Universiti Malaysia PAHANG
Engineering • Technology • Creativity

DUM 2413 STATISTICS & PROBABILITY

CHAPTER 3
PROBABILITY

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EXPECTED OUTCOMES

- Able to apply three different approaches in determining the probability of an event
- Able to determine the probability using basic probability rules and theory
- Able to solve the application problems using Bayes' theorem
- Able to determine the probability of an event using basic counting rules

CONTENT

3.1
BASIC IDEA AND CONSIDERATION

3.2
MUTUALLY EXCLUSIVE

BASIC OF PROBABILITY

EXPERIMENT

- Any activity with an observable result or outcome

OUTCOME (SAMPLE POINT)

- The results obtained from an experiment

SAMPLE SPACE

- The set of all possible outcomes or sample points of an experiment.

EVENT

- An event is a subset of the sample space.
  - Simple Event : An outcomes or an event that cannot be further broken down into simpler components.
  - Compound Event : Any event combining two or more simple event.

THREE DIFFERENT APPROCHES TO FINDING THE PROBABILITY OF AND EVENT

PROBABILITY

Definition: The measure of the likelihood that the event will occur as a result of an experiment

APPROCH

Classical Probability

The probability obtained from an experiment, which contains n simple events A that are having an equally likely to happen

Empirical Probability

The proportion of the number of outcomes for a specified event A to the number of trials in an experiment

Subjective Probability

The probability of an event A, which estimated based on the knowledge of the relevant conditions

Classical Probability

$$P(\mathrm{A}) = \frac{\text{number of ways A can occur}}{\text{number of different simple event}}$$

Empirical Probability

$$P(\mathrm{A}) = \frac{\text{number of times event A occured}}{\text{number of times the experiment was repeated}}$$

EXAMPLE 3.1
CLASSICAL PROBABILITY

Identify the possible outcomes, sample space and probability of the event for the given experiment below.

1. **Miss Ezzatul spun an equal 4 sectors spinner with coloured red , yellow, green and blue, respectively. What is the probability of landing on each colour?**

SOLUTION

Outcomes: red, yellow, green, blue

Sample space: {red, yellow, green, blue}

Event : {red}; {yellow}; {green}; {blue}

$$P(\text{red})=\frac{1}{4};\ P(\text{yellow})=\frac{1}{4};\ P(\text{green})=\frac{1}{4};P(\text{blue})=\frac{1}{4}$$

2. **Mr. Adam rolled a single virtual 6-sided dice.**
   (i) What is the probability of rolling a prime number?
   (ii) What is the probability of rolling not a prime number?

SOLUTION

Outcomes: 1, 2, 3, 4, 5, 6

Sample space : {1, 2, 3, 4, 5, 6}

i

Let A is the event of rolling a prime number.
Therefore, A : {2, 3, 5}

$$P(\text{A})=\frac{n(\text{A})}{n(\text{S})}=\frac{3}{6}=\frac{1}{2}$$

ii

Let B is the event of rolling not a prime number.
Therefore, B : {1, 3, 5}

$$P(\text{B})=\frac{n(\text{B})}{n(\text{S})}=\frac{3}{6}=\frac{1}{2}$$

EXAMPLE 3.1-CONTINUE

3. A box contains 6 red, 3 yellow, 5 green, and 8 blue balls. Rahman is randomly drawn a single ball from the box, what is the probability that he draws

(i) a red ball?
(ii) a green ball?
(iii) a blue ball?
(iv) a yellow ball?

