Universiti Malaysia PAHANG
Engineering • Technology • Creativity

DUM 2413 STATISTICS & PROBABILITY

CHAPTER 3
PROBABILITY

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EXPECTED OUTCOMES

- Able to apply three different approaches in determining the probability of an event
- Able to determine the probability using basic probability rules and theory
- Able to solve the application problems using Bayes' theorem
- Able to determine the probability of an event using basic counting rules

CONTENT

3.1 Basic Idea and Consideration

3.2 Mutually Exclusive

3.3 Independent Event

3.4 Conditional Probability

3.5 Counting Rules and Probability

3.3
INDEPENDENCE EVENTS

3.2
CONDITIONAL PROBABILITY

THEORY OF PROBABILITY

THEORY OF PROBABILITY

- INDEPENDENCE PROBABILITY
  - INDEPENDENT
  - DEPENDENT
- CONDITIONAL PROBABILITY
  - BAYES'S THEOREM

Bridge the conditional probability to it inverses

INDEPENDENCE PROBABILITY

Two events, A and B are independent if and only if

$$P(\mathrm{A}\cap\mathrm{B}) = P(\mathrm{A}).P(\mathrm{B})$$

If two events are not independent. Therefore, they are dependent .

CONDITIONAL PROBABILITY

If A and B are any two events in a sample space S, and $P(\mathrm{A}) \neq 0$, therefore conditional probability of events B given A is

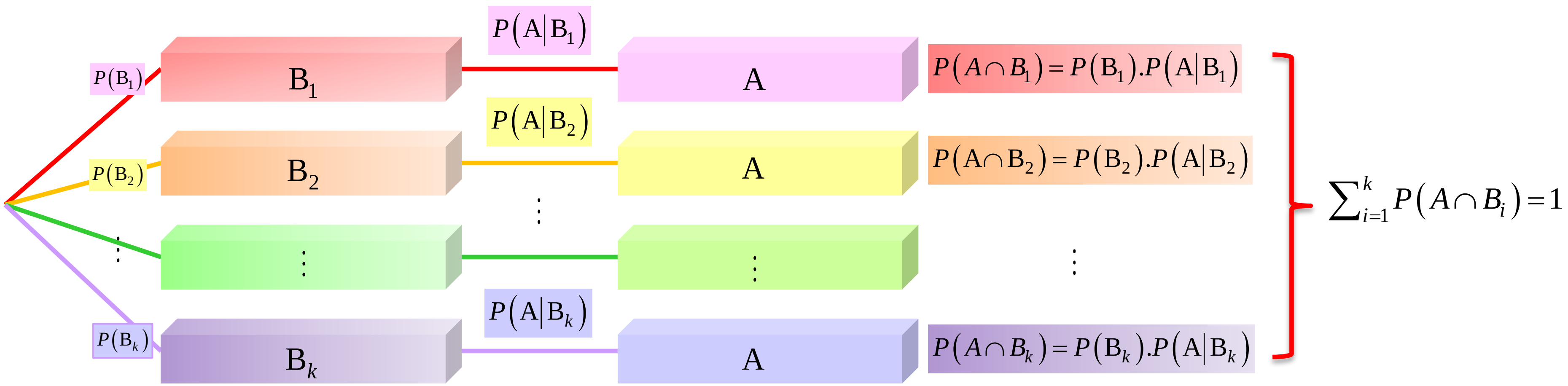
$$P(\mathrm{B}|\mathrm{A}) = \frac{P(\mathrm{A}\cap\mathrm{B})}{P(\mathrm{A})}$$

BAYES'S THEOREM

BAYES'S THEOREM

If $\mathrm{B}_1,\ \mathrm{B}_2,\ \ldots,\ \mathrm{B}_k$ **constitute a partition of the sample space** S **and** $P(\mathrm{B}_i) \neq 0$ **for** $i = 1, 2, \ldots, k,$ **then for any event** A **in** S **such that** $P(\mathrm{A}) \neq 0.$ **Therefore,**

$$P(\mathrm{B}_r|\mathrm{A}) = \frac{P(\mathrm{B}_r).P(\mathrm{A}|\mathrm{B}_i)}{\sum_{i=1}^{k} P(\mathrm{B}_i).P(\mathrm{A}|\mathrm{B}_i)}$$



