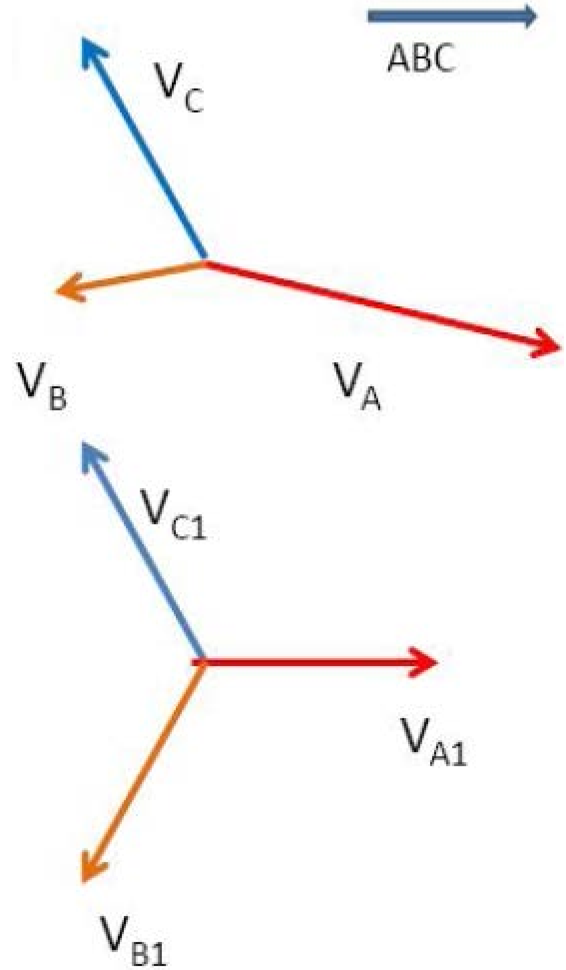
- 3 types of unbalanced faults occurred at transmission lines: 1) single line to ground fault 2) line-to-line fault 3) double line to ground fault
- → causes unbalanced circuits, so cannot used single line diagram simplifies solution that used in balanced 3Φ fault analysis
- → symmetrical components method is used to solve the unbalanced fault problem
- symmetrical components allow unbalanced phase quantities such as currents and voltages to be replaced by three separate balanced symmetrical components
- Any unsymmetrical set of 3Φ voltages or currents could be broke down into three symmetrical sets of balanced 3Φ components

# Fundamental of Symmetrical Components

A set of **positive-sequence components** consisting of three phasors

- equal in magnitude,
- displaced from each other by **120°**,
- and having the **same phase sequence** as the original power system.

