UMP OPEN COURSEWARE

Universiti Malaysia PAHANG
Engineering • Technology • Creativity

CHAPTER 5

BEE3143:POWER SYSTEM ANALYSIS- Power flow solution- Fast Decoupled

Expected Outcomes

Able to solve power flow solution using Fast-Decoupled technique

Fast-Decoupled Power Flow Solution

- **Transmission lines and transformers have high *X/R* ratios**
  - Real power change, ΔP
    - is less sensitive to changes in the voltage magnitude, $\Delta|V|$
    - is more sensitive to changes in the phase angle, $\Delta\delta$
  - Reactive power changes, ΔQ
    - is less sensitive to changes in the phase angle , $\Delta\delta$
    - is more sensitive to changes in the voltage magnitude, $\Delta|V|$
  - Jacobian submatrices J_{Qd} and J_{PV} tend to be much smaller in magnitude compared to J_{Pd} and J_{QV}
- **Jacobian submatrices J_{Qd} and J_{PV} can be set to zero**

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{Pd} & 0 \\ 0 & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta d \\ \Delta|V| \end{bmatrix}$$

$$\Delta P = J_{Pd} \cdot \Delta d = {}^{\partial P}/_{\partial d}\, \Delta d$$

$$\Delta Q = J_{QV} \cdot \Delta|V| = {}^{\partial Q}/_{\partial |V|}\, \Delta|V|$$

...Fast-Decoupled Power Flow Solution

- **J_{PV} elements**

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos(q_{ij} - d_i + d_j)$$

$$q_{ij} \approx 90^\circ \quad d_i \approx d_j$$

$$\frac{\partial P_i}{\partial |V_j|} \approx |V_i||Y_{ij}|\cos(90^\circ) = 0.0$$

- **J_{Qd} elements**

$$\frac{\partial Q_i}{\partial |d_j|} = -|V_i||V_j||Y_{ij}|\cos(q_{ij} - d_i + d_j)$$

$$q_{ij} \approx 90^\circ \quad d_i \approx d_j$$

$$\frac{\partial Q_i}{\partial |d_j|} \approx -|V_i||V_j||Y_{ij}|\cos(90^\circ) = 0.0$$

...Fast-Decoupled Power Flow Solution

- **The matrix equation is separated into two decoupled equations**
  - requires considerably less time to solve compared to the full Newton-Raphson method
  - J_{Pd} and J_{QV} submatrices can be further simplified to eliminate the need for recomputing of the submatrices during each iteration
    - some terms in each element are relatively small and can be eliminated
    - the remaining equations consist of constant terms and one variable term
    - the one variable term can be moved and coupled with the change in power variable
    - the result is a Jacobian matrix with constant term elements

...Fast-Decoupled Power Flow Solution

Jacobian J_{Pd} Diagonal Terms

$$\frac{\partial P_i}{\partial d_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_i||V_j||Y_{ij}| \sin\left(q_{ij} - d_i + d_j\right)$$

$$= -|V_i|^2 |Y_{ii}| \sin\left(q_{ii}\right) + \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin\left(q_{ij} - d_i + d_j\right)$$

$$\frac{\partial P_i}{\partial d_i} = -|V_i|^2 |Y_{ii}| \sin\left(q_{ii}\right) - Q_i \qquad Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin\left(q_{ij} - d_i + d_j\right)$$

$$|Y_{ii}| \sin\left(q_{ii}\right) = B_{ii} \qquad B_{ii} >> Q_i \qquad \frac{\partial P_i}{\partial d_i} = -|V_i|^2 B_{ii}$$

$$|V_i|^2 \approx |V_i| \qquad \Rightarrow \qquad \frac{\partial P_i}{\partial d_i} = -|V_i| B_{ii}$$