Youtube Link (Bayes' Theorem): https://www.youtube.com/watch?v=pPTLK5hFGnQ

EXAMPLE 3.7

Given that A and B are independent events, which $P(\mathrm{A})=0.70$ and $P(\mathrm{B})=0.20.$ Find the probability that

(i) $P(\mathrm{A}\cap\mathrm{B})$

(ii) $P(\mathrm{A}\cup\mathrm{B})$

(iii) $P(\mathrm{A}'\cup\mathrm{B}')$

(i)

$$P(\mathrm{A}\cap\mathrm{B})=P(A)\cdot P(\mathrm{B})=0.70\cdot 0.20=0.14$$

(ii)

$$P(\mathrm{A}\cup\mathrm{B})=P(\mathrm{A})+P(\mathrm{B})-P(\mathrm{A}\cap\mathrm{B})=0.70+0.20-0.14=0.76$$

(iii)

$$\begin{aligned}P(\mathrm{A}'\cup\mathrm{B}')&=P(\mathrm{A}')+P(\mathrm{B}')-P(\mathrm{A}'\cap\mathrm{B}')\\&=(1-0.70)+(1-0.20)-(1-0.70)\cdot(1-0.20)\\&=0.86\end{aligned}$$

EXAMPLE 3.8

A two virtual 6-sided dice tossed twice in succession. Denote that A is the event that an even number comes up on the first toss, B is the event that an even number comes up on the second toss, and C is the event that both tosses result in the same number. Based on the given information, determine

(i) the pairwise independence among events A, B and C.

(ii) whether three events A, B, and C are independent.

SOLUTION

Second Roll						
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)

First Roll

Event C
{(1,1),(2,2),(3,3),(4,4),(5,5), (6,6)}

Event B
{(1,2),(2,4),(2,6),(2,2),(2,4),(2,6)
(3,2),(3,4),(3,6),(4,2),(4,4),(4,6)
(5,2),(5,4),(5,6),(6,2),(6,4),(6,6)}

Event A
{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

EXAMPLE 3.8-CONTINUE

$$P(\mathrm{A})=\frac{n(\mathrm{A})}{n(\mathrm{S})}=\frac{18}{36}=\frac{1}{2};\ P(\mathrm{B})=\frac{n(\mathrm{B})}{n(\mathrm{S})}=\frac{18}{36}=\frac{1}{2};\ P(\mathrm{C})=\frac{n(\mathrm{C})}{n(\mathrm{S})}=\frac{6}{36}=\frac{1}{6}$$

$$P(\mathrm{A}\cap\mathrm{B})=\frac{n(\mathrm{A}\cap\mathrm{B})}{n(\mathrm{S})}=\frac{9}{36}=\frac{1}{4};\ P(\mathrm{A}\cap\mathrm{C})=\frac{n(\mathrm{A}\cap\mathrm{C})}{n(\mathrm{S})}=\frac{3}{36}=\frac{1}{12};\ P(\mathrm{B}\cap\mathrm{C})=\frac{n(\mathrm{B}\cap\mathrm{C})}{n(\mathrm{S})}=\frac{3}{36}=\frac{1}{12}$$

$$\left(P(\mathrm{A}\cap\mathrm{B})=\frac{1}{4}\right)=\left(P(\mathrm{A})\cdot P(\mathrm{B})=\frac{1}{4}\right)$$

$\therefore$ A and B are indepedendent events.

$$\left(P(\mathrm{B}\cap\mathrm{C})=\frac{1}{12}\right)=\left(P(\mathrm{B})\cdot P(\mathrm{C})=\frac{1}{12}\right)$$

$\therefore$ A and C are indepedendent events.

$$\left(P(\mathrm{A}\cap\mathrm{C})=\frac{1}{12}\right)=\left(P(\mathrm{A})\cdot P(\mathrm{C})=\frac{1}{12}\right)$$

$\therefore$ A and C are indepedendent events.