SOLUTION

Outcomes: red, yellow, green, blue

Sample space : {red, yellow, green, blue}

i. Let A is the event of drawing a red ball

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{6+5+8+3} = \frac{3}{11}$$

ii. Let B is the event of drawing a green ball

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{6+5+8+3} = \frac{5}{22}$$

iii. Let C is the event of drawing a blue ball

$$P(C) = \frac{n(C)}{n(S)} = \frac{8}{6+5+8+3} = \frac{4}{11}$$

iv. Let D is the event of drawing a yellow ball

$$P(D) = \frac{n(D)}{n(S)} = \frac{3}{6+5+8+3} = \frac{3}{22}$$

EXAMPLE 3.2
EMPIRICAL PROBABILITY

1. **Eight coins are tossed simultaneously and this procedure has been repeated for 270 times. The table below shows the frequencies of the number of tails are appearing.**

Number of tails	0	1	2	3	4	5	6	7	8
Frequency	2	15	29	57	70	59	20	10	8

Calculate the probability of occurred tailed.

(i) Equal to five.

(ii) Less than four.

(iii) More than six.

SOLUTION

i

$n(A) = 59;$

$n(S) = 2 + 15 + 29 + 57 + 70 + 59 + 20 + 10 + 8 = 270$

$P(A) = \frac{59}{270} = 0.2185$

iii

$n(A) = 10 + 8 = 18;$

$n(S) = 2 + 15 + 29 + 57 + 70 + 59 + 20 + 10 + 8 = 270$

$P(A) = \frac{18}{270} = 0.0667$

ii

$n(A) = 2 + 15 + 29 + 57 = 103;$

$n(S) = 2 + 15 + 29 + 57 + 70 + 59 + 20 + 10 + 8 = 270$

$P(A) = \frac{103}{270} = 0.3815$

EXAMPLE 3.2-CONTINUE

2. The table below illustrates the distribution marks obtained by 1200 students in a particular examination.

Marks	0-19	20-39	40-59	60-79	80-100
Number of students	63	142	500	320	175

If one student is randomly selected, find the probability he marks is

(i) Below 40.

(ii) Above 80.

(iii) Between 40 and 100.

SOLUTION

i

$$n(\text{A}) = 63 + 142 = 205;$$
$$n(\text{S}) = 1200$$
$$P(\text{A}) = \frac{205}{1200} = 0.1708$$

ii

$$n(\text{A}) = 175;$$
$$n(\text{S}) = 1200$$
$$P(\text{A}) = \frac{175}{1200} = 0.1458$$

iii

$$n(\text{A}) = 500 + 320 + 175 = 995;$$
$$n(\text{S}) = 1200$$
$$P(\text{A}) = \frac{995}{1200} = 0.8292$$

EXERCISES 3.1

Miss Laila rolled the two virtual 6-sided dice simultaneously. Find the probability that she will obtain

(i) the total sum of both dice is 6.
(ii) the one dice being twice the value of another dice.
(iii) the total sum of both dice is greater than 9.

SOLUTION

(6,1) 6+1=7	(6,2) 6+2=8	(6,3) 6+3=9	(6,4) 6+4=10	(6,5) 6+5=11	(6,6) 6+6=12
(5,1) 5+1=6	(5,2) 5+2=7	(5,3) 5+3=8	(5,4) 5+4=9	(5,5) 5+5=10	(5,6) 5+6=11
(4,1) 4+1=5	(4,2) 4+2=6	(4,3) 4+3=7	(4,4) 4+4=8	(4,5) 4+5=9	(4,6) 4+6=10
(3,1) 3+1=4	(3,2) 3+2=5	(3,3) 3+3=6	(3,4) 3+4=7	(3,5) 3+5=8	(3,6) 3+6=9
(2,1) 2+1=3	(2,2) 2+2=4	(2,3) 2+3=5	(2,4) 2+4=6	(2,5) 2+5=7	(2,6) 2+6=8
(1,1) 1+1=2	(1,2) 1+2=3	(1,3) 1+3=4	(1,4) 1+4=5	(1,5) 1+5=6	(1,6) 1+6=7

i

$$P(\mathrm{A})=\frac{n(\mathrm{A})}{n(\mathrm{S})}=\frac{5}{36}$$

ii

$$P(\mathrm{A})=\frac{n(\mathrm{A})}{n(\mathrm{S})}=\frac{6}{36}=\frac{1}{6}$$

iii

$$P(\mathrm{A})=\frac{n(\mathrm{A})}{n(\mathrm{S})}=\frac{6}{36}=\frac{1}{6}$$

EXERCISES 3.2

According to the hospital records at Kuantan last month, it found that 127 maternity patients stayed in the hospital for the number of days shown in the table below.

Number of days stayed	Frequency
3	15
4	32
5	56
6	19
7	5

If an administration officer randomly selected a record of the patient, find the probabilities that the patients who

(i) stayed exactly 5 days.

(ii) stayed less than 6 days.

(iii) stayed at most 4 days.

(iv) stayed at least 5 days.

EXERCISES 3.2-CONTINUE

(i)

$$P(A)=\frac{n(A)}{n(S)}=\frac{56}{127}$$

(ii)

$$P(A)=\frac{n(A)}{n(S)}=\frac{15+32+56}{127}=\frac{103}{127}$$

(iii)

$$P(A)=\frac{n(A)}{n(S)}=\frac{15+32}{127}=\frac{47}{127}$$

(iv)

$$P(A)=\frac{n(A)}{n(S)}=\frac{56+19+5}{127}=\frac{80}{127}$$

3 AXIOMS (ASSUMPTIONS)

&

SOME IMPORTANT BASIC RULES OF PROBABILILTY

KOLMOGOTOV AXIOMS (ASSUMPTIONS) OF PROBABILITY

AXIOMS OF PROBABILITY

AXIOM 1

The probability of an event is nonnegative real number; that is, $P(\mathrm{A}) \geq 0$ for any subset A of S.

AXIOM 2

$P(\mathrm{S}) = 1$

AXIOM 3

If $\mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3, \ldots$, is finite or infinite sequence of mutually exclusive events of S, then

$$P(\mathrm{A}_1 \cup \mathrm{A}_2 \cup \mathrm{A}_3 \cup \ldots) = P(\mathrm{A}_1) + P(\mathrm{A}_2) + P(\mathrm{A}_3) + \ldots$$

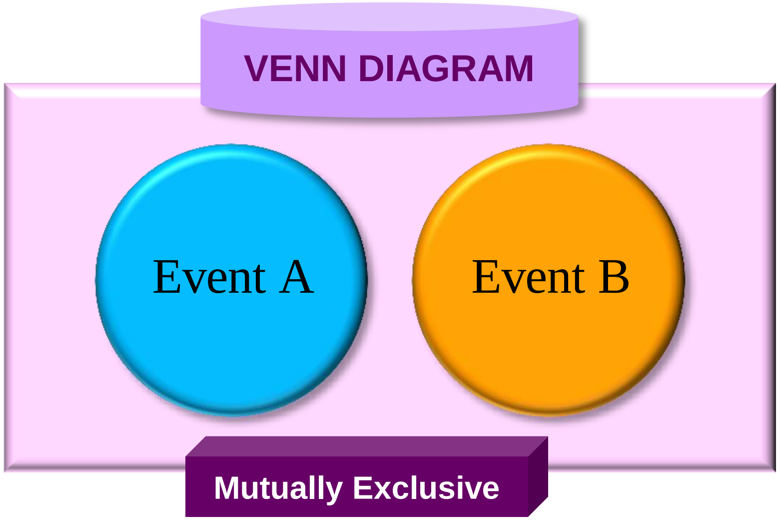
A is an event A

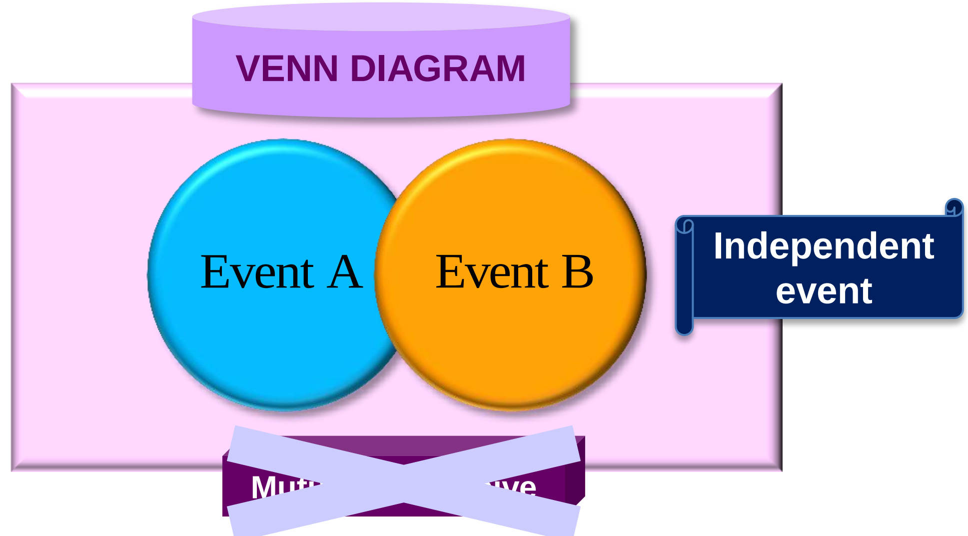
$P(\mathrm{A})$ is probability of event A

S is discrete sample space

What is "mutually exclusive"?

MUTUALLY EXCLUSIVE





Two events A and B are said to be **mutually exclusive** if (when the experiment is performed a single time) the occurrence of one of the events **excludes the possibility of the occurrence** of the other events

If two events A and B are **mutually exclusive**, the **probability of the intersection** of A and B

$$P(A \cap B) = \varnothing$$

EXAMPLE 3.3

An experiment has five possible outcomes, namely *A*1, *A*2, *A*3, *A*4, and *A*5. Given that these five possible outcomes are mutually exclusive, determine whether the following assignments of probabilities are permissible and give a reason.

(i) $P(A1)=0.20;\ P(A2)=0.20;\ P(A3)=0.20;\ P(A4)=0.20;\ P(A5)=0.20.$

Permissible. **Reason:** $P(S)=1$

(ii) $P(A1)=0.21;\ P(A2)=0.26;\ P(A3)=0.58;\ P(A4)=0.01;\ P(A5)=0.06.$

Not Permissible **Reason:** $P(S)>1$

(iii) $P(A1)=0.18;\ P(A2)=0.19;\ P(A3)=0.20;\ P(A4)=0.21;\ P(A5)=0.22.$

Permissible. **Reason:** $P(S)=1$

(iv) $P(A1)=0.10;\ P(A2)=0.30;\ P(A3)=0.10;\ P(A4)=0.06;\ P(A5)=-0.10.$

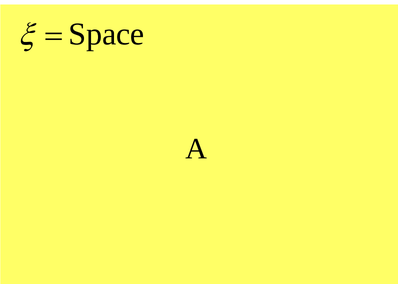
Not Permissible. **Reason:** $P(E)=-0.10$

(v) $P(A1)=0.23;\ P(A2)=0.12;\ P(A3)=0.05;\ P(A4)=0.50;\ P(A5)=0.08.$

Not Permissible. **Reason:** $P(S)<1$

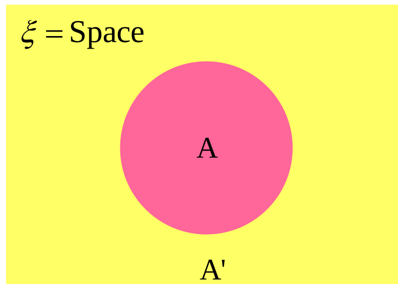
REVIEWS OF VENN DIAGRAM

Set Theory



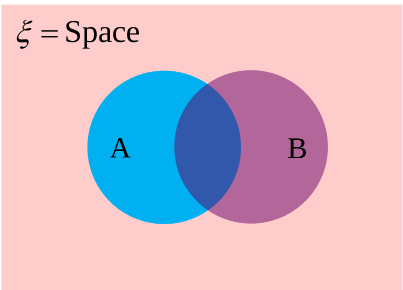
Set A

Set Theory



A' is complement of set A

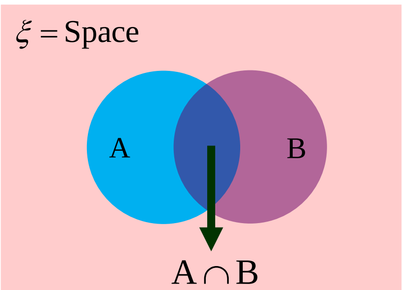
Set Theory



$A \cup B$ is

Set A union Set B

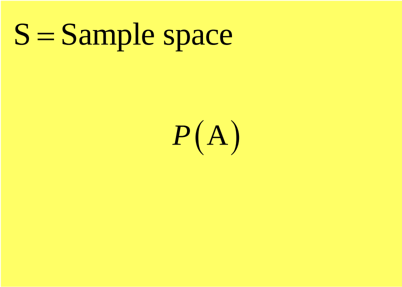
Set Theory



$A \cap B$ is

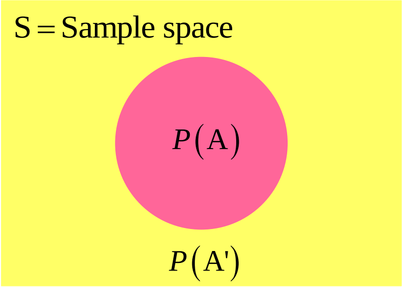
Set A intersect Set B

Probability Theory



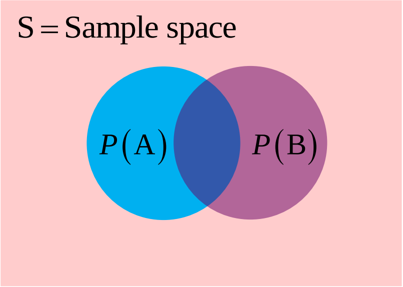
Probability of event A

Probability Theory



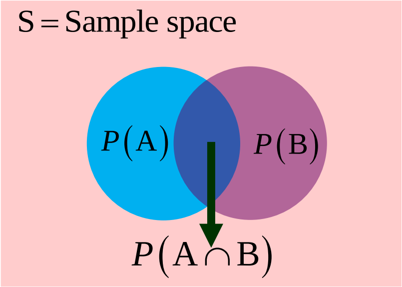
$P(A')$ is probability of complement of event A

Probability Theory



$P(A \cup B)$ is probability of event A union event B

Probability Theory



$P(A \cap B)$ is probability of event A intersect event B

Keywords: "OR"

Keywords: "AND"

EXAMPLE 3.4

If S={1, 2, 3, 4, 5, 6, 7, 8, 9}, A={1, 3, 5, 7}, B={6, 7, 8, 9}, C={2, 4, 8}, and D={1, 5, 9}, list the elements of the subsets of S corresponding to the following events.

(i) $A'\cap B$; **(ii)** $(A'\cap B)\cap C$; **(iii)** $B'\cup C$; **(iv)** $(B'\cup C)\cap D$; **(iv)** $A'\cap C$; **(v)** $(A'\cap C)\cap D$;

