Calculus
Applications of Integration

By

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Description

Aims

This chapter is aimed to :

1. Consider various application of integration
2. evaluate the definite and indefinite integral
3. explain the basic properties of integral

Expected Outcomes

1. Students should be able to use definite integral to find area between two curves
2. Students should be able to sketch graph to find area between curve and surface area
3. Students should be able to determine the length of a plane curve

References

1. Abdul Wahid Md Raji, Hamisan Rahmat, Ismail Kamis, Mohd Nor Mohamad, Ong Chee Tiong*. **The First Course of Calculus for Science & Engineering Students***, Second Edition, UTM 2016.

Content

1. Volume : Slicing Method
2. Volume : Disk Method
3. Volume : Cylindrical Shells

Volume: Slicing Method

- When the surface area is revolved about a line, it generates a volume.
- The slicing method is a way of computing the volume of a solid.
- This method use cross section (or slices) in planes perpendicular to x-axis or y-axis.
- Recall that, to find the area of a region, we divide the region into thin strips, approximate the area of each strip by the area of rectangle, and form Riemann sums to produce an integral for the area.
- Likewise, to find the volume of a solid, we divide the solid into thin slabs, approximate the volume of each slab, and form Riemann sums to produce an integral for the volume.

Volume: Slicing Method

Definition – The cross section is perpendicular to x-axis :

The volume of a solid of known integrable cross-sectional area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b

$$V = \int_a^b A(x)dx = \int_a^b \pi\left[R(x)\right]^2 dx$$

where

$$A(x) = \pi(\text{radius})^2 = \pi\left[R(x)\right]^2$$

Volume: Slicing Method

Definition – The cross section is perpendicular to y-axis :

The volume of a solid of known integrable cross-sectional area $A(y)$ from $y = c$ to $y = d$ is the integral of A from c to d

$$V = \int_c^d A(y)dy = \int_c^d \pi\left[R(y)\right]^2 dy$$

where

$$A(y) = \pi(\text{radius})^2 = \pi\left[R(y)\right]^2$$

Volume: Disks Method

Definition – Volume by disk perpendicular to the x-axis :

Suppose R is the region bounded by $y = f(x)$, the x-axis, $x = a$ and $x = b$. The volume of the solid revolution that is generated by revolving the region R about the x-axis is

$$V = \int_a^b \pi \left[f(x) \right]^2 dx$$

”

Volume: Disks Method

Definition – Volume by disk perpendicular to the y-axis :

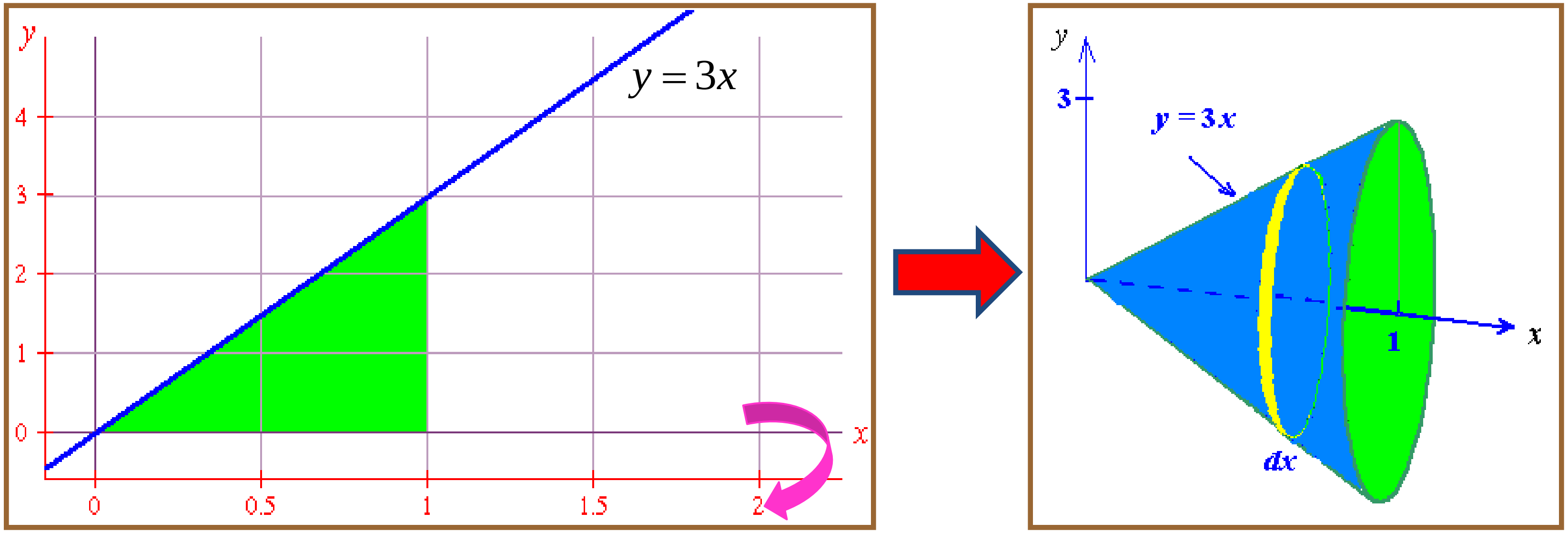
Suppose R is the region bounded by $x = f(y)$, the y-axis, $y = c$ and $y = d$. The volume of the solid revolution that is generated by revolving the region R about the y-axis is

