UMP OPEN COURSEWARE

Universiti Malaysia PAHANG
Engineering • Technology • Creativity

CHAPTER 2

BEE3143:POWER SYSTEM ANALYSIS- Power flow solution (*Ybus*)

Expected Outcomes

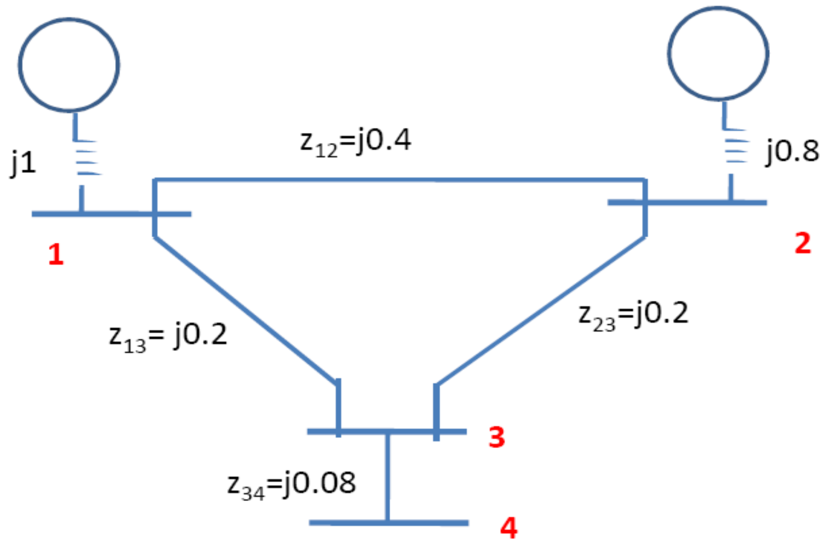
Able to develop bus admittance matrix to solve power flow solution

Bus admittance matrix, *Ybus*

- The simples way to perform power-flow calculations is by iteration.
- We create a bus admittance matrix ***Ybus*** for the power system.
- Make an initial estimate for the voltages at each bus
- Update the voltage estimate for each bus in the system, one at a time, based on the estimates for the voltages and power flows at every other bus and the values of the bus admittance matrix.
- The updated voltage will not be correct but will usually be closer to the correct answer than the original guess.
- The process is repeated until the solution has converged to the correct answer.

...Bus admittance matrix, *Ybus*

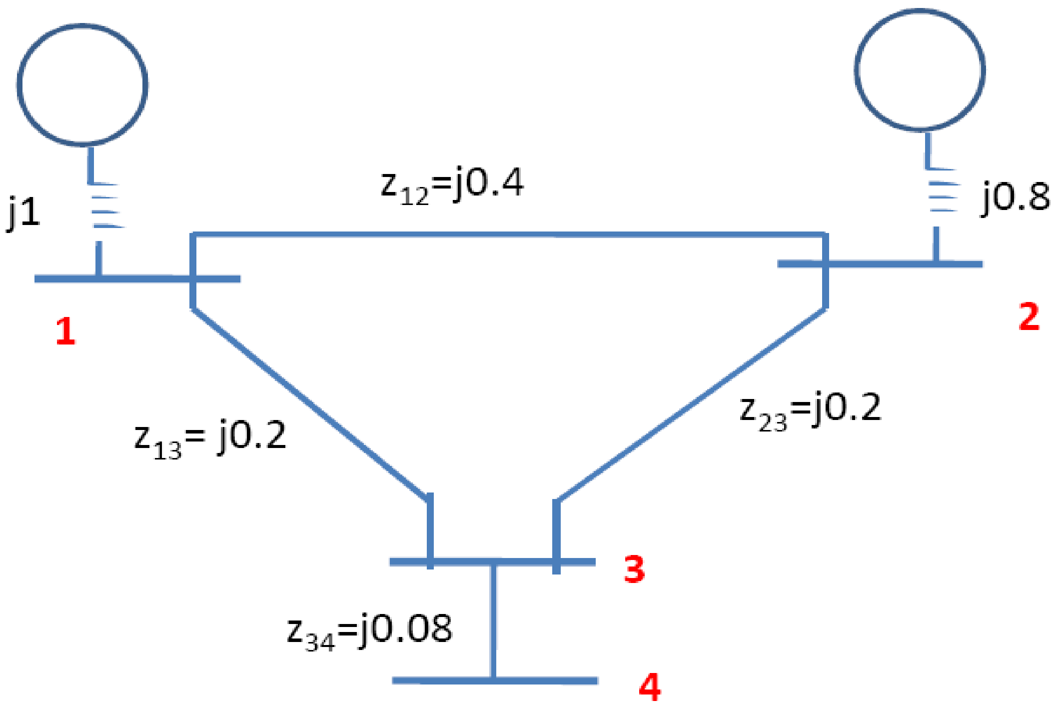
- In order to obtain the node-voltage equations, consider a simple power system:
  - Impedances are expressed in p.u. on a common MVA base
  - sometimes, for simplicity, resistances are neglected
  - Impedance are converted to admittance:



