Exercise 9: Applications of Integrations

Topic 5.1 : Area

1. Find the area of the region bounded by the curve $y = 2 - x^2$, the $x-$axis and the lines $x = 0$ and $x = 1$
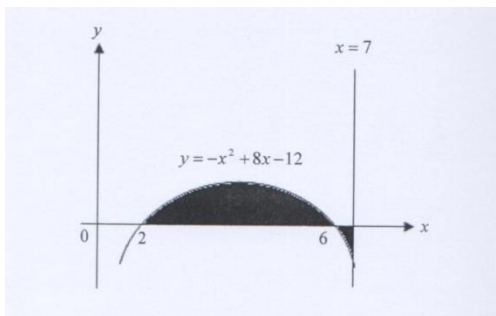
$$\left[\frac{5}{3}\right]$$

2. Find the area bounded by the lines $y = 2 - x$, $x = 3$, $x = 4$, and $x-$axis

$$\left[\frac{3}{2}\right]$$

3. Find the area of the region bounded by the curve $y = x^3$, $x-$axis with the lines $x = -1$ and $x = 1$
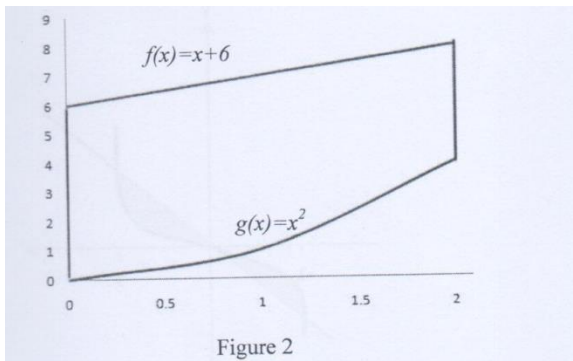
$$\left[\frac{1}{2}\right]$$

4. The following diagram shows the curve $y = -x^2 + 8x - 12$ and the straight line $x = 7$. Find the area of the shaded region.

$$[13]$$

5. Figure 2 shows a region bounded by $f(x) = x + 6$, $g(x) = x^2$, $x = 0$ and $x = 2$. Find the area of the region

Figure 2

$$\left[\frac{34}{3}\right]$$

6. Show that the area of the region bounded by the curve $y = 4 - x^2$ and the line $y + 2x = 4$ is $1\frac{1}{3}$

7. Find the area of the region bounded by the curves $y = 4 - x^2$ and $y = x^2 - 2x$

[Ans: 9]

8. Find the area of the region bounded by the curves $y = x^2 + 2$ and $y = 4 - x$

[Ans: $\frac{9}{2}$]

9. Find the area of the region bounded by the curve $y = x^2 + 2$ and the lines $y = -x$, $x = 0$ and $x = 1$

[Ans: $\frac{17}{6}$]

10. Find the area of the region bounded by the curve $y = 8x - x^2 - 12$ and the $x-$ axis

[Ans: $\frac{32}{3}$]

11. Find the area of the region bounded by the curve $y = 2 - x^2$ and the $x-$ axis and the lines $x = -1$ and $x = 0$

[Ans: $\frac{5}{3}$]

Topic 9.2 : Surface Area

1. Find the surface area of a sphere if the semicircle $y = \sqrt{4 - x^2}$ is rotated $360°$ about the $x-$ axis.

$[16\pi]$

2. Determine the surface area of the solid obtained by rotating $y = \sqrt{9 - x^2}$, $-2 < x < 2$ about the x-axis.

$[24\pi]$

3. Determine the surface area of the solid obtained by rotating $y = \sqrt{x}$, $1 \le x \le 4$ about the $x-$axis.

[Ans: (a) $\dfrac{4\pi}{3}\left(\dfrac{7}{4}\sqrt{\dfrac{7}{4}} - \dfrac{1}{4}\sqrt{\dfrac{1}{4}}\right)$]

4. Determine the surface area of the solid obtained $y = 2x$, $-3 \le x \le -1$ about the x-axis.

[Ans: $16\sqrt{5}\pi$]

5. Determine the surface area of the solid obtained by rotating $y = x^2$, $0 \le x \le 1$ about the y-axis.

[Ans: $\dfrac{4\pi}{3}\left(\dfrac{\left(5\sqrt{5}-1\right)}{6}\pi\right)$]