FACULTY OF INDUSTRIAL SCIENCES & TECHNOLOGY
FINAL EXAMINATION

COURSE	**:**	**CALCULUS**
COURSE CODE	**:**	**DUM1123**
LECTURER	**:**	**NORHAFIZAH MD SARIF**
DATE	**:**	**08 JUNE 2016**
DURATION	**:**	**3 HOURS**
SESSION/SEMESTER	**:**	**SESSION 2015/2016 SEMESTER II**
PROGRAMME CODE	**:**	**DAA/DCS/DEE/DKK**

INSTRUCTIONS TO CANDIDATE

1. This question paper consists of **FIVE (5)** questions. Answer **ALL** questions.
2. All answers to a new question should start on new page.
3. All the calculations and assumptions must be clearly stated

EXAMINATION REQUIREMENTS

1. Scientific Calculator
2. **APPENDIX**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **EIGHT (8)** printed pages including front page.

QUESTION 1

(a) Evaluate numerically

$$\lim_{x \to 5} \frac{3x-15}{\sqrt{x^2-10x+25}}$$

(5 Marks)

(b) Evaluate the following limits analytically.

(i) $\lim_{x \to -1} \dfrac{2x^2-3x+1}{x^3+2}$

(ii) $\lim_{x \to \infty} \dfrac{x+3}{\sqrt{9x^2-5x}}$

(5 Marks)

(c) Consider a function

$$f(x) = \begin{cases} x^2+x+m\ , & x<1 \\ x^3, & x \geq 1 \end{cases}$$

Find a value of m so that the function is continuous at $x=1$.

(5 Marks)

[15 Marks, CO1/PO1]

QUESTION 2

(a) Differentiate each of the following functions

(i) $y = \cos^3(4x - 1)$.

(3 Marks)

(ii) $y = \left(x^2 + 1\right)\left(x - 5 - \dfrac{1}{x}\right)$.

(3 Marks)

(iii) $y = \dfrac{x^2 - 2x}{\sqrt{x}}$.

(3 Marks)

[9 Marks, CO1/PO1]

(b) Consider the equation

$$y^2 - xy = 8.$$

(i) Find $\dfrac{dy}{dx}$ by using implicit differentiation.

(4 Marks)

(ii) Show that

$$\frac{d^2y}{dx^2} = \frac{2y(y-x)}{\left(2y-x\right)^3}.$$

(7 Marks)

[11 Marks, CO1/PO1]

QUESTION 3

(a) Evaluate

$$\int \frac{\left(\sqrt{x}+2\right)^3}{\sqrt{x}}\, dx$$

by using appropriate substitution.

(5 Marks)

(b) Evaluate using integration by parts

$$\int_1^2 x^3 \ln x dx.$$

(6 Marks)

(c) Use partial fraction to evaluate

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\, dx.$$

(8 Marks)

[19 Marks, CO2/PO1]

QUESTION 4

(a) The parametric equations of a curve is given by

$$x = 1 + 3\sin t, \quad y = 2 - 5\cos t$$

Find the equation of the tangent line to the curve at point $t = \frac{\pi}{6}$.

(7 Marks)

(b) Given a function

$$y = x^3 - 12x + 3.$$

(i) Find all the critical points of the function.

(4 Marks)

(ii) Locate all the maximum and minimum points by using second derivative test.

(3 Marks)

(iii) Determine the inflection point(s) (if any).

(2 Marks)

(iv) Sketch the graph of the function.

(2 Marks)

(c) A 10-foot ladder leans against the side of a building. The bottom of the ladder is pulled away from the wall at the rate of 3 ft/s.

(i) Find the rate at which the top of the ladder is sliding when the bottom is 8 feet from the wall.

(6 Marks)

(ii) Find the rate at which the angle between the ladder and the ground is changing when the bottom of the ladder is 8 feet from the wall.