$$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + jx_{ij}}$$

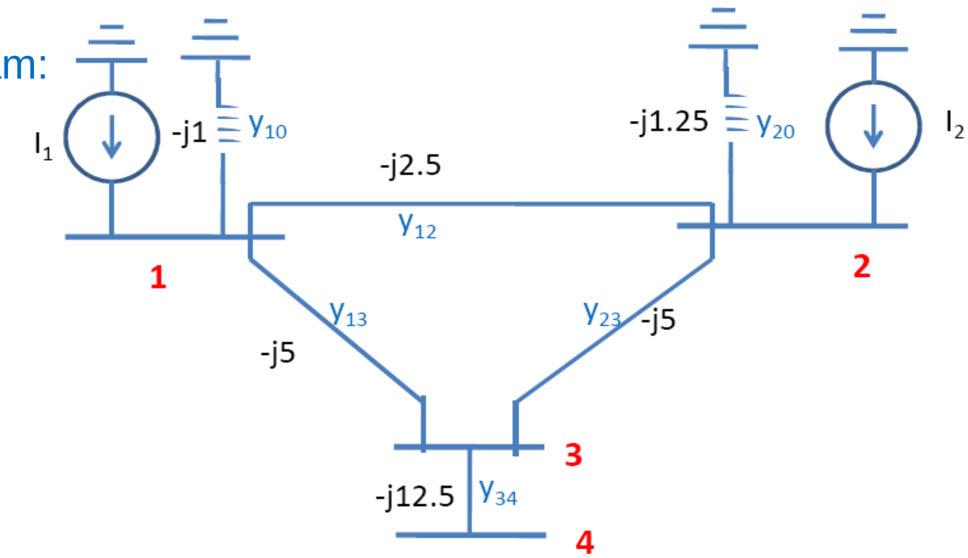
Example 1

- From the impedance diagram shown below, convert all impedances to admittance and draw the admittance diagram.



Solution

The admittance diagram:



Applying at each bus:

Bus 1 $$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$$

Bus 2 $$I_2 = y_{20}V_2 + y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3)$$

Bus 3 $$0 = y_{23}\left(V_3 - V_2\right) + y_{13}(V_3 - V_1) + y_{34}(V_3 - V_4)$$

Bus 4 $$0 = y_{34}\left(V_4 - V_3\right)$$

... Solution

Rearranging

$$I_1 = (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3$$

$$I_2 = -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3$$

$$0 = -y_{13}V_1 - y_{23}V_2 + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4$$

$$0 = -y_{34}V_3 + y_{34}V_4$$

Introducing the following admittance

$$Y_{11} = y_{10} + y_{12} + y_{13}$$

$$Y_{22} = y_{20} + y_{12} + y_{23}$$

$$Y_{33} = y_{13} + y_{23} + y_{34}$$

$$Y_{44} = y_{34}$$

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{13} = Y_{31} = -y_{13}$$

$$Y_{23} = Y_{32} = -y_{23}$$

$$Y_{34} = Y_{43} = -y_{34}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$