$$V = \int_c^d \pi \left[f(y) \right]^2 dy$$

”

Disk Method

If the plane bounded by the curve $y = f(x)$, the x–axis and $a \le x \le b$ is rotating through a complete revolutions about the x–axis, it will generate a solid symmetrical.



Example

Find the volume that is generated by revolving the portion of the curve $y=\frac{2}{3}x$ between $0\le x\le 3$ about x - axis

Volume for the above problem is given by

$$\int_0^3 \pi y^2 dx = \int_0^3 \pi \left(\frac{2}{3}x\right)^2 dx$$

$$= \int_0^3 \pi \frac{4}{9} x^2 dx$$

$$= \frac{4}{9}\pi \left.\frac{x^3}{3}\right|_0^3$$

$$= 4\pi \text{ unit}^3$$

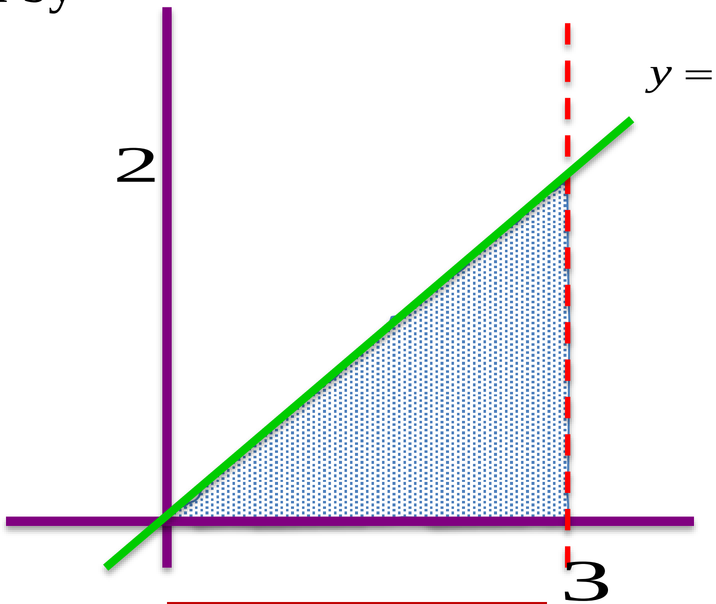


Figure 14

Example

Find the volume that is generated by revolving the portion of the curve $y = x - x^2$ between $0 \le x \le 1$ about x - axis