$A'=\{2,4,6,8,9,10\}$ $B=\{6,7,8,9\}$

$A'\cap B=\{6,8,9\}$

$A'\cap B=\{6,8,9\}$ $C=\{2,4,8\}$

$(A'\cap B)\cap C=\{8\}$

$B'=\{1,2,3,4,5\}$ $C=\{2,4,8\}$

$B'\cup C=\{1,2,3,4,5,8\}$

$B'\cup C=\{1,2,3,4,5\}$ $D=\{1,5,9\}$

$(B'\cup C)\cap D=\{1,5\}$

$A'=\{2,4,6,8,9\}$ $C=\{2,4,8\}$

$A'\cap C=\{2,4,8\}$

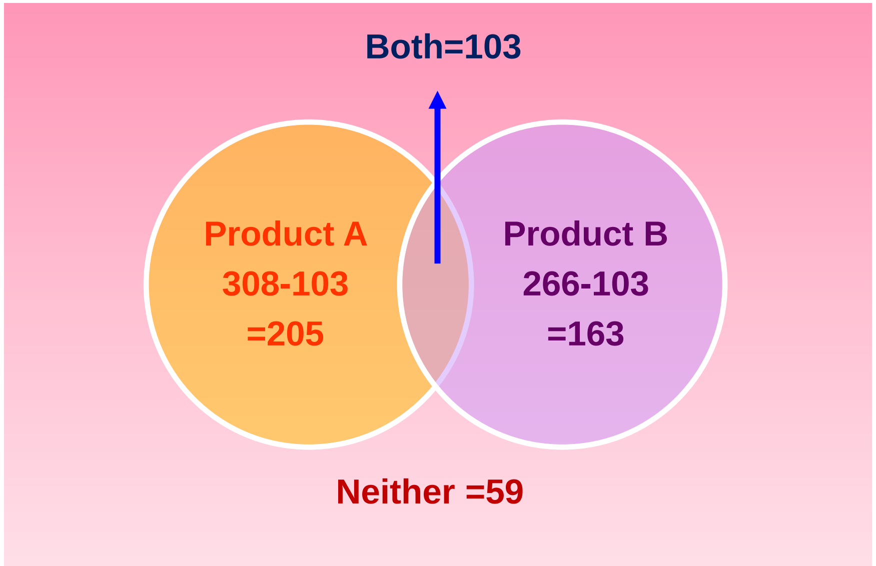
$A'\cap C=\{2,4,8\}$ $D=\{1,5,9\}$

$(A'\cap C)\cap D=\varnothing$

EXERCISES 3.3

A marketing analyst claimed that among 700 consumers, 310 consumers will regularly purchase the Product A, 280 consumers will regularly purchase Product B, 120 consumers regularly buy both products and the rest of consumers buy neither on a regular basis. Based on the given information and using the Venn diagram, verify whether the results of this study should be questioned and state the reason.

SOLUTION



Based on the Venn diagram, the total shoppers 205+103+163+59=530

Since the total shoppers based on Venn diagram are greater than 500 as claimed by researcher, therefore this study should be questioned.

THE BASIC RULES OF PROBABILITY

THE RULES OF PROBABILITY

- SUBSTRATION RULE (COMPLEMENTARY)
- ADDITION RULE
- MULTIPLICATION RULE

SUBSTRATION RULES

If A and A' are complementary events in a sample space S, then

$$P(\mathrm{A}) = 1 - P(\mathrm{A}')$$

ADDITION RULES

If A and B are any two events in a sample S, then

$$P(\mathrm{A} \cup \mathrm{B}) = P(\mathrm{A}) + P(\mathrm{B}) - P(\mathrm{A} \cap \mathrm{B})$$

Special Case

If A and B are two mutually events in a sample space S, then

$$P(\mathrm{A} \cup \mathrm{B}) = P(\mathrm{A}) + P(\mathrm{B})$$

This is due to $P(\mathrm{A} \cap \mathrm{B}) = \varnothing$

MULTIPLICATION RULES

If A and B are any two events in a sample space S, and $P(\mathrm{A}) \neq 0$. Then,

$$P(\mathrm{A} \cap \mathrm{B}) = P(\mathrm{A}) \cdot P(\mathrm{B} | \mathrm{A})$$

EXAMPLE 3.5

The probabilities that the supportability of a new cutting machine will be rated very difficult, difficult, average, easy or very easy are 0.12, 0.17, 0.34, 0.29 or 0.08, respectively. Find the probabilities that the supportability of this machine will be rated

(i) Difficult or very difficult.
(ii) Neither very difficult nor very easy.
(iii) Average, difficult or very difficult.
(iv) Average, easy or very easy.