$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4$$

$$I_4 = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4$$

[1] H. Saadat, *Power System Analysis,* 2nd Edition, McGraw-Hill, 2004

...Solution

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$

$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4$$

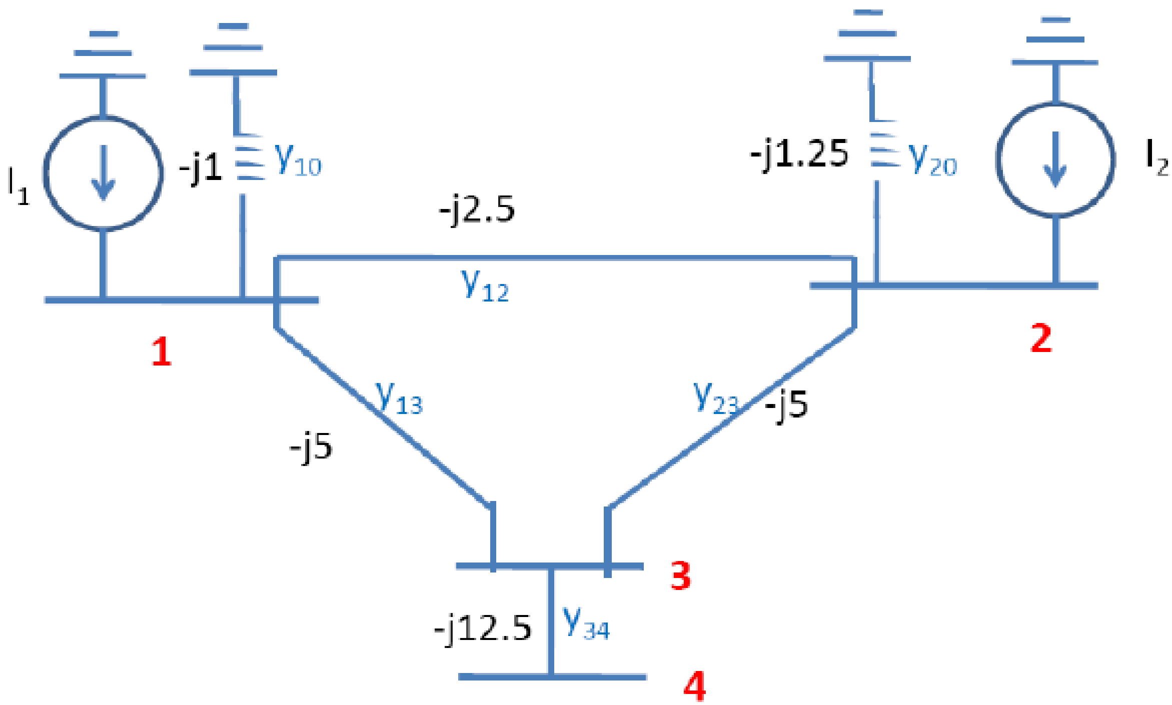
$$I_4 = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4$$

$$I_{bus} = Y_{bus}\ V_{bus}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

In this example, since there is no connection between bus 1 and 4, $Y_{14} = Y_{41} = 0$, similarly $Y_{24} = Y_{42} = 0$.

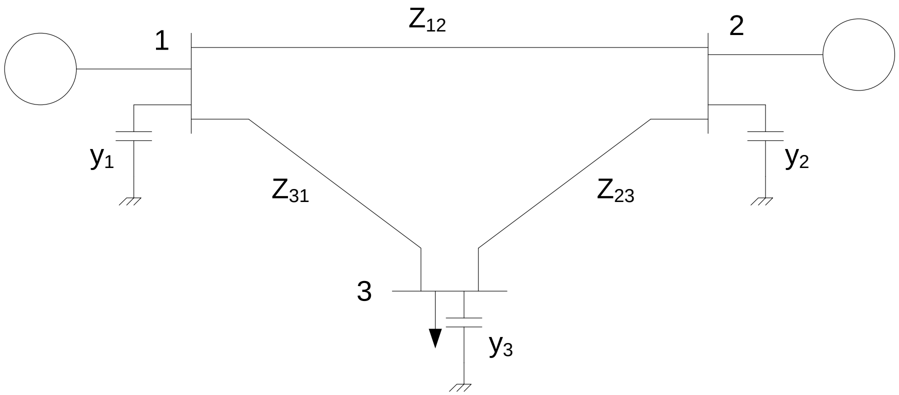
...Solution



$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} -j8.5 & j2.5 & j5 & 0 \\ j2.5 & -j8.75 & j5 & 0 \\ j5.0 & j5.0 & -j22.5 & j12.5 \\ 0 & 0 & j12.5 & -j12.5 \end{bmatrix}$$

Exercise 1

Find the bus admittance matrix, Y_{bus}.



The line data for 3 buses power system above

Line	Z p.u
1 - 2	0.08 + j0.24
1 - 3	0.02 + j0.06
2 - 3	0.06 + j0.18


والله أعلم