Universiti Malaysia PAHANG

FACULTY OF INDUSTRIAL SCIENCES & TECHNOLOGY
FINAL EXAMINATION

COURSE	**:**	**CALCULUS**
COURSE CODE	**:**	**DUM1123**
LECTURER	**:**	**NORHAFIZAH MD SARIF** **MOHD ZUKI SALLEH** **NAWWARAH SUHAIMY**
DATE	**:**	**28 DECEMBER 2015**
DURATION	**:**	**3 HOURS**
SESSION/SEMESTER	**:**	**SESSION 2015/2016 SEMESTER I**
PROGRAMME CODE	**:**	**DAA/DCS/DEE/DKK/DMM**

INSTRUCTIONS TO CANDIDATES:

1. This question paper consists of **FIVE (5)** questions. Answer **ALL** questions.
2. All answers to a new question should start on new page.
3. All the calculations and assumptions must be clearly stated.

EXAMINATION REQUIREMENTS

1. Scientific Calculator
2. **APPENDIX**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **NINE (9)** printed pages including front page.

QUESTION 1

(a) Find

$$\lim_{x \to 0} \frac{\sin 3x - \sin 2x}{4x}$$

by using numerical method.

(5 Marks)

(b) Given

$$\lim_{x \to a} \frac{a + 7x}{3\sqrt{a} - \sqrt{x}} = 4\sqrt{2}.$$

Find the value of a.

(3 Marks)

(c) A function f is given as

$$f(x) = \begin{cases} e^{-2x} & , \quad x < 0 \\ -1 & , \quad x = 0 \\ x + 1 & , \quad x > 0. \end{cases}$$

(i) Find $f(0)$.

(1 Mark)

(ii) Find $\lim_{x \to 0} f(x)$.

(5 Marks)

(iii) Is $f(x)$ continuous at $x = 0$? Give a reason to your answer.

(2 Marks)

QUESTION 2

(a) Given two parametric equations

$$y = \frac{1}{7 - t^2} \text{ and } x = (3t^2 - t + 1)^3.$$

Find $\frac{dy}{dx}$.

(6 Marks)

(b) Suppose $y = xe^{3x}$. Show that

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0.$$

(7 Marks)

(c) Find $\frac{dy}{dx}$ for the implicit function

$$\cos 2x - 4x^2 y = 2y^3.$$

(6 Marks)

QUESTION 3

(a) Evaluate

$$\int \left(e^{2x+1} - \frac{1}{\sqrt{x}} \right) dx.$$

(3 Marks)

(b) Evaluate the following function

$$\int t^4 \sqrt{3-5t^5}\ dt.$$

(6 Marks)

(c) Use partial fractions to evaluate

$$\int \frac{7x+3}{x(x-1)^2}\ dx.$$

(8 Marks)

QUESTION 4

(a) The parametric equation of a curve is given by

$$x = t^2 + 1, \quad y = t^3.$$

(i) Show that $(5, -8)$ is a point on the curve.

(4 Marks)

(ii) Find the equation of the tangent line to the curve at that point.

(6 Marks)

(b) Given a function

$$y = x^3 (4 - x).$$

(i) Find all the critical points.

(5 Marks)

(ii) Locate all the maximum and minimum points by using the first derivative test.

(2 Marks)

(iii) Find the inflection point(s).

(4 Marks)

(c) A ball is thrown up vertically from ground level. The position of the ball, s at time, t is given by

$$s = -5t^2 + 20t.$$

(i) Find the velocity at $t = 1$.

(2 Mark)

(ii) Calculate the time for the ball to reach the maximum point.

(2 Mark)

(iii) Find the velocity of the ball when it hits the ground.

(2 Marks)

QUESTION 5

(a) **Figure 1** shows a region bounded by the curves $y = 2x^2 + 10$ and $y = 4x + 16$ for $-2 \le x \le 5$.

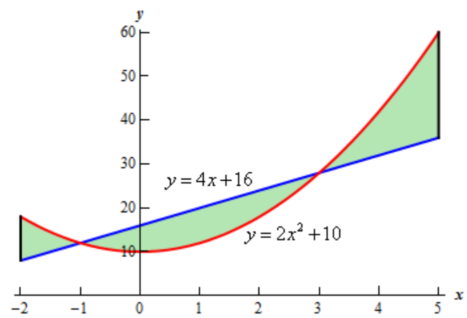


Figure 1

(i) Find the intersection points between the two curves.

(5 Marks)

(ii) Find the area of the shaded region.

(7 Marks)

(b) The region bounded by the curves $y = x^2 - 4x + 5$, $x = 1$, $x = 4$ and the x-axis are illustrated in **Figure 2**. Find the volume of the solid of revolution when the region bounded revolves about x-axis.

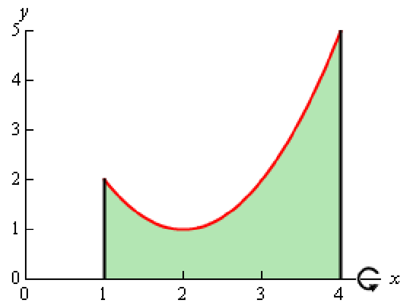


Figure 2

(9 Marks)

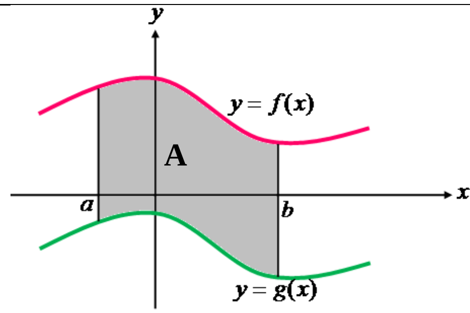
END OF QUESTION PAPER

APPENDIX

Derivatives and Integration of Commonly Used Functions

Function $y = f(x)$	**Derivatives formulae** $f'(x)$	**Integration Formulae** $\int f(x)dx$
constant, k	0	$kx + C$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln\lvert x\rvert + C$
e^x	e^x	$e^x + C$
$\ln x$	$\frac{1}{x}$	$x \ln x + C$
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	$-\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x$	$\ln\lvert\sec x\rvert + C$
$\sec x$	$\sec x \tan x$	$\sec x \tan x + C$

Chain Rule	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
Product Rule	If $y = u(x) \cdot v(x)$, then $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$
Quotient Rule	If $y = \frac{u(x)}{v(x)}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Parametric Rule	If $y = f(t)$ and $x = f(t)$ then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Integration by Parts	$\int u\,dv = uv - \int v\,du$
Area between Two Curves	 $A = \int_a^b [\,f(x) - g(x)]\,dx$
Surface Area	$S = \int_a^b 2\pi y\sqrt{1+\left[y'(x)\right]^2}\,dx$
Volume of Revolution	$V = \pi\int_a^b y^2\,dx$