(i)

$$P(\mathrm{A}\cap\mathrm{B}\cap\mathrm{C})=\frac{n(\mathrm{A}\cap\mathrm{B}\cap\mathrm{C})}{n(S)}=\frac{3}{36}=\frac{1}{12}$$

$$\left(P(\mathrm{A}\cap\mathrm{B}\cap\mathrm{C})=\frac{1}{12}\right)\neq\left(P(\mathrm{A})\cdot P(\mathrm{B})\cdot P(\mathrm{C})=\frac{1}{24}\right)$$

$\therefore$ A, B, C are depedent events.

(ii)

EXAMPLE 3.9

A consumer organization Malaysia conducted a surveyed for 50 new car dealers about the services under warranty. The collected data are summarized as table below.

Duration of business	Good service under warranty (G)	Poor service under warranty (G')
At least 10 years (T)	16	4
Less than 10 years (T')	10	20

(i) If a new car dealer is randomly selected, what is the probability that the dealer who provided a good service under warranty?

(ii) If a new car dealer who has been in business for at least 10 years, what is the probability that the dealer who provided good service under warranty?

(iii) What is the probability that a new car dealer who will be providing good service under warranty given the dealer has been in business less than 10 years?

EXAMPLE 3.9

SOLUTION

(i)

$$P(\mathrm{G})=\frac{16+10}{16+10+4+20}=\frac{26}{50}=\frac{13}{25}$$

(ii)

$$P(G\cap \mathrm{T})=\frac{16}{16+10+4+20}=\frac{8}{25};\ P(\mathrm{T})=\frac{16+4}{16+10+4+20}=\frac{2}{5}$$

$$P(\mathrm{G}|\mathrm{T})=\frac{P(G\cap \mathrm{T})}{P(\mathrm{T})}=\frac{4}{5}$$

(iii)

$$P(G\cap \mathrm{T}')=\frac{10}{16+10+4+20}=\frac{1}{5};\ P(\mathrm{T}')=\frac{10+20}{16+10+4+20}=\frac{3}{5}$$

$$P(\mathrm{G}|\mathrm{T}')=\frac{P(G\cap \mathrm{T}')}{P(\mathrm{T}')}=\frac{1}{3}$$

EXERCISE 3.6

(i) A rocket has a built-in redundant systems with three components, namely C1, C2 and C3. In this system, if the component C1 fails to function, the component C2 will be used. If component C2 fails and the component C3 will be used. Given that the probability of failure of any one of these components is 0.15 and the failures of these components are independent events, find the probability that the system does not fail.

(ii) The probability the occurrence of the operator injury due to an accident in a month is (0.01)m, where m is the number of days in the particular month. If the company's year starts in January, find the probability that the first accident is in April. Assume that there is 28 days in February and the numbers of accidents occurred is independently among the months.

(iii) Three football players attempt to kick a field goal from the 25-yard line. The probability of the player 1, player 2 and player 3 to score the field goal is P(player 1)=0.50, P(player 2)=0.70, and P(player 3)=0.60, respectively. Assume that the players scored for the field goal are mutually independent, find the probability that

(a) exactly one player will be scored the field goal.

(b) exactly two players will be scored the field goal.

EXERCISE 3.7-CONTINUE

SOLUTION

(i)

$$P\left(\left(A_1\cap A_2\cap A_3\right)'\right)=1-P\left(A_1\cap A_2\cap A_3\right)=1-\left(0.15\right)^3=0.9966$$

(ii)

$$\begin{aligned}P\left(J'\cap F'\cap M'\cap A\right)&=\left(1-P\left(J\right)\right)\cdot\left(1-P\left(F\right)\right)\cdot\left(1-P\left(M\right)\right)\cdot P\left(A\right)\\&=\left(1-\left(0.01\right)\left(31\right)\right)\cdot\left(1-\left(0.01\right)\left(28\right)\right)\cdot\left(1-\left(0.01\right)\left(31\right)\right)\cdot\left(\left(0.01\right)\left(30\right)\right)\\&=0.1028\end{aligned}$$

J:January; F:February; M:March; A:April

(iii)(a)

$$\begin{aligned}&P\left(A1\cap A2'\cap A3'\right)+P\left(A1'\cap A2\cap A3'\right)+P\left(A1'\cap A2'\cap A3\right)\\&=P\left(A1\right)P\left(A2'\right)P\left(A3'\right)+P\left(A1'\right)P\left(A2\right)P\left(A3'\right)+P\left(A1'\right)P\left(A2'\right)P\left(A3\right)\\&=\left(0.50\right)\left(0.30\right)\left(0.40\right)+\left(0.50\right)\left(0.70\right)\left(0.40\right)+\left(0.50\right)\left(0.30\right)\left(0.60\right)\\&=0.2900\end{aligned}$$

