Exercise 10: Applications of Integrations

Topic 10.1 : Volume

1. Find the volume of the solid that results by revolving the region enclosed by the line $x+y=8$ with $0\le x\le 8$.

[Ans: $\frac{512}{3}\pi$]

2. Find the volume of the solid of revolution when the region bounded by the curve $y^2=8x$ and $y=x^2$ revolves at 360° about x-axis.

[Ans: $\frac{48}{5}\pi$]

3. Find the volume of the solid of revolution when the region bounded by the following curves revolves 360° about x-axis.
   a) $y=x^2,\ x=0,\ x=2$ and $y=0$
   b) $y=1+x^3,\ x=1,\ x=2$ and $y=0$
   c) $y=9-x^2$ and $y=0$
   d) $y=x^2$ and $y=4x$

[Ans: (a) $\frac{32\pi}{5}$, (b) $\frac{373\pi}{14}$ (c) $\frac{1296\pi}{5}$, (d) $\frac{2048}{15}\pi$]

4. Find the volume of the solid of revolution when the region bounded by the following curves revolves 360° about y-axis
   a) $y=x^3,\ x=0,$ and $y=1$
   b) $y=1+x^3,\ x=1,$ and $y=9$
   c) $x=y^2$ and $x=y+2$
   d) $x=\sqrt{1+y}$, $x=0$ and $y=3$

[Ans: (a) $\frac{3\pi}{5}$, (b) $\frac{58\pi}{5}$ (c) $\frac{72\pi}{5}$, (d) 8π]

5. Sketch the region bounded by the curve $x=y^2+2$ and the line $x=6$. Find the volume of the solid of revolution when the region bounded revolves at 360° about *x-axis.*

[Ans: 6π]

6. Find the volume of the solid of revolution when the region bounded by the curve $y = 2(x^2 + 1)$, the x-axis and the lines $x = 1, x = 2$ revolves at 360° about x-axis.

[Ans: $\frac{712}{15}\pi$]

7. Find the volume of the solid of revolution when the region bounded by the curve $y = x - 1$, $y = 4 - x$ and $y = 0$ revolves at 360° about x-axis.

[Ans: $\frac{9}{4}\pi$]

8. Consider a region R in the first quadrant bounded by two curves $y = \frac{1}{4}\left(4 - x^2\right)$ and $y = \frac{1}{2}\left(4 - x^2\right)$ and the y-axis. Find the

   a) area of the region R

   b) volume of the solid of revolution when the bounded region R revolves 360° about y-axis.

[Ans: (a) $\frac{4}{3}$, (b) 2π]