...Fast-Decoupled Power Flow Solution

Jacobian J_{Pd} Off-diagonal Terms

$$\frac{\partial P_i}{\partial d_j} = -|V_i||V_j||Y_{ij}|\sin(q_{ij} - d_i + d_j)$$

$$d_j - d_i \approx 0$$

$$\frac{\partial P_i}{\partial d_i} = -|V_i||V_j||Y_{ij}|\sin(q_{ij})$$

$$|Y_{ij}|\sin(q_{ij}) = B_{ij} \qquad |V_j| \approx 1$$

$$\frac{\partial P_i}{\partial d_i} = -|V_i|B_{ij}$$

...Fast-Decoupled Power Flow Solution

Jacobian J_{QV} Diagonal Terms

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin(q_{ii}) - \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_j||Y_{ij}|\sin(q_{ij} - d_i + d_j)$$

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}|\sin(q_{ii}) - |V_i|^{-1}\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(q_{ij} - d_i + d_j)$$

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}|\sin(q_{ii}) + |V_i|^{-1} Q_i \qquad Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(q_{ij} - d_i + d_j)$$

$$|Y_{ii}|\sin(q_{ii}) = B_{ii} \qquad B_{ii} >> Q_i \qquad \Rightarrow \qquad \frac{\partial Q_i}{\partial |V_i|} = -|V_i| B_{ii}$$

...Fast-Decoupled Power Flow Solution

Jacobian J_{QV} Off-diagonal Terms

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}|\sin(q_{ij} - d_i + d_j)$$

$$d_j - d_i \approx 0$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}|\sin(q_{ij})$$

$$|Y_{ij}|\sin(q_{ij}) = B_{ij}$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i|B_{ij}$$

...Fast-Decoupled Power Flow Solution

- **Individual power change equations in J_{Pd} and J_{QV}**

$$\Delta P_i = \sum_{j=1}^{n} -|V_i| B_{ij} \Delta d_j \qquad \Rightarrow \qquad \frac{\Delta P_i}{|V_i|} = \sum_{j=1}^{n} -B_{ij} \Delta d_j$$

$$\Delta Q_i = \sum_{j=1}^{n} -|V_i| B_{ij} \Delta |V_j| \qquad \Rightarrow \qquad \frac{\Delta Q_i}{|V_i|} = \sum_{j=1}^{n} -B_{ij} \Delta |V_j|$$

- **Matrix equation for J_{Pd} and J_{QV}**

$$\frac{\Delta P}{|V_i|} = -\mathbf{B}' \Delta d \qquad \Rightarrow \qquad \Delta d = -[\mathbf{B}']^{-1} \frac{\Delta P}{|V|}$$

$$\frac{\Delta Q}{|V_i|} = -\mathbf{B}'' \Delta |V| \qquad \Rightarrow \qquad \Delta |V| = -[\mathbf{B}'']^{-1} \frac{\Delta Q}{|V|}$$

...Fast-Decoupled Power Flow Solution

- **Individual power change equations in J_{Pd} and J_{QV}**

$$\Delta P_i = \sum_{j=1}^{n} -\left|V_i\right| B_{ij} \Delta d_j \quad \Rightarrow \quad \frac{\Delta P_i}{\left|V_i\right|} = \sum_{j=1}^{n} -B_{ij} \Delta d_j$$

$$\Delta Q_i = \sum_{j=1}^{n} -\left|V_i\right| B_{ij} \Delta\left|V_j\right| \quad \Rightarrow \quad \frac{\Delta Q_i}{\left|V_i\right|} = \sum_{j=1}^{n} -B_{ij} \Delta\left|V_j\right|$$

- **Matrix equation for J_{Pd} and J_{QV}**

$$\frac{\Delta P}{\left|V_i\right|} = -\mathbf{B}' \Delta d \quad \Rightarrow \quad \Delta d = -\left[\mathbf{B}'\right]^{-1} \frac{\Delta P}{\left|V\right|}$$

$$\frac{\Delta Q}{\left|V_i\right|} = -\mathbf{B}'' \Delta\left|V\right| \quad \Rightarrow \quad \Delta\left|V\right| = -\left[\mathbf{B}''\right]^{-1} \frac{\Delta Q}{\left|V\right|}$$


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