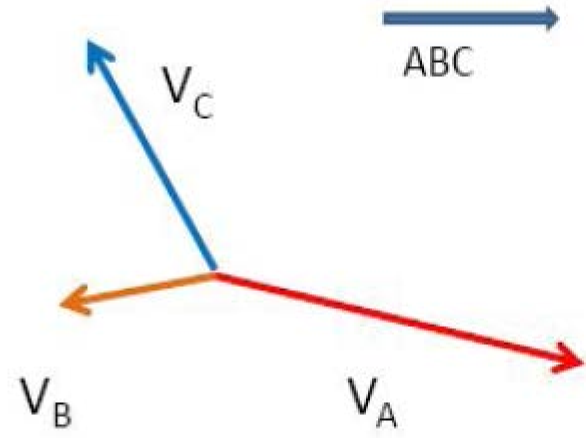


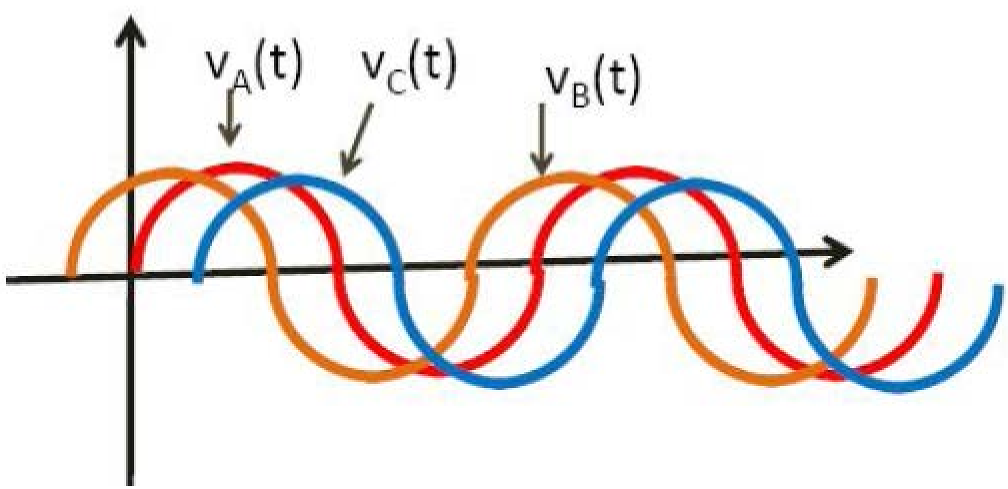



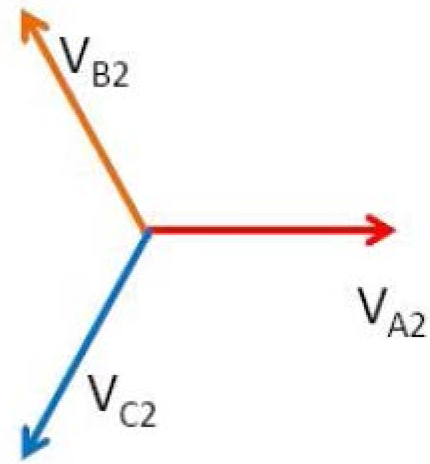
# ...Fundamental of Symmetrical Components

A set of **negative-sequence components** consisting of three phasors

- equal in magnitude,
- displaced from each other by **120°**,
- and having the **opposite phase sequence** from the original power system.

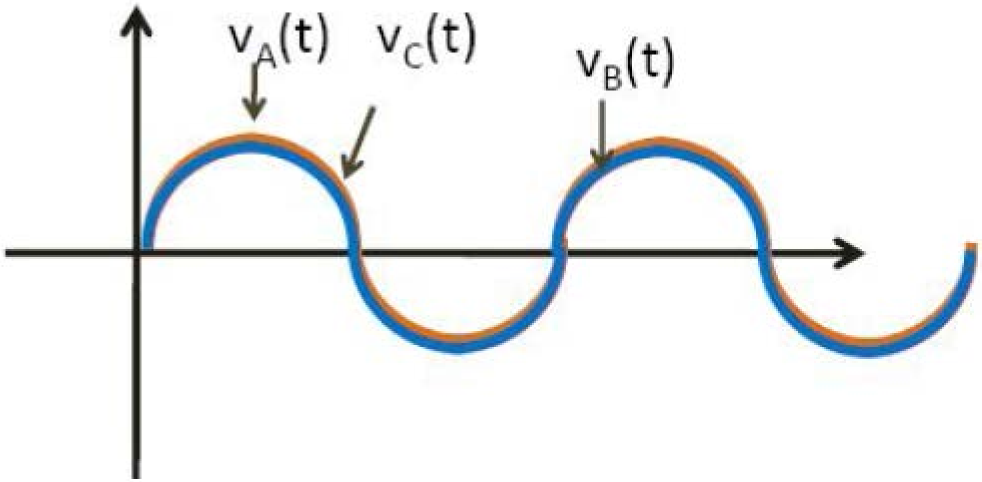


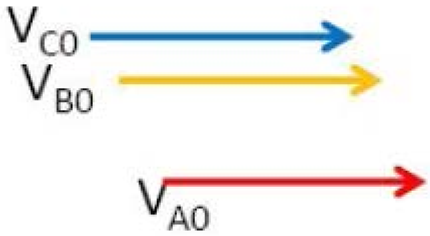



# ...Fundamental of Symmetrical Components

A set of **zero-sequence components** consisting of three phasors equal in magnitude and phase.