(iii)(b)

$$\begin{aligned}&P\left(A1\cap A2\cap A3'\right)+P\left(A1\cap A2'\cap A3\right)+P\left(A1'\cap A2\cap A3\right)\\&=P\left(A1\right)P\left(A2\right)P\left(A3'\right)+P\left(A1\right)P\left(A2'\right)P\left(A3\right)+P\left(A1'\right)P\left(A2\right)P\left(A3\right)\\&=\left(0.50\right)\left(0.70\right)\left(0.40\right)+\left(0.50\right)\left(0.70\right)\left(0.60\right)+\left(0.50\right)\left(0.30\right)\left(0.60\right)\\&=0.4400\end{aligned}$$

EXERCISE 3.8

A Ministry of Health of a particular country conducted a study on community health. The detailed results of the study obtained from 1000000 residents who are given a common AIDS test are illustrated as table below.

	B_1 Carry AIDS Virus	B_2 Do not Carry AIDS Virus	Total
A_1 Test Positive	4885	73630	78515
A_2 Test Negative	115	921370	921485
Total	5000	99500	1000000

If one of the residents is selected, find the following probabilities:

(i) $P(B_1)$

(ii) $P(A_1)$

(iii) $P(A_1|B_2)$

(iv) $P(B_1|A_1)$

SOLUTION

(i)

$$P(B_1)=\frac{n(B_1)}{n(S)}=\frac{5000}{1000000}=0.0050$$

(ii)

$$P(A_1)=\frac{n(A_1)}{n(S)}=\frac{78515}{1000000}=0.0785$$

(iii)

$$P(A_1|B_2)=\frac{n(A_1\cap B_2)/n(S)}{n(B_2)/n(S)}=\frac{73630}{99500}=0.7400$$

(iv)

