

CHAPTER 4

BEE3143:POWER SYSTEM ANALYSIS- Power flow solution- Gauss-Seidel

Expected Outcomes

Able to solve power flow solution using Gauss-Seidel technique

Gauss-Seidel Power Flow Solution

Step 1

- Calculate the bus admittance matrix
- Include the admittance of all transmission lines, transformers, between lines, but exclude the admittance of the loads or generators themselves

Step 2

- Select a slack bus
- One of the buses in the power system should be chosen as the slack bus
- Its voltage will be assumed to be $1.0 \angle 0$

Step 3

- Select initial estimates for all buses voltages
- The estimate should be reasonable as poor choice may result in convergence to incorrect values (usually start with $1.0 \angle 0$ (flat start))

...Gauss-Seidel Power Flow Solution

Step 4

- Write voltage equation for every other bus in the system

$$V_i = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ (k \neq i)}}^N Y_{ik} V_k \right)$$

or

$$V_i = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^*} - \sum_{k=1}^{i-1} Y_{ik} V_k - \sum_{k=i+1}^N Y_{ik} V_k \right)$$

...Gauss-Seidel Power Flow Solution

Step 5

- Calculate and update estimate of the voltage at each load bus in sequence using the voltage equation

Step 6

- Compare the difference between the old voltage and the estimates
- If the difference between the estimates less than the specified tolerance for all buses, we are done

Step 7

- Check that the result is reasonable
- Typical value of phase range is less than 45°
- Larger ranges may indicate the system converged to incorrect solution
- Change the initial condition, try again.. 😊

...Gauss-Seidel Power Flow Solution

$$P_i^{(k+1)} = \Re \left\{ V_i^{*(k)} \left[V_i^{(k)} \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij} V_j^{(k)} \right] \right\} \quad j \neq i$$

$$Q_i^{(k+1)} = -\Im \left\{ V_i^{*(k)} \left[V_i^{(k)} \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij} V_j^{(k)} \right] \right\} \quad j \neq i$$

y_{ij} = actual admittance in pu

P_i^{sch} = net real power in pu

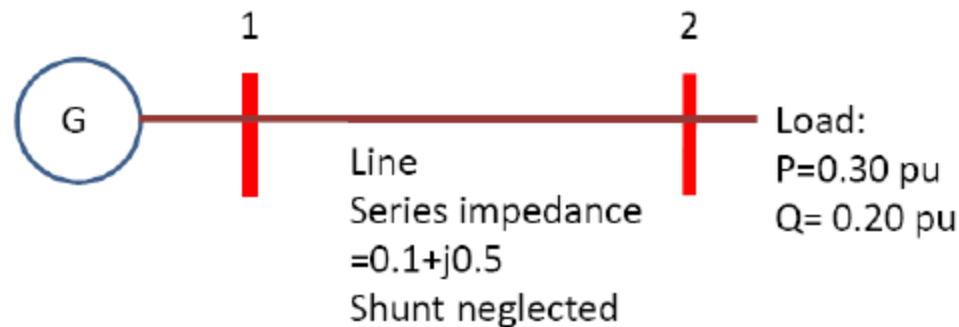
Q_i^{sch} = reactive power in pu

KCL : current entering bus i was assumed positive

- for P & Q are injected into the bus such as generator, P & Q have positive values
- for load buses where P & Q are flowing away from the bus, P & Q have negative values

Example 1

Determine the voltage at each bus for the specified load condition



Step 1: Calculate the bus admittance matrix

$$Y_{line} = \frac{1}{Z_{line}} = \frac{1}{0.10 + j0.50} = 0.3846 - j1.9231$$

$$Y_{11} = 0.3846 - j1.9231$$

$$Y_{22} = 0.3846 - j1.9231$$

$$Y_{12} = -0.3846 + j1.9231$$

$$Y_{21} = -0.3846 + j1.9231$$

$$Y_{bus} = \begin{bmatrix} 0.3846 - j1.9231 & -0.3846 + j1.9231 \\ -0.3846 + j1.9231 & 0.3846 - j1.9231 \end{bmatrix}$$

Solution

Step 2: Determine slack bus

Bus 1 : Slack bus.

$$V_1 = 1.0 \angle 0 \text{ pu}$$

Step 3: Select initial values of all bus voltages

Bus 2 is a Load bus

Choose $V_2 = 1.0 \angle 0 \text{ pu}$ as initial estimate

...Solution

Step 4: Write voltage equation for every other bus in the system

Voltage equation for bus 2

$$V_2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_{2,old}^*} - (Y_{21}V_1) \right]$$

Real and reactive power at Bus 2:

$$S_i = S_{iG} - S_{iL}$$

$$P_2 = -0.3 \text{ pu}; Q_2 = -0.2 \text{ pu}$$

$$V_2 = \frac{1}{0.3846 - j1.9231} \left[\frac{-0.30 + j0.2}{V_{2,old}^*} - [(-0.3846 + j1.9231)V_1] \right]$$

$$V_2 = \frac{1}{1.9612 \angle -78.8} \left[\frac{0.3603 \angle 146.3}{V_{2,old}^*} - [(1.9612 \angle 101.3)1 \angle 0] \right]$$

$$V_2^{k+1} = \frac{1}{1.9612 \angle -78.8} \left[\frac{0.3603 \angle 146.3}{V_2^{*k}} - [(1.9612 \angle 101.3)1 \angle 0] \right]$$

...Solution

Step 5: Calculate an updated estimate of the voltage at each load bus

Initial estimate $V_2^0 = 1.0 \angle 0$ pu

$$\begin{aligned} V_2^1 &= \frac{1}{1.9612 \angle -78.8} \left[\frac{0.3603 \angle 146.3}{V_2^{*0}} - [(1.9612 \angle 101.3) 1 \angle 0] \right] \\ &= \frac{1}{1.9612 \angle -78.8} \left[\frac{0.3603 \angle 146.3}{1 \angle 0} - [(1.9612 \angle 101.3) 1 \angle 0] \right] \\ &= ? \end{aligned}$$

Second estimate

$$\begin{aligned} V_2^2 &= \frac{1}{1.9612 \angle -78.8} \left[\frac{0.3603 \angle 146.3}{V_2^{*1}} - [(1.9612 \angle 101.3) 1 \angle 0] \right] \\ &= ? \end{aligned}$$

Third iteration

$$\begin{aligned} V_2^3 &= \frac{1}{1.9612 \angle -78.8} \left[\frac{0.3603 \angle 146.3}{V_2^{*2}} - [(1.9612 \angle 101.3) 1 \angle 0] \right] \\ &= ? \end{aligned}$$

...Solution

Step 6: Compare the difference between old and new estimates

If the magnitude of the voltage is barely changing, we consider this value is close enough to the correct answer. Iteration stops.

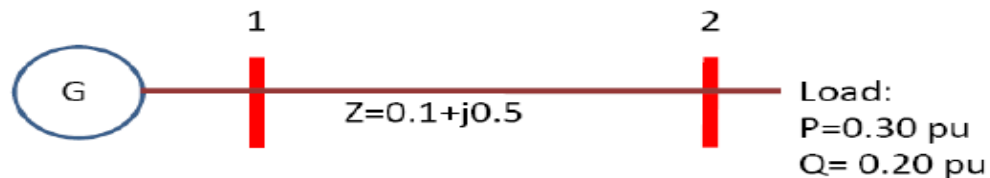
Step 7: Confirm that the solution is reasonable

$$V_1 = 1.0 \angle 0 \text{ pu}$$

$$V_2 = 0.8315 \angle -8.994 \text{ pu}$$

← assumed value for discussions

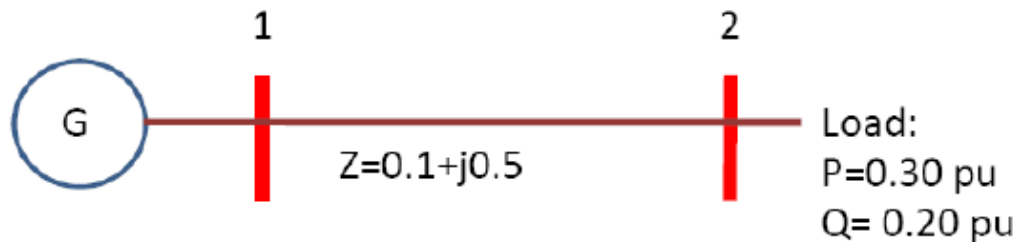
The phase angles differ by only 10° , these result appear reasonable



Calculate current flowing in the transmission line from Bus 1 to Bus 2

$$\begin{aligned} I_{1,2} &= \frac{(V_1 - V_2)}{Z_{Line}} \\ &= \frac{1 \angle 0 - 0.8315 \angle -8.994}{0.10 + j0.50} \\ &= 0.4333 \angle -42.65 \end{aligned}$$

Solution



$$V_1 = 1.0 \angle 0 \text{ pu}$$

$$I_{1,2} = 0.4333 \angle -42.65$$

$$V_2 = 0.8315 \angle -8.994 \text{ pu}$$

Calculate power supplied by the transmission line to Bus 2

$$S = VI^*$$

$$= (0.8315 \angle -8.994)(0.4333 \angle -42.65)^*$$

$$= 0.2999 + j0.1997$$

This is almost equal to the power being consumed by the loads. Thus the solution appears to be correct.

