FACULTY OF INDUSTRIAL SCIENCES & TECHNOLOGY
FINAL EXAMINATION

COURSE	**:**	**CALCULUS**
COURSE CODE	**:**	**DUM1123**
LECTURER	**:**	**NORHAFIZAH BINTI MD SARIF** **NADIRAH BINTI MOHD NASIR** **INTAN SABARIAH BINTI SABRI**
DATE	**:**	**29 DECEMBER 2014**
DURATION	**:**	**3 HOURS**
SESSION/SEMESTER	**:**	**SESSION 2014/2015 SEMESTER I**
PROGRAMME CODE	**:**	**DEE/DMM/DAA/DCS/DKK/DSH**

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of **FIVE (5)** questions. Answer **ALL QUESTIONS**.
2. All the calculations and assumptions must be clearly stated.
3. Your final answers must in **FOUR (4) decimal places** (if any).
4. Candidates are not allowed to bring any material other than those allowed by the invigilator into the examination room.

EXAMINATION REQUIREMENTS

1. Scientific calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **NINE (9)** printed pages including front page.

QUESTION 1

(a) Use numerical method to make a conjecture about the value of

$$\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}$$

(5 Marks)

(b) Evaluate the following

(i) $\lim_{x\to -1}\dfrac{2x^2-4x+1}{5x^3+7}$

(ii) $\lim_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1}$

(iii) $\lim_{x\to \infty}\dfrac{9x^2+2x-1}{x+5}$

(11 Marks)

(c) A function f is given as

$$f(x)=\frac{x^2-x-12}{x+3}$$

(i) Find $f(-3)$

(ii) Find $\lim_{x\to -3} f\left(x\right)$.

(iii) Does $f\left(x\right)$ continuous at $x=-3$? Give a reason to your answer.

(7 Marks)

QUESTION 2

(a) Given two parametric equations

$$y = \frac{t}{1+t} \text{ and } x = (1+t)^{-2}.$$

Find $\frac{dy}{dx}$.

(7 Marks)

(b) If $y = 4x^2 + \frac{2}{x^3}$, show that $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 6y = 0$.

(6 Marks)

(c) Find $\frac{dy}{dx}$ for the implicit function $e^{-3x} - 3xy^2 = y^3$.

(6 Marks)

QUESTION 3

(a) A curve has equation

$$2x^2 + 7y - 3 = 0 .$$

Find slope for the tangent line at point $(1,4)$.

(4 Marks)

(b) The volume of a spherical balloon is increasing at a constant rate of $5\text{m}^3/\text{s}$. Find the rate of change of its radius when the volume of the balloon is $\frac{32}{3}\pi\text{m}^3$.

Given, volume of sphere $V = \frac{4}{3}\pi r^3$.

(6 Marks)

(c) Given a function of $f(x) = 2x^3 - 9x^2 + 12x - 3$

(i) Find the critical point, local maximum and minimum points of function

(ii) Determine the point of inflection of $f(x)$ function (if any), hence sketch the graph.

(11 Marks)

QUESTION 4

(a) Evaluate

$$\int \frac{x+1}{x^2}\, dx$$

(3 Marks)

(b) Find the integration of

$$\int_0^1 e^{2x}(1+2x)\, dx$$

(7 Marks)

(c) Integrate

$$\int \frac{x^2}{(x+1)(x-1)}\, dx$$

(7 Marks)

QUESTION 5

(a) Figure 1 shows a region bounded by $y = x + 3$ and $y = x^2 + 1$.

(i) Determine the point of intersection.

(ii) Find the area of the region.

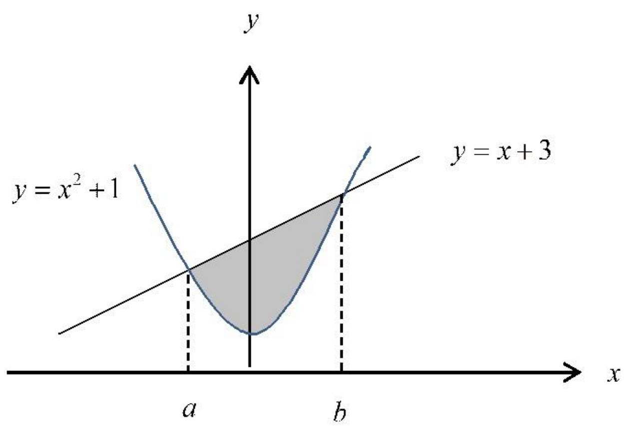

Figure 1

(9 Marks)

(b) Find the volume of the solid generated by revolving the shaded region in Figure 2 about the x-axis.

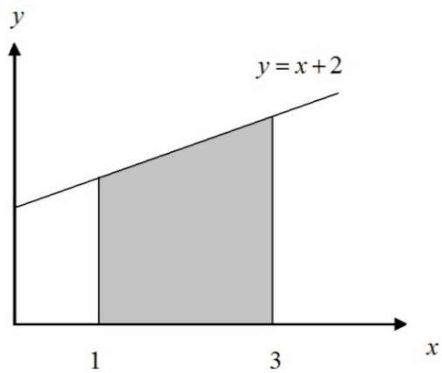

Figure 2

(11 Marks)

END OF QUESTION PAPER

APPENDIX

Derivatives of Commonly Used Functions

Function $y = f(x)$	**Derivatives formulae** $f'(x)$
constant, k	0
x	1
x^n	nx^{n-1}
$kf(x)$	$kf'(x)$
e^x	e^x
e^{-x}	$-e^{-x}$
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Product Rule

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Quotient Rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Parametric Rule

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Integration of Commonly Used Functions

Function $y = f(x)$	**Integration Formulae** $\int f(x)dx$
constant, k	$kx + C$
x^n	$\frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
e^x	$e^x + C$
e^{-x}	$-e^{-x} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\tan x$	$\ln\lvert\sec x\rvert + C$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

Area between Two Curves

$$A = \int_a^b \left[f(x) - g(x) \right] dx$$

Surface Area

$$S = \int_a^b 2\pi y \sqrt{1 + \left[f'(x) \right]^2} \, dx$$

Volume of Revolution

$$V = \pi \int_a^b y^2 \, dx$$