Exercise 6: Applications of Differentiation

Topic 6.1 : Maximum & Minimum

1. Given $f(x)=x^3-6x^2+9x+1$, find if exist, the point of inflection, the minimum and maximum point using a) first derivative test b) second derivative test

   [(1,5) max point, (3,1) min point, (2,3) point of inflection]

2. Given $f(x)=2x^4-8x^3+45$, find:
   (i) the critical points and determine their nature
   (ii) the point of inflection

   [(3,–9) min point, (0,45) and (2,13) point of inflection]

3. Given $f(x)=x(5-x)^2$
   (i) find the stationary points and determine their nature
   (ii) find the point of inflection
   (iii) hence, sketch the graph

   $\left[\left(\frac{5}{3},\frac{500}{27}\right)\text{max point, } (5,0) \text{ min point, } \left(\frac{10}{3},\frac{250}{27}\right) \text{ point of inflection}\right]$

4. Find if exist, the point of inflection, the minimun and maximum points for each of the following curves
   (i) $f(x)=x^2-6x+8$
   (ii) $f(x)=-x^2+x+3$
   (iii) $f(x)=x^3-12x$
   (iv) $f(x)=-x^3+3x^2$

   [(a) (3,-1) min point, (b) $\left(\frac{1}{2},\frac{13}{4}\right)$max point,
   (c) (2,-16) min point, $(-2,16)$ max point, (0,0) point of inflection
   (d) (0,0) min point, $(2,4)$ max point, (1,2) point of inflection]

5. Given $f(x)=x^3-9x^2+15x$, find if exist, the point of inflection, the minimum and maximum point using a) first derivative test b) second derivative test. Hence sketch the graph.

   [(1,7) max point, (5,-25) min point, (3,-9) point of inflection]

6. Given $f(x)=x^3-6x^2+9x+1$, find if exist, the point of inflection, the minimum and maximum point using a) first derivative test b) second derivative test

$[(1,5)$ max point, $(3,1)$ min point, $(2,3)$ point of inflection$]$

7. