FACULTY OF INDUSTRIAL SCIENCES & TECHNOLOGY
FINAL EXAMINATION

COURSE	**:**	**CALCULUS**
COURSE CODE	**:**	**DUM1123**
LECTURER	**:**	**NORHAFIZAH BINTI MD SARIF** **YUHANI BINTI YUSOF**
DATE	**:**	**22 AUGUST 2016**
DURATION	**:**	**3 HOURS**
SESSION/SEMESTER	**:**	**SESSION 2015/2016 SEMESTER III**
PROGRAMME CODE	**:**	**DCS/DMM**

INSTRUCTIONS TO CANDIDATE

1. This question paper consists of **FIVE (5)** questions. Answer **ALL** questions.
2. All answers to a new question should start on new page.
3. All the calculations and assumptions must be clearly stated

EXAMINATION REQUIREMENT

1. Scientific Calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **EIGHT (8)** printed pages including front page.

QUESTION 1

(a) Find each of the following limits analytically.

(i) $\lim_{x \to 4} \frac{x^2 - x - 12}{x - 4}$.

(3 Marks)

(ii) $\lim_{x \to \infty} \frac{\sqrt{x} - 2}{2x^2 + 1}$.

(4 Marks)

(iii) $\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16}$.

(4 Marks)

(b) Given a piecewise function

$$f(x) = \begin{cases} x^2 & \text{if} \quad x < 0 \\ -2x & \text{if} \quad x \geq 0. \end{cases}$$

(i) Sketch the graph of $f(x)$.

(4 Marks)

(ii) Determine the continuity of the function $f(x)$ at $x = 0$ by using continuity test.

(8 Marks)

QUESTION 2

(a) Differentiate

$$y = -2\cos^2 z$$

by using chain rule.

(6 Marks)

(b) Given a function defined by the following parametric equations

$$x = (1+3t)^2 \text{ and } y = \frac{-2}{1+t}.$$

Find $\dfrac{dy}{dx}$.

(6 Marks)

(c) Given the implicit function

$$e^{2x} + \ln(3y) = -2 + x^2 y.$$

Find:

(i) $\dfrac{dy}{dx}$

(6 Marks)

(ii) $\left.\dfrac{dy}{dx}\right|_{(2,3)}$.

(2 Marks)

QUESTION 3

(a) Evaluate the following integral

$$\int 4x(2x^2-3)^6 \; dx$$

by using appropriate substitution.

(5 Marks)

(b) Evaluate the following integral

$$\int_1^2 x^2 \ln x dx$$

by using integration by parts.

(7 Marks)

(c) Use partial fraction to evaluate

$$\int \frac{x^2}{(x+1)(x-1)^2} \; dx.$$

(8 Marks)

QUESTION 4

(a) The motion of a bike at any time t is described by

$$s(t) = 2t^3 + 14t^2 - 2 .$$

(i) What is the velocity function?

(2 Marks)

(ii) What is the velocity at $t = 2$?

(2 Marks)

(iii) Determine the acceleration of the bike when $t = 7$.

(3 Marks)

(b) Given

$$y = x^3 - x^2 - 8x.$$

(i) Find the critical point(s).

(9 Marks)

(ii) Locate all the maximum and minimum points by using second derivative test.

(4 Marks)

(iii) Determine the inflection point(s) (if any).

(3 Marks)

QUESTION 5

(a) **Figure 1** shows a region bounded by the curves $y = \sqrt{x}$ and $y = x^2$. Find the area of the bounded region.

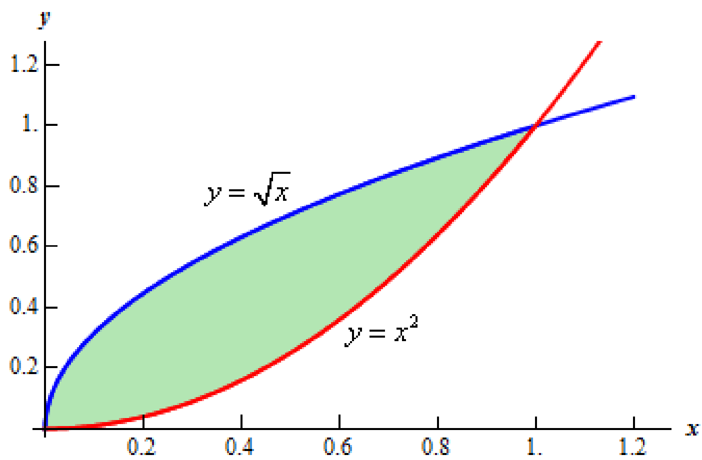


Figure 1

(7 Marks)

(b) A region bounded by the curves $y = \sqrt[3]{x}$, $y = 0$ for $0 \le x \le 8$ is illustrated in **Figure 2**. Find the volume of the solid of revolution when the bounded region is revolved about the *x*-axis.

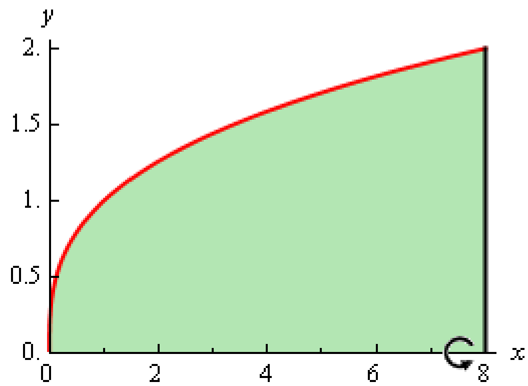


Figure 2

(7 Marks)

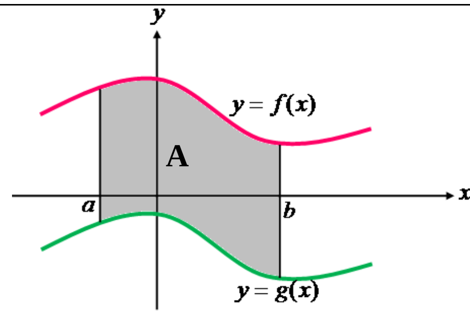
END OF QUESTION PAPER

APPENDIX

Derivatives and Integration of Commonly Used Functions

Function $y = f(x)$	**Derivatives Formulae** $f'(x)$	**Integration Formulae** $\int f(x)dx$
constant, k	0	$kx + C$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln\lvert x\rvert + C$
e^x	e^x	$e^x + C$
$\ln x$	$\frac{1}{x}$	$x\ln x + C$
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	$-\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x$	$\ln\lvert\sec x\rvert + C$
$\sec x$	$\sec x \tan x$	$\sec x \tan x + C$

Chain Rule	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
Product Rule	If $y = u(x) \cdot v(x)$, then $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$
Quotient Rule	If $y = \frac{u(x)}{v(x)}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Parametric Rule	If $y = f(t)$ and $x = f(t)$ then $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$
Integration by Parts	$\int u\,dv = uv - \int v\,du$
Area between Two Curves	 $A = \int_a^b [f(x) - g(x)]\,dx$
Surface Area	$S = \int_a^b 2\pi y\sqrt{1+\left[y'(x)\right]^2}\,dx$
Volume of Revolution	$V = \pi\int_a^b x^2\,dy$