SOLUTION

Let
P(Very difficult)=P(A)=0.12; P(Difficult)=P(B)=0.17;
P(Average)=P(C)=0.34;
P(Easy)=P(D)=0.29; P(Vey Easy)=P(E)=0.08.

i

$$P(\mathrm{A}\cup\mathrm{B}) = P(\mathrm{A}) + P(\mathrm{B})$$
$$= 0.12 + 0.17$$
$$= 0.29$$

ii

$$P(\mathrm{A}\cup\mathrm{E})' = 1 - P(\mathrm{A}\cup\mathrm{E}) = 1 - \left(P(\mathrm{A}) + P(\mathrm{E})\right)$$
$$= 1 - (0.12 + 0.08)$$
$$= 0.80$$

iii

$$P(\mathrm{A}\cup\mathrm{B}\cup\mathrm{C}) = P(\mathrm{A}) + P(\mathrm{B}) + P(\mathrm{C})$$
$$= 0.12 + 0.17 + 0.34$$
$$= 0.63$$

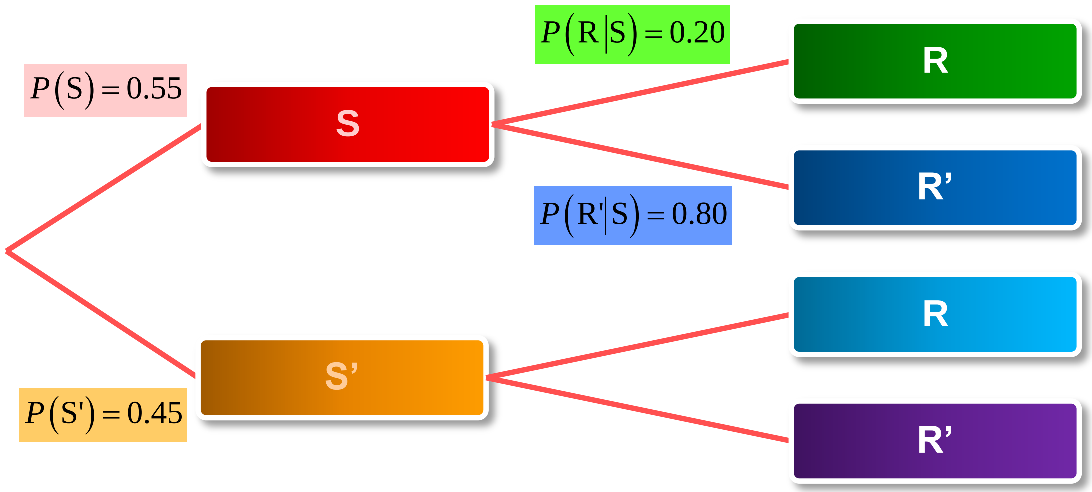
iv

$$P(\mathrm{C}\cup\mathrm{D}\cup\mathrm{E}) = P(\mathrm{C}) + P(\mathrm{D}) + P(\mathrm{E})$$
$$= 0.34 + 0.29 + 0.08$$
$$= 0.71$$

EXAMPLE 3.6

According to a surgical specialist, the probability of a patient surviving after a heart transplant surgery is 0.55. If a patient survives after the surgery, the probability that the body reject the transplant within a month is 0.20. What is the probability that the patient is surviving and do not reject the transplant?

SOLUTION



$$P(S \cap R') = P(S) \cdot P(R'|S)$$
$$= 0.55 \cdot 0.80$$
$$= 0.44$$

EXERCISES 3.4

Abu required to replace a new tyres for his private car. The probabilities that he replaced the tyres with brand of Dunlop, Continental, Goodyear, Maxxis, Bridgestone or Michelin are 0.15, 0.24, 0.03, 0.28, 0.22 and 0.08, respectively. Find the probabilities that he will replace the tyres with brands of

(i) Continental or Bridgestone tyres.