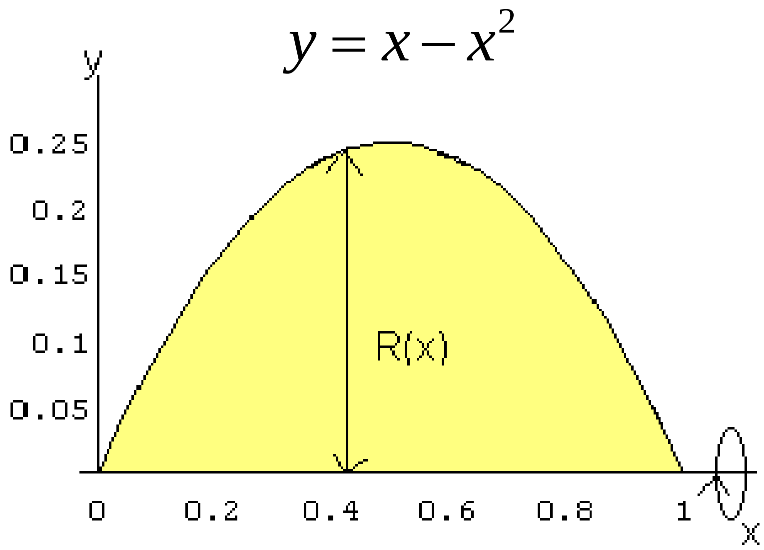


Figure 15

By inserting the function into the volume's formula, we obtain

$$\int_0^1 \pi y^2 dx = \int_0^1 \pi \left(x - x^2\right)^2 dx$$

$$= \int_0^1 \pi \left(x^2 - 2x^3 + x^4\right) dx$$

$$= \pi \left(\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5} \right) \Bigg|_0^1$$

$$= \frac{1}{30} \pi \, \text{unit}^3$$

Example

Determine the volume of the solid obtain bounded by $y = x^2 - 5x + 7$, $x = 1$
$x = 5$ about x - axis

In this example, the radius is simply the distance from x -axis to the curve

$$R = x^2 - 5x + 7$$

Hence
$$V = \int_a^b \pi \left[R(x)\right]^2 dx$$
$$= \int_1^5 \pi \left(x^2 - 5x + 7\right) dx$$
$$= \pi \int^5 x^4 - 10x^3 + 39x^2 - 70x + 49dx$$
$$= \frac{164}{5}\pi$$

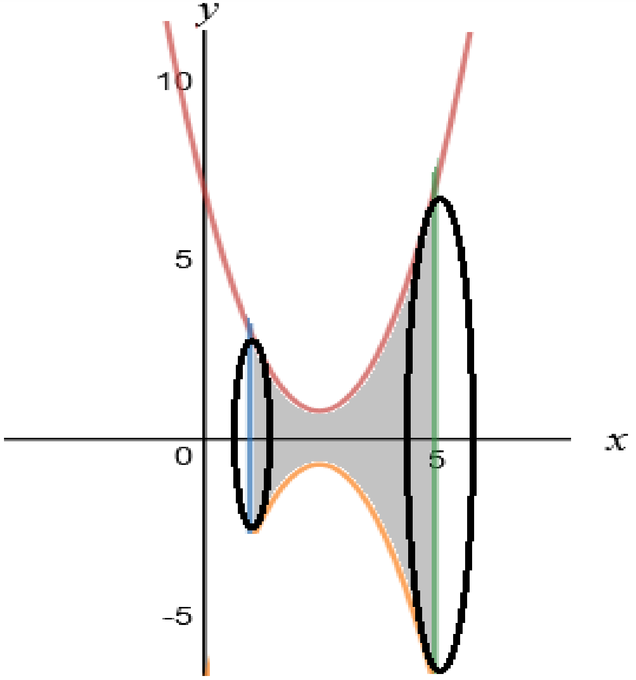


Figure 16

Example

Find the volume of the solid generated by revolving the region between the y - axis and the curve, $y = \dfrac{3}{x}$ where $1 \le y \le 6$.