(4 Marks)

[28 Marks, CO2/PO1]

QUESTION 5

(a) **Figure 1** shows a region bounded by curves $y = \sin x$, and $y = \cos x$ for $0 \le x \le \frac{\pi}{2}$. Find the area of the bounded region.

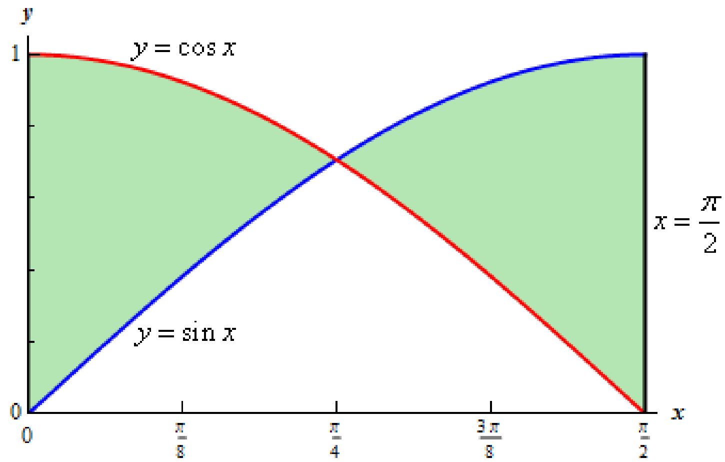


Figure 1

(9 Marks)

(b) Region bounded by curves $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant are illustrated in **Figure 2**. Find the volume of the solid of revolution when the region bounded revolves about *y-axis.*

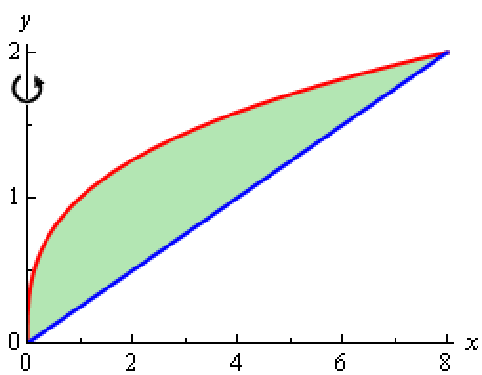


Figure 2

(9 Marks)

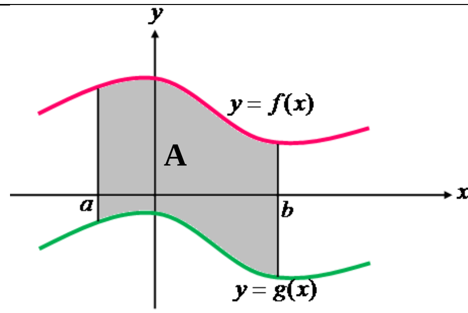
END OF QUESTION PAPER

APPENDIX

Derivatives and Integration of Commonly Used Functions

Function $y = f(x)$	**Derivatives Formulae** $f'(x)$	**Integration Formulae** $\int f(x)dx$
constant, k	0	$kx + C$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln\lvert x \rvert + C$
e^x	e^x	$e^x + C$
$\ln x$	$\frac{1}{x}$	$x \ln x + C$
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	$-\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x$	$\ln\lvert \sec x \rvert + C$
$\sec x$	$\sec x \tan x$	$\sec x \tan x + C$

Chain Rule	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
Product Rule	If $y = u(x) \cdot v(x)$, then $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$
Quotient Rule	If $y = \frac{u(x)}{v(x)}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Parametric Rule	If $y = f(t)$ and $x = f(t)$ then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Integration by Parts	$\int u\,dv = uv - \int v\,du$
Area between Two Curves	 $A = \int_a^b [\,f(x) - g(x)]\,dx$
Surface Area	$S = \int_a^b 2\pi y \sqrt{1 + [y'(x)]^2}\,dx$
Volume of Revolution	$V = \pi \int_a^b x^2\,dy$