$$P(B_1|A_1)=\frac{n(B_1\cap A_1)/n(S)}{n(A_1)/n(S)}=\frac{4885}{78515}=0.0622$$

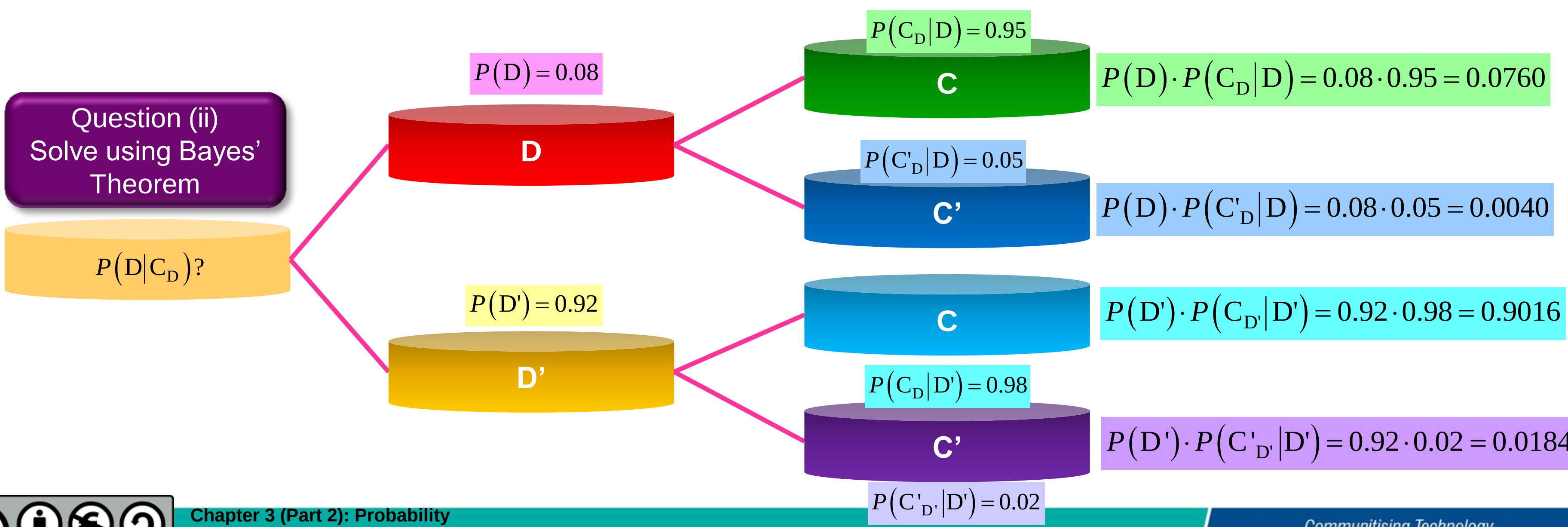
EXERCISE 3.8

Based on previous records, there are 8% of all adults over 50 have diabetes in a community. If a public health service officer in this community correctly diagnoses 95% of all adults over 50 with diabetes as having the disease, and incorrectly diagnoses 2% of all adults over 50 without diabetes as having the disease. Find the probability that

(i) the community health service will diagnose an adult over 50 as having the disease.

(ii) a person over 50 diagnosed by the health service as having the disease actually has the disease.

SOLUTION



EXERCISE 3.8-CONTINUE

(i)

$$P(\mathrm{C_D}) = P(\mathrm{D})P(\mathrm{C_D}|\mathrm{D}) + P(\mathrm{D'})P(\mathrm{C'_{D'}} \cap \mathrm{D'})$$

$$P(\mathrm{C_D}) = 0.0760 + 0.0184$$

$$P(\mathrm{C_D}) = 0.0944$$

(ii)

$$P(\mathrm{D}|\mathrm{C_D}) = \frac{P(\mathrm{D})P(\mathrm{C_D}|\mathrm{D})}{P(\mathrm{D})P(\mathrm{C_D}|\mathrm{D}) + P(\mathrm{D'})P(\mathrm{C'_{D'}}|\mathrm{D'})}$$

$$P(\mathrm{D}|\mathrm{C_D}) = \frac{0.0760}{0.0944}$$

$$P(\mathrm{D}|\mathrm{C_D}) = 0.8051$$

EXERCISE 3.9

Chong (C), Siti (S), and Murugan (M) are the three stock clerks in mail-order (O) house. One of their job scope is to pull items from shelves and assemble them for subsequent verification and packaging. Based on previous records, C, S and M can be processed 30%, 40% and 30% the mail-orders per day, respectively. Suppose that the chances of C, S and M makes a mistake in processing the daily mail-orders is one time in a hundred, five times in a hundred, and three times in a hundred, respectively. What are the probability

(i) a mistake will be made in an order?

(ii) the mail-order was filled by C given that a mistake is made in an order?

(iii) the mail-order was filled by S knowing that a mistake is made in a mail-order?

EXERCISE 3.9-CONTINUE

SOLUTION

C

- **O**: $P(\mathrm{O_C}|\mathrm{C}) = 0.01$ → $P(\mathrm{C}) \cdot P(\mathrm{O_C}|\mathrm{C}) = 0.0030$
- **O’**: $P(\mathrm{O_c}'|\mathrm{C}) = 0.99$ → $P(\mathrm{C}) \cdot P(\mathrm{O_C}'|\mathrm{C}) = 0.2970$

S

$P(\mathrm{S}) = 0.40$

- **O**: $P(\mathrm{O_S}|\mathrm{S}) = 0.05$ → $P(\mathrm{S}) \cdot P(\mathrm{O_S}|\mathrm{S}) = 0.0200$
- **O’**: $P(\mathrm{O_S}'|\mathrm{S}) = 0.95$ → $P(\mathrm{S}) \cdot P(\mathrm{O_S}'|\mathrm{S}) = 0.3800$

M

$P(\mathrm{M}) = 0.30$

- **O**: $P(\mathrm{O_M}|\mathrm{M}) = 0.03$ → $P(\mathrm{M}) \cdot P(\mathrm{O_M}|\mathrm{M}) = 0.0090$
- **O’**: $P(\mathrm{O_M}'|\mathrm{M}) = 0.97$ → $P(\mathrm{M}) \cdot P(\mathrm{O_M}'|\mathrm{M}) = 0.2910$

EXERCISE 3.9-CONTINUE

i

$$P(O) = P(\mathrm{C}) \cdot P(\mathrm{O_C}|\mathrm{C}) + P(\mathrm{S}) \cdot P(\mathrm{O_S}|\mathrm{S}) + P(\mathrm{M}) \cdot P(\mathrm{O_M}|\mathrm{M})$$
$$= 0.0030 + 0.0200 + 0.0090$$
$$= 0.0320$$

ii

$$P(\mathrm{C}|\mathrm{O}) = \frac{P(\mathrm{C}) \cdot P(\mathrm{O_C}|\mathrm{C})}{P(\mathrm{C})}$$
$$= \frac{0.0030}{0.0320}$$
$$= 0.09375$$

iii

$$P(\mathrm{S}|\mathrm{O}) = \frac{P(\mathrm{S}) \cdot P(\mathrm{O_S}|\mathrm{S})}{P(\mathrm{O})}$$
$$= \frac{0.0200}{0.0320}$$
$$= 0.6250$$

3.5
COUNTING RULES AND PROBABILITY

FUNDAMENTAL COUNTING RULES IN PROBABILITY

FUNDAMENTAL COUNTING RULES

Objective: To determine the number of outcomes associated with an event in a random experiment

PERMUTATION

The number of permutation of n distinct objects taken r at a time is

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

- Repetition is allowed
- Order matters

COMBINATION

The number of combination of n distinct objects taken r at a time is

$$^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

- Repetition is not allowed
- Order don't matters

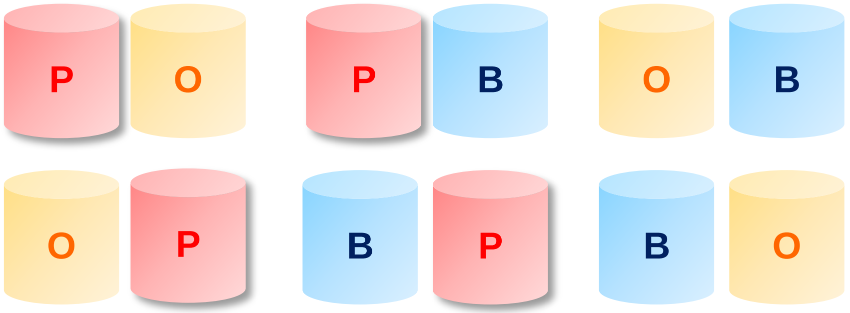
WHAT'S IS DIFFERENCE?

PERMUTATION vs. COMBINATION

Given that there are three difference colour of paint, including pink (P), orange (B) and blue (B).

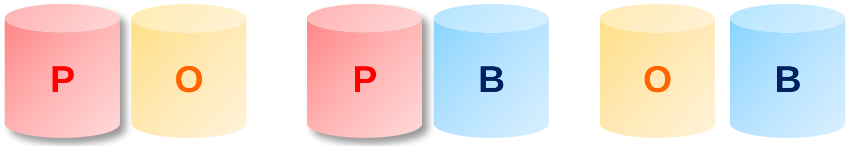
Selects 2 different colour of paint using permutation and combination rules, respectively.

PERMUTATION



$$^{n}P_{r} = {}^{3}P_{2} = 6 \text{ ways}$$

COMBINATION



$$\binom{n}{r} = {}^{n}C_{r} = {}^{3}C_{2} = 3 \text{ ways}$$

EXAMPLE 3.10

Given the following problem statements, identify whether the problem involved permutation or combination and explain the reason. Hence, solve the problem based on permutation or combination rules.

(i) How many groups of five songs can be selected from a list of 35 songs?

Combination; Reason: The order of the songs is not important; $^{35}C_5 = 324632$ ways

(ii) How many ways can a person chosen three different desserts from a dessert tray of 8 desserts.

Combination; Reason: The order of the desserts is not important; $^{8}C_3 = 56$ ways

(iii) How many ways are there to place seven distinct pieces of art in a row?

Permutation; Reason: The order of each piece of art is important; $^{7}P_7 = 7! = 5040$ ways

(iv) How many ways can the manager arrange all 10 TV sets if he is concerned about the order they were placed in?