In this case, the radius is simply the distance from y -axis to the curve

$$R = \frac{3}{y}$$

Hence

$$V = \int_c^d \pi \left[R(y)\right]^2 dy = \int_1^6 \pi \left(\frac{3}{y}\right)^2 dy$$

$$= \pi \int_1^6 \frac{9}{y^2} dy$$

$$= 9\pi \left[-\frac{1}{y}\right]_1^6$$

$$= \frac{15}{2}\pi$$

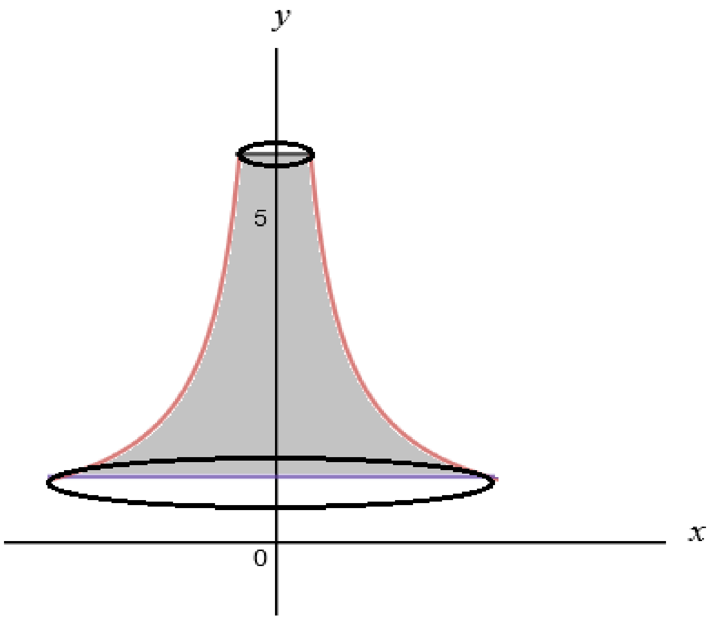


Figure 17

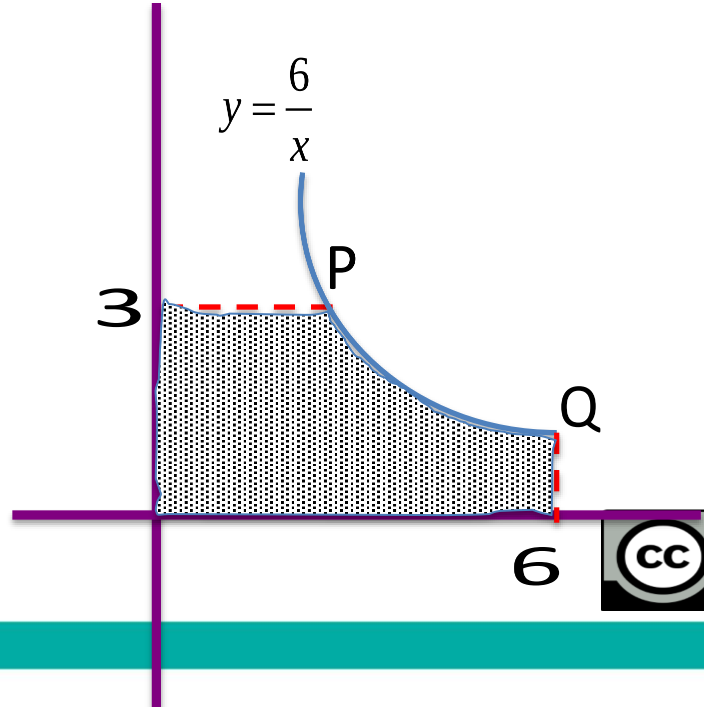
Example

Figure below shows the curve $y = \frac{6}{x}$ and the lines $x = 6,\ y = 3.$

Find the

a) coordinates of points P and Q

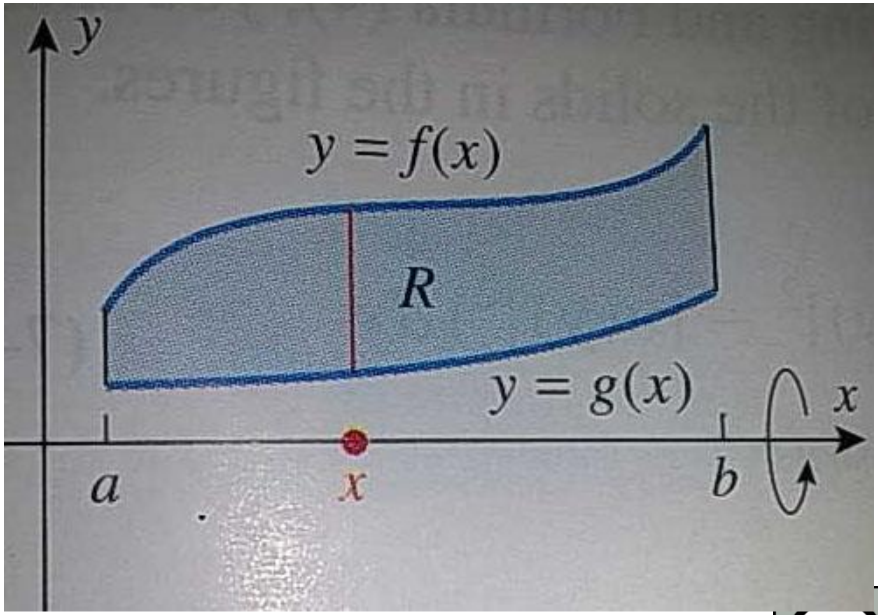
b) volume of the revolution of the solid generated when the shaded region revolves 360° about the $x-$axis.

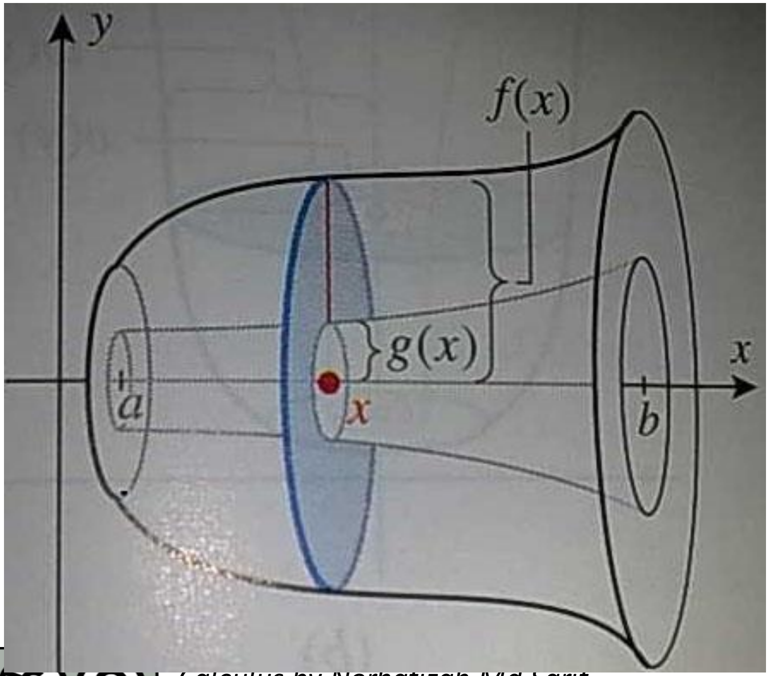


Answer : 30π unit^3

Washers Method

With a small modification of the disk method, we can find the volume of a solid figure generated by revolving about the x-axis the region between two curves $y = f(x)$ and $y = g(x)$, where $f(x) \geq g(x)$ for $a \leq x \leq b$.





Washers Method

Definition – Volume by Washer perpendicular to x-axis :

Suppose R is the region bounded by from $y = f(x)$, $y = g(x)$ $x = a$ and $x = b$ where $f(x) \geq g(x)$ in $[a,b]$. The volume of the solid revolution that is generated by revolving the region R about the x-axis is

$$V = \int_a^b \pi\left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right)dx$$

Washers Method

Definition – Volume by Washer perpendicular to y-axis :

Suppose R is the region bounded by from $x = f(y)$, $x = g(y)$ $y = c$ and $y = d$ where $f(y) \geq g(y)$ in [c,d]. The volume of the solid revolution that is generated by revolving the region R about the axis is

$$V = \int_c^d \pi\left(\left[f(y)\right]^2 - \left[g(y)\right]^2\right)dy$$

Example

Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x$ that lies in the first quadrant about the x- axis.

