# Exercise 8: Techniques of Integration

## Topic 8.1 : Integration by Substitution

1. Find the following integrals

a) $\int 3x^2\left(x^3-2\right)^4\,dx$

b) $\int \frac{-4x^3}{(7-x^4)^2}\,dx$

c) $\int \frac{x}{\sqrt{x^2+2}}\,dx$

d) $\int \frac{(\ln x)^3}{x}\,dx$

e) $\int \frac{x}{\sqrt{4x^2+3}}\,dx$

f) $\int_4^5 \frac{4x^3+2}{x^4+2x-1}dx$

g) $\int_{0.5}^1 \left(3e^x\right)^3 dx$

h) $\int_1^2 \frac{e^{2x}}{1+e^{2x}}dx$

i) $\int_1^2 \frac{1}{\sqrt[3]{x^2}}e^{\sqrt[3]{x}+1}dx$

j) $\int\left(1-\sin^2 x\right)\cos x dx$

k) $\int xe^{x^2}\left(e^{x^2}-1\right)dx$

l) $\int 5r\left(r+6\right)^4\,dr$

m) $\int\left(3x-2\right)^2\,dx$

n) $\int \frac{(2+\ln x)^2}{x}\,dx$

o) $\int_4^5 \frac{4x^3+2}{x^4+2x-1}dx$

[ Ans: (a) $\frac{\left(x^3-2\right)^5}{5}+C$, (b) $-\frac{1}{7-x^4}+C$, (c) $\sqrt{x^2+2}+C$, (d) $\frac{(\ln x)^4}{4}+C$,
(e) $\frac{\sqrt{4x^2+3}}{4}+C$, (f) 0.8799 (g) 140.4343 (h) 0.9456, (i) -3.2899]

[ Ans: (j) $\sin x-\frac{\sin^3 x}{3}+C$, (k) $\frac{1}{4}e^{x^2}-\frac{1}{2}e^{x^2}+\frac{1}{4}+C$, (l) $\frac{5(r+6)^6}{6}-6\left(r+6\right)^5+C$,
(m) $\frac{(3x-2)^3}{9}+C$, (n) $\frac{(2+\ln x)^3}{3}+C$ ]

## Topic 8.2 : Integration by Parts

2. Evaluate the following integrals

a) $\int xe^{-2x}dx$

b) $\int x^2e^{x^2}dx$

c) $\int x^3\left(\ln x\right)^2\,dx$

d) $\int x^2\sin 3x dx$

e) $\int_3^4 x\sin x dx$

f) $\int_2^3 x^4\ln x dx$

g) $\int_{0.2}^{0.5} 3x^5e^{x^3+1}dx$

h) $\int_0^2\left(\frac{x^3}{4}+x\right)dx$

i) $\int_0^2 \frac{e^x+e^{-3x}}{e^x}dx$

[Ans: (a) $-\frac{e^{-2x}}{2}\left(x+\frac{1}{2}\right)+C$, (b) $\frac{e^{x^2}}{2x}\left(x^2-1\right)+C$, (c) $\frac{x^4}{4}\left(\left(\ln x\right)^2-\frac{\ln x}{2}-\frac{1}{8}\right)+C$, (d) $-\frac{x^2\cos 3x}{3}+\frac{2x\sin 3x}{9}+\frac{2\cos 3x}{27}+C$, (e) -1.2534 (f) 65.34 (g) $5.5288\times10^{-3}$ ]

Topic 8.2 : Integration by Partial Fraction

3. Evaluate the following integrals

a) $\int\frac{8x+1}{(x+3)(x+1)}dx$ b) $\int\frac{3x^2+x}{(x+3)(x-1)(x-5)}dx$ c) $\int\frac{1}{x(x-2)^2}dx$

d) $\int\frac{4}{x(2+x^2)}dx$ e) $\int\frac{e^{2x}}{e^{2x}-e^x-6}dx$ f) $\int_2^3\frac{3x^2+x-2}{x^3-x^2}dx$

g) $\int_4^5\frac{2x-14}{x^2-2x-3}dx$ h) $\int_3^4\frac{3x-4}{x^2-4x+4}dx$ i) $\int_5^6\frac{x^2+4x+12}{(x-2)(x^2+4)}dx$

[Ans: (a) $\frac{23}{2}\ln|x+3|-\frac{7}{2}\ln|x+1|+C$, (b) $\frac{3}{4}\ln|x+3|-\frac{1}{4}\ln|x-1|+\frac{5}{2}\ln|x-5|+C$, (c) $\frac{1}{4}\ln x-\frac{1}{4}\ln|x-2|-\frac{1}{2(x-2)}+C$, (d) $2\ln|x|-\ln|2+x^2|+C$, (e) $\frac{3}{5}\ln|e^x-3|+\frac{2}{5}\ln|e^x+2|+C$ (f) 1.4584, (g) -0.6571 (h) 3.0794 (i) 0.5415 ]

4. Evaluate the integral

$$\int\frac{x}{2x^2-x-3}dx$$

[Ans: $\frac{3}{10}\ln|2x-3|+\frac{1}{5}\ln|x+1|+C$ ]