Permutation; Reason: The order of each TV sets is important; $^{10}P_{10} = 10! = 3628800$ ways

EXAMPLE 3.11

(i) The manufacturer supplied ten computers to the computer shoppers in Kuantan, which include three defective computers. If a computer shopper purchase four of these computers, how many possible ways that he will receive at least two defective computers.

(ii) Dr. Roshini plan to have a Paris tour for five days four nights end of this year. She manages to book and stay at four different sojourn selected from among ten sojourn along her tour. How many different selections can be made by Dr. Roshini

(a) if the order of the sojourn is matters.

(b) if the order of the sojourn does not matters.

(iii) How many different bridge hands are possible containing five spades, three diamonds, three clubs, and two hearts?

(iv) Determine the number of selections can be made by a supplier who want to select two out of fifteen warehouse to ship an order from his customers.

(v) A manufacturer wants to solder three out of twenty integrated-circuit chips sorted on his table to become a larger electronic component. How many selections he can make?

EXAMPLE 3.11-CONTINUE

TWO DEFECTIVE	D	D	D'	D'
THREE DEFECTIVE	D	D	D	D'

SOLUTION

i

Number of ways:

$$= \left({}^{3}C_2 \cdot {}^{7}C_2\right) + \left({}^{3}C_3 \cdot {}^{7}C_1\right)$$

$$= 70 \text{ ways}$$

ii(a)

Number of ways:

$$= {}^{10}P_4$$

$$= 5040 \text{ ways}$$

ii(b)

Number of ways:

$$= {}^{10}C_4$$

$$= 210 \text{ ways}$$

iii

Number of ways:

$$= {}^{13}C_5 \cdot {}^{13}C_3 \cdot {}^{13}C_3 \cdot {}^{13}C_2$$

$$= 8211173256 \text{ ways}$$

iv

Number of ways:

$$= {}^{15}C_2$$

$$= 105 \text{ ways}$$

v

Number of ways:

$$= {}^{20}C_3 \cdot 3!$$

$$= 6840 \text{ ways}$$

EXERCISE 3.10

(i) Last week, Zulkhibri played 10 soccer games online. Based on these 10 games, how many possible ways that he will win in five games, four losses, and one tie?

(ii) Atikah want to form four-letter code words based on letters "HOPE", how many she can be formed if

(a) the letters may not be repeated.

(b) the letters may be repeated.

(iii) A public safety committee of 5 persons to be formed from 6 men and 4 women. How many possible committee can be formed

(a) when at least two women should be included.

(b) when at most two women should be included.

(iv) Based on 5 different consonants and 4 vowels, how many words can be formed using 3 consonants and 2 vowels by regardless the meaning of the words?

EXERCISE 3.10-CONTINUE

SOLUTION

i

Number of ways:

$$= {}^{10}C_5 \cdot {}^{5}C_4 \cdot {}^{1}C_1$$

$$= 1260 \text{ ways}$$

ii(a)

Number of ways:

$$= 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 24 \text{ ways}$$

ii(b)

Number of ways:

$$= 4 \cdot 4 \cdot 4 \cdot 4$$

$$= 256 \text{ ways}$$

iii(a)

Number of ways:

$$= \left({}^{4}C_2 \cdot {}^{6}C_3\right) + \left({}^{4}C_3 \cdot {}^{6}C_2\right) + \left({}^{4}C_4 \cdot {}^{6}C_1\right)$$

$$= 186 \text{ ways}$$

iii(b)

Number of ways:

$$= \left({}^{4}C_0 \cdot {}^{6}C_5\right) + \left({}^{4}C_1 \cdot {}^{6}C_4\right) + \left({}^{4}C_2 \cdot {}^{6}C_3\right)$$

$$= 186 \text{ ways}$$

iv

Number of ways:

$$= {}^{5}C_3 \cdot {}^{4}C_2 \cdot {}^{5}P_5$$

$$= 7200 \text{ ways}$$

EXAMPLE 3.12

FINDING PROBABILITY USING PERMUTATION AND COMBINATION

(i) There are 5 boys and 2 girls in a nursery class. If a teacher randomly selects 4 of them to form a group, what is the probability that the students in the group are all boys?

(ii) Azrin repeatedly flipped a fair coin by 7 times, what is the probability that he obtained exactly 4 tails?