The intersection between two curves:

$$x^2 = x$$
$$x(x-1) = 0$$
$$x = 0,1$$

The outer radius is $y = x$ and the inner radius is $y = x^2$. Thus, the volume is

$$V = \int_0^1 \pi\left[(x)^2 - \left(x^2\right)^2\right]dx$$
$$= \pi\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = \frac{2}{15}\pi$$

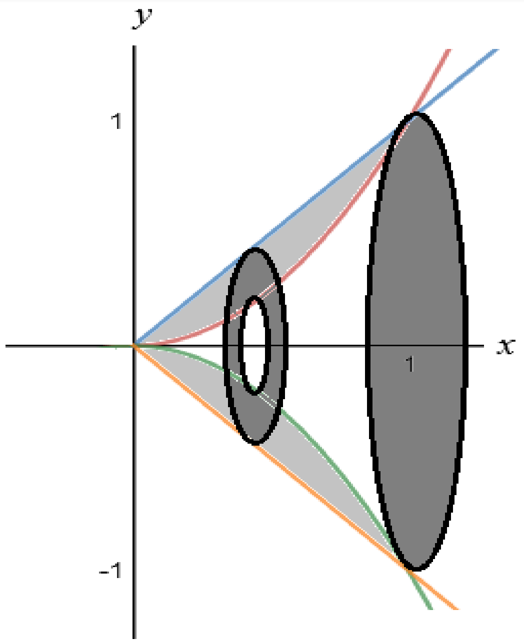


Figure 19

Example

Find the volume of the solid obtained by rotating the region bounded by

$y = x^2$ and $y = x$ that lies in the first quadrant about the y- axis

The intersection between two curves:

$$x^2 = x$$
$$x(x-1) = 0$$
$$x = 0, \quad x = 1$$
$$y = 0, \quad y = 1$$

The outer radius is $x = \sqrt{y}$ and the inner radius is $x = y$. Thus, the volume is

$$V = \int_0^1 \pi \left[\left(\sqrt{y} \right)^2 - (y)^2 \right] dx$$

$$= \pi \left[\frac{y^2}{2} - \frac{y^3}{3} \right]_0^1 = \frac{1}{6}\pi$$

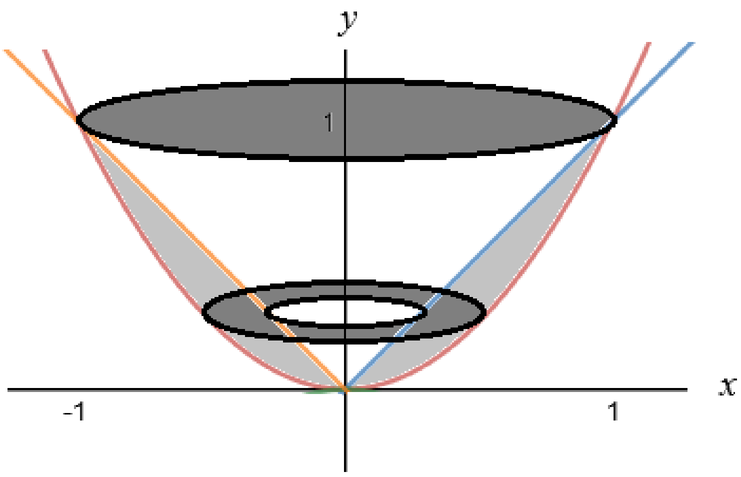


Figure 20

Example

Find the volume of the solid revolution when the region bounded by the curve $y^2 = 8x$ and $y = x^2$ revolves at 360° about the x -axis

The intersection between two curves:

$$x^4 = 8x$$
$$x(x^3 - 8) = 0$$
$$x = 0, \quad x = 2$$

The outer radius is $y = \sqrt{8x}$ and the inner radius is $y = x^2$. Thus, the volume is

$$V = \int_0^2 \pi \left[\left(\sqrt{8x}\right)^2 - \left(x^2\right)^2 \right] dx$$

$$= \pi \left[4x^2 - \frac{x^5}{5} \right]_0^2 = \frac{48}{5}\pi$$

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