# ...Fundamental of Symmetrical Components

## The a constant

- For simplicity we denote the sequence components ($V_{A1}$, $V_{A2}$ and $V_{A0}$) as ($V_1$, $V_2$ and $V_0$)
- $V_A$, $V_B$ and $V_C$ are defined by the following transformation:

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

With constant a represents

$$a = 1\angle 120^\circ$$

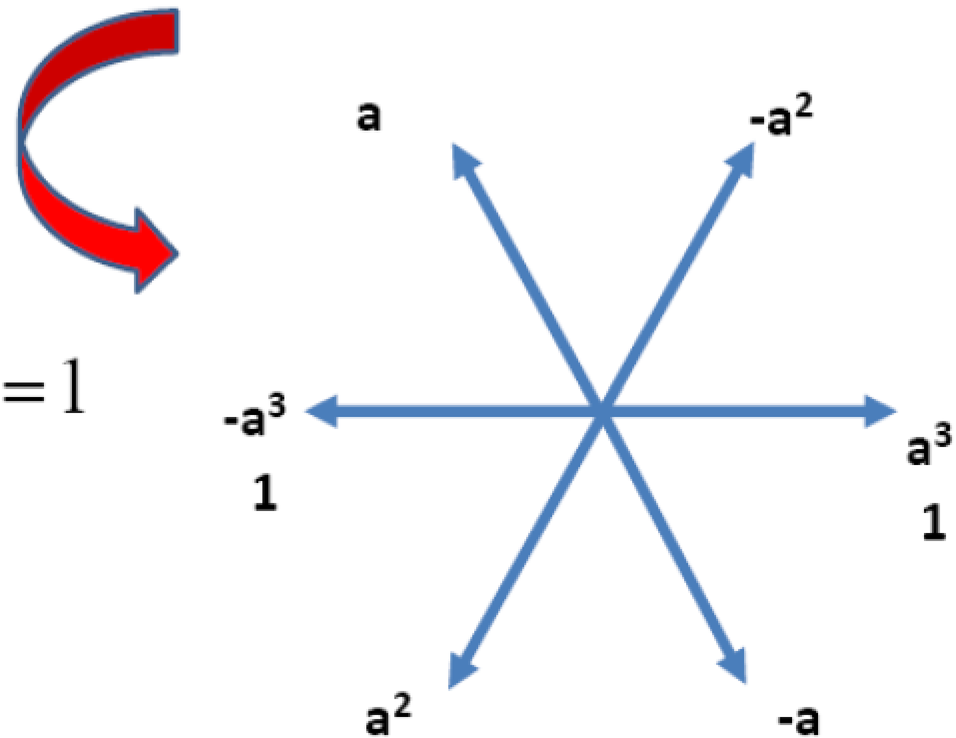
# ...Fundamental of Symmetrical Components