(iii) An urn contains six purple colour balls and five red colour balls. A sample of four colour balls is randomly selected from the urn. What is the probability that the sample contains two purple and two red colour balls?

(iv) Yusoff has one Apple iPad, who want to set a 4-digit passcode to lock the device.
  (a) How many different 4-digit passcode can Yusoff set if he didn't interested to repeat the same digits?
  (b) If the 4-digit passcode are randomly selected without replacement, what is the probability that Yusoff will set the passcode as 1234?
  (c) Aiman is the best for Yusoff, who also has one Apple iPad. What is the probability that both of them will set a similar passcode, namely 1234?

EXAMPLE 3.12-CONTINUE

SOLUTION

i

$$\text{Probability} = \frac{n(A)}{n(S)} = \frac{{}^5C_4 \cdot {}^2C_0}{{}^7C_4} = \frac{1}{7}$$

ii

$$\text{Probability} = \frac{n(A)}{n(S)} = \frac{{}^7C_4 \cdot {}^3C_3}{2^7} = \frac{35}{128}$$

iii

$$\text{Probability} = \frac{n(A)}{n(S)} = \frac{{}^6C_2 \cdot {}^5C_2}{{}^{11}C_4} = \frac{5}{11}$$

iii(a)

Number of ways:

$$= \left({}^{10}C_4\right) \times 4!$$

$$= 5040 \text{ ways}$$

iii(b)

$$\text{Probability} = \frac{n(A)}{n(S)} = \frac{1 \cdot 1 \cdot 1 \cdot 1}{{}^{10}P_4} = \frac{1}{5040}$$

iv

$$\text{Probability} = \frac{n(A)}{n(S)} = \frac{1 \cdot 1 \cdot 1 \cdot 1}{{}^{10}P_4} \cdot \frac{1 \cdot 1 \cdot 1 \cdot 1}{{}^{10}P_4} = \frac{1}{25401600}$$

EXERCISE 3.11

(i) Section 1 of Statistics & Probability class consists of 15 male and 14 female students. During the tutorial class, the lecturers would like to random select 6 of them to demonstrate how they solve the exercises given last week on the whiteboard. What is the probability that all 6 of the students selected are females?

(ii) An international high school Kuantan planning to held the musical festival. There are 20 talented singers auditioning for this musical festival. However, the principle is looking for two talented singers who could sing a good duet.

(a) Azie and Mazni are two talented singers who could sing a good duet. What is the probability that both of them will be selected by the director?

(b) The principle also interested to choose a lead singer and a backup talented duet singer. What is the probability that Azie will be selected as the lead singer while Mazni selected as the backup singer?

(iii) Nine modern-style pool balls with a number from 1 to 9 are put into a basket. Mr. Hafiz has randomly picked three balls from the basket without replacement in order to form a 3-digit number.

(a) How many possible 3-digit numbers can Mr. Hafiz forms?

(b) How many 3-digit numbers can Mr. Hafiz forms, which start with the digit 1?

(c) What is the probability that the 3-digit formed less than 200?

EXERCISE 3.11-CONTINUE

SOLUTION

i

$$\text{Probability} = \frac{n(A)}{n(S)} = \frac{{}^{14}C_6 \cdot {}^{15}C_0}{{}^{29}C_6} = \frac{11}{1740}$$

ii(a)

$$\text{Probability} = \frac{n(A)}{n(S)} = \frac{1 \cdot 1}{{}^{20}C_2} = \frac{1}{190}$$

ii(b)

$$\text{Probability} = \frac{n(A)}{n(S)} = \frac{1 \cdot 1}{{}^{20}P_2} = \frac{1}{380}$$

iii(a)

Number of ways:

$$= 9 \cdot 8 \cdot 7 = {}^{9}P_3$$

$$= 504 \text{ ways}$$

iii(b)

Number of ways:

$$= 1 \cdot 8 \cdot 7 = {}^{8}P_2$$

$$= 56 \text{ ways}$$

iii(c)

$$\text{Probability} = \frac{n(A)}{n(S)} = \frac{1 \cdot 8 \cdot 7}{9 \cdot 8 \cdot 7} = \frac{{}^{8}P_2}{{}^{9}P_3} = \frac{1}{9}$$

THANK YOU

END OF CHAPTER 3 (PART 2)