(ii) Dunlop, Goodyear, or Bridgestone tyres.

(iii) Goodyear or Michelin tyres.

(iv) Dunlop, Goodyear, Maxxis, or Bridgestone tyres.

SOLUTION

Let
P(Dunlop)=P(A)=0.15; P(Continental)=P(B)=0.24;
P(Goodyear)=P(C)=0.03; P(Maxxis)=P(D)=0.28;
P(Bridgestone)=P(E)=0.22; P(Michelin)=P(F)=0.08.

i

$$P(\mathrm{B}\cup\mathrm{E})=P(\mathrm{B})+P(\mathrm{E})$$
$$=0.24+0.22$$
$$=0.46$$

ii

$$P(\mathrm{A}\cup\mathrm{C}\cup\mathrm{E})=P(\mathrm{A})+P(\mathrm{C})+P(\mathrm{E})$$
$$=0.15+0.03+0.22$$
$$=0.40$$

iii

$$P(\mathrm{C}\cup\mathrm{F})=P(\mathrm{C})+P(\mathrm{F})$$
$$=0.03+0.08$$
$$=0.11$$

iv

$$P(\mathrm{A}\cup\mathrm{C}\cup\mathrm{D}\cup\mathrm{E})=P(\mathrm{A})+P(\mathrm{C})+P(\mathrm{D})+P(\mathrm{E})$$
$$=0.15+0.03+0.28+0.22$$
$$=0.68$$

EXERCISE 3.5

A container contains 20 red colour balls label with numbers from 1 through 20, 10 orange colour balls label with a number from 1 to 10, 40 green colour balls label a with number from 1 through 40, and 10 blue colour balls label with a number from 1 through 10. If Chuan drawn a colour ball from the container, find the probability that the ball he draw is

- **(i) blue or red colour.**
- **(ii) with a number 1, 2, 3, 4, or 5.**
- **(iii) orange or green colour and label with a number 1, 2, 3, or 4.**
- **(iv) with a number of 5, 15, 25, or 35.**
- **(v) red colour and with a number higher than 12 or green colour and with the number higher than 26.**

EXERCISE 3.5-CONTINUE

SOLUTION

(i)

$$P(\text{blue}\cup\text{red})=\frac{n(\text{blue}\cup\text{red})}{n(S)}=\frac{10+20}{80}=\frac{3}{8}$$

(ii)

$$P(\text{A}_{\text{red}}\cup\text{A}_{\text{orange}}\cup\text{A}_{\text{green}}\cup\text{A}_{\text{blue}})=\frac{n(\text{A}_{\text{red}}\cup\text{A}_{\text{orange}}\cup\text{A}_{\text{green}}\cup\text{A}_{\text{blue}})}{n(S)}=\frac{5+5+5+5}{80}=\frac{1}{4}$$

(iii)

$$P(\text{B}_{\text{orange}}\cup\text{B}_{\text{green}})=\frac{n(\text{B}_{\text{orange}}\cup\text{B}_{\text{green}})}{n(S)}=\frac{4+4}{80}=\frac{1}{10}$$

(iv)

$$P(\text{C}_{\text{red}}\cup\text{C}_{\text{orange}}\cup\text{C}_{\text{green}}\cup\text{C}_{\text{blue}})=\frac{n(\text{C}_{\text{red}}\cup\text{C}_{\text{orange}}\cup\text{C}_{\text{green}}\cup\text{C}_{\text{blue}})}{n(S)}=\frac{2+1+4+1}{80}=\frac{1}{10}$$

(v)

$$P(\text{D}_{\text{red}}\cup\text{E}_{\text{green}})=\frac{n(\text{D}_{\text{red}}\cup\text{E}_{\text{green}})}{n(S)}=\frac{8+14}{80}=\frac{11}{40}$$

THANK YOU

END OF CHAPTER 3 (PART 1)