Phasor diagram showing relationships among the various powers of **a**

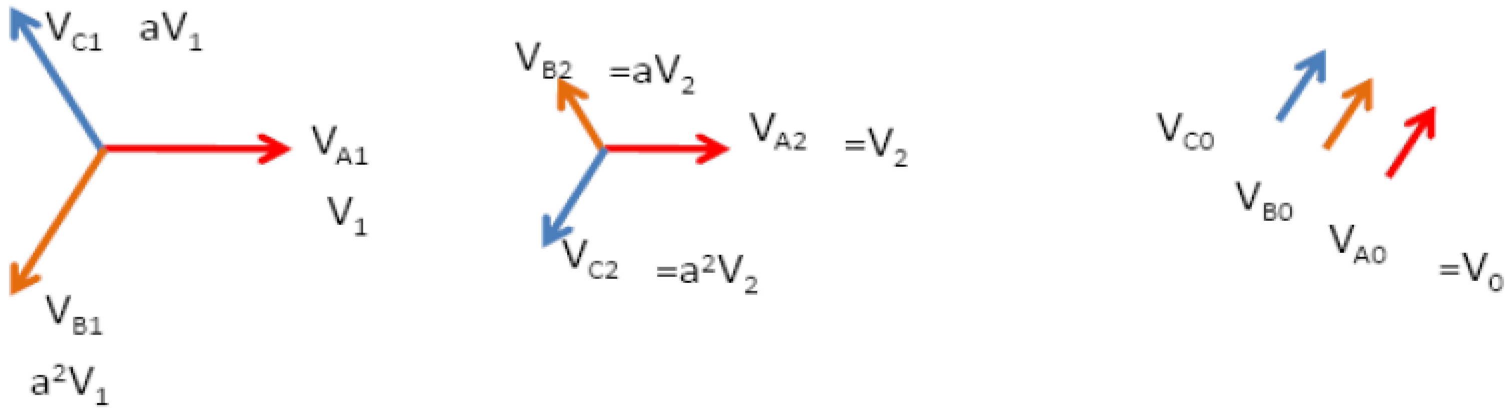
$$a = 1\angle 120°$$

$$a^2 = 1\angle 240°$$

$$a^3 = 1\angle 360° = 1\angle 0° = 1$$

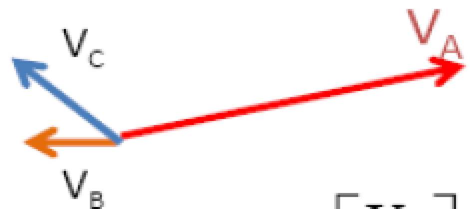


# ...Fundamental of Symmetrical Components




$V_C = V_{C1}+V_{C2}+V_{C0}$

$V_C = aV_1+a^2V_2+V_0$




$V_A = V_{A1}+V_{A2}+V_{A0}$

$V_A = V_1+V_2+V_0$

$V_B = V_{B1}+V_{B2}+V_{B0}$

$V_B = a^2V_1+aV_2+V_0$

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

# ...Fundamental of Symmetrical Components

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \Rightarrow \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = A \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

A

The symmetrical components of the unbalanced three-phase voltage can be expressed as

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = A^{-1} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

# ...Fundamental of Symmetrical Components

Symmetrical components of an unbalanced three-phase current

$I_C = I_{C1} + I_{C2} + I_{C0}$

$I_C = aI_1 + a^2 I_2 + I_0$


$I_A = I_{A1} + I_{A2} + I_{A0}$

$I_A = I_1 + I_2 + I_0$

$I_B = I_{B1} + I_{B2} + I_{B0}$

$I_B = a^2 I_1 + aI_2 + I_0$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

# Example

- Obtain the symmetrical components of a set of unbalanced currents:

