FACULTY OF INDUSTRIAL SCIENCES & TECHNOLOGY
FINAL EXAMINATION

COURSE	**:**	**CALCULUS**
COURSE CODE	**:**	**DUM1123**
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DATE	**:**	**10 JUNE 2015**
DURATION	**:**	**3 HOURS**
SESSION/SEMESTER	**:**	**SESSION 2014/2015 SEMESTER II**
PROGRAMME CODE	**:**	**DEE/DMM/DKK/DCS/DAA**

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of **FIVE (5)** questions. Answer **ALL** questions.
2. All answers to a new question should start on new page.
3. All the calculations and assumptions must be clearly stated.
4. Candidates are not allowed to bring any material other than those allowed by the invigilator into the examination room.

EXAMINATION REQUIREMENTS

1. APPENDIX
2. Scientific calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **EIGHT (8)** printed pages including front page.

QUESTION 1

(a) Find

$$\lim_{x \to +\infty} \left(\sqrt{x^2 + 3x} - x \right)$$

by using numerical method.

(CO1,PO1/3 Marks)

(b) Find

(i) $\lim_{x \to -1} \dfrac{-4x^2 - 2x - 8}{-2x^3 - 7}$.

(ii) $\lim_{x \to \infty} \dfrac{7x^2 + 5x - 2}{x^2 - 3}$.

(CO1,PO1/5 Marks)

(c) Given a piece-wise function

$$f(x) = \begin{cases} x^2 & , \quad x < 2 \\ cx + 3 & , \quad 2 \le x \le 4 \\ 10 & , \quad x > 4. \end{cases}$$

(i) Find the value of c if $f(x)$ is continuous at $x = 2$.

(ii) Determine whether the function is continuous at $x = 4$.

(CO1,PO1/10 Marks)

QUESTION 2

(a) Given parametric equations

$$x = (1-2t)^3, \quad y = \frac{1}{1-t}.$$

Find $\frac{dy}{dx}$.

(CO1,PO1/6 Marks)

(b) Given the function

$$y = 3e^{-2x} + 4e^{3x}.$$

Show that

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$$

(CO1,PO1/5 Marks)

(c) Find $\frac{dy}{dx}$ for the implicit function

$$\sin y - 3xy = 2y^3.$$

(CO1,PO1/6 Marks)

QUESTION 3

(a) Evaluate

$$\int \frac{x^2}{x^3+3}\,dx.$$

(CO2,PO1/5 Marks)

(b) Use integration by part to evaluate

$$\int e^{-3x}(1-2x)^2\,dx.$$

(CO2,PO1/10 Marks)

(c) Use partial fractions to evaluate

$$\int \frac{2(x-1)}{x^2+x}dx.$$

(CO2,PO1/8 Marks)

QUESTION 4

(a) Given a function

$$y = \frac{(x+1)^2}{x-1}.$$

(i) Find all the critical points.

(ii) Locate all the maximum and minimum points by using the first derivative test.

(CO2,PO1/10 Marks)

(b) Two resistors with resistances R_1 and R_2 are connected in parallel as shown in **Figure 1.**

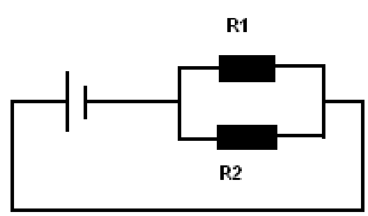


Figure 1

The electrical behavior of R_1 and R_2 is related to a resistor of resistance R such that

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

The change of R_1 with respect to time is denoted as $\frac{dR_1}{dt}$ and R_2 is a constant. Show that

$$\frac{dR}{dt} = \left(\frac{R_2}{R_1 + R_2}\right)^2 \frac{dR_1}{dt}.$$

(CO2,PO1/8 Marks)

QUESTION 5

(a) Find the surface area that is generated by revolving the curve $y = \sqrt{9 - x^2}$ between $-2 < x < 2$ about the x-axis.

(CO2,PO1/8 Marks)

(b) **Figure 2** shows that the curve $y = x^{3/2} - 1$ intersects the x-axis at (1,0). The tangent line to the curve touches the curve at the point (4,7).

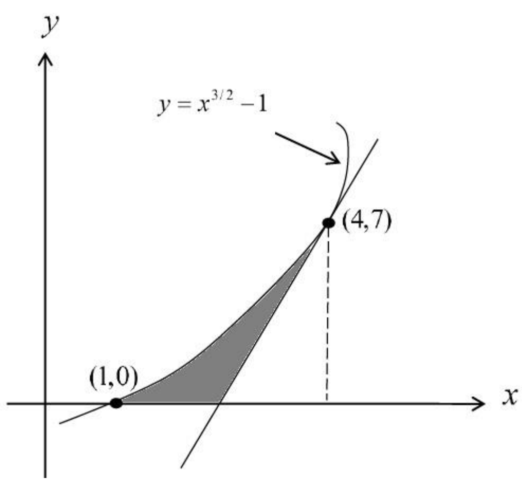


Figure 2

(i) Show that

$$\int_1^4 \left(x^{3/2} - 1\right) dx = \frac{47}{5}.$$

(ii) Find the area of the shaded region enclosed by the curve, the tangent line and the x-axis.

(CO2,PO1/16 Marks)

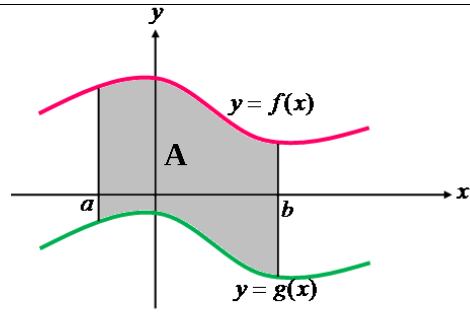
END OF QUESTION PAPER

APPENDIX

Derivatives and Integration of Commonly Used Functions

Function $y = f(x)$	**Derivatives formulae** $f'(x)$	**Integration Formulae** $\int f(x)dx$
constant, k	0	$kx + C$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln\lvert x\rvert + C$
e^x	e^x	$e^x + C$
$\ln x$	$\frac{1}{x}$	$x \ln x + C$
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	$-\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x$	$\ln\lvert\sec x\rvert + C$
$\sec x$	$\sec x \tan x$	$\sec x \tan x + C$

Chain Rule	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
Product Rule	If $y = u(x) \cdot v(x)$, then $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$
Quotient Rule	If $y = \frac{u(x)}{v(x)}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Parametric Rule	If $y = f(t)$ and $x = f(t)$ then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Integration by Parts	$\int u\,dv = uv - \int v\,du$
Area between Two Curves	 $A = \int_a^b [f(x) - g(x)]\,dx$
Surface Area	$S = \int_a^b 2\pi y \sqrt{1 + [y'(x)]^2}\,dx$
Volume of Revolution	$V = \pi \int_a^b y^2\,dx$