QUIZ 5

NAME	
COURSE CODE	DUM 2413 STATISTICS AND PROBABILITY
DURATION	10 MINUTES

The probability density function of X is given by

$$f(x)=\begin{cases}\frac{3}{2}(x-1)^2, & 0\le x\le 1\\ 0, & \text{elsewhere}\end{cases}$$

(i) Find $P\left(\frac{1}{2}<X\le\frac{3}{4}\right)$.

(ii) Determine $\mathrm{E}(3X+2)$.

(iii) Determine $\mathrm{Var}(X)$.

ANSWERS:

(i)
$$P\left(\frac{1}{2} < X \le \frac{3}{4}\right) = \frac{3}{2}\int_{\frac{1}{2}}^{\frac{3}{4}} (x-1)^2 dx$$
$$P\left(\frac{1}{2} < X \le \frac{3}{4}\right) = \frac{1}{2}\Big[(x-1)^3\Big]_{\frac{1}{2}}^{\frac{3}{4}}$$
$$P\left(\frac{1}{2} < X \le \frac{3}{4}\right) = \frac{1}{2}\left(\left(\frac{3}{4}-1\right)^3 - \left(\frac{1}{2}-1\right)^3\right)$$
$$P\left(\frac{1}{2} < X \le \frac{3}{4}\right) = \frac{7}{128}$$

(ii)
$$\mathrm{E}(X) = \frac{3}{2}\int_{-\infty}^{\infty} \left(x^3 - 2x^2 + x\right) dx$$
$$\mathrm{E}(X) = \frac{3}{2}\int_{-\infty}^{0} \left(x^3 - 2x^2 + x\right) dx + \frac{3}{2}\int_{0}^{1} \left(x^3 - 2x^2 + x\right) dx + \frac{3}{2}\int_{1}^{-\infty} \left(x^3 - 2x^2 + x\right) dx$$
$$\mathrm{E}(X) = \frac{3}{2}\left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right]_0^1$$
$$\mathrm{E}(X) = \frac{3}{2}\left[\left(\frac{(1)^4}{4} - \frac{2(1)^3}{3} + \frac{(1)^2}{2}\right) - \left(\frac{(0)^4}{4} - \frac{2(0)^3}{3} + \frac{(0)^2}{2}\right)\right]$$
$$\mathrm{E}(X) = \frac{1}{8}$$

(iii)
$$\mathrm{E}(X^2) = \frac{3}{2}\int_{-\infty}^{\infty} \left(x^4 - 2x^3 + x^2\right) dx$$
$$\mathrm{E}(X^2) = \frac{3}{2}\int_{-\infty}^{0} \left(x^4 - 2x^3 + x^2\right) dx + \frac{3}{2}\int_{0}^{1} \left(x^4 - 2x^3 + x^2\right) dx + \frac{3}{2}\int_{1}^{-\infty} \left(x^4 - 2x^3 + x^2\right) dx$$
$$\mathrm{E}(X^2) = \frac{3}{2}\left[\frac{x^5}{5} - \frac{1x^4}{2} + \frac{x^3}{3}\right]_0^1$$
$$\mathrm{E}(X^2) = \frac{3}{2}\left[\left(\frac{(1)^5}{5} - \frac{1(1)^4}{2} + \frac{(1)^3}{3}\right) - \left(\frac{(0)^5}{5} - \frac{1(0)^4}{2} + \frac{(0)^3}{3}\right)\right]$$
$$\mathrm{E}(X^2) = \frac{1}{20}$$
$$\mathrm{Var}(X) = E(X^2) - (E(X))^2$$
$$\mathrm{Var}(X) = \frac{1}{20} - \left(\frac{1}{8}\right)^2$$
$$\mathrm{Var}(X) = \frac{11}{320}$$