$$I_a = 1.6\angle 25^\circ$$

$$I_b = 1.0\angle 180^\circ$$

$$I_c = 0.9\angle 132^\circ$$

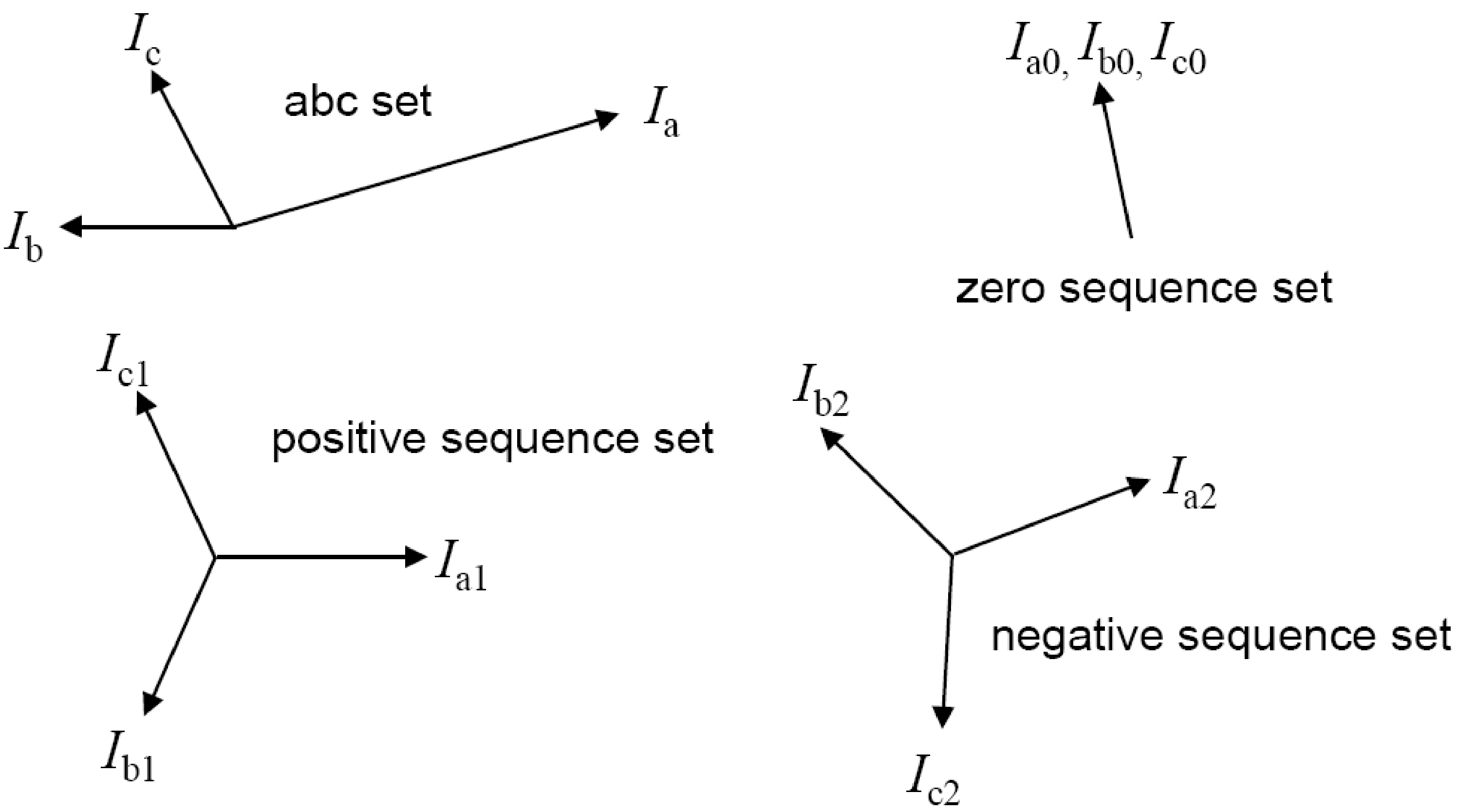
# Solution

$$I_{a0} = \frac{(1.6\angle 25^\circ) + (1.0\angle 180^\circ) + (0.9\angle 132^\circ)}{3} = 0.45\angle 96.5^\circ$$

$$I_{a1} = \frac{(1.6\angle 25^\circ) + a(1.0\angle 180^\circ) + a^2(0.9\angle 132^\circ)}{3} = 0.94\angle -0.1^\circ$$

$$I_{a2} = \frac{(1.6\angle 25^\circ) + a^2(1.0\angle 180^\circ) + a(0.9\angle 132^\circ)}{3} = 0.60\angle 22.3^\circ$$

# ...Solution



$I_c$
abc set
$I_a$
$I_b$
$I_{a0}, I_{b0}, I_{c0}$
zero sequence set
$I_{c1}$
positive sequence set
$I_{a1}$
$I_{b1}$
$I_{b2}$
$I_{a2}$
negative sequence set
$I_{c2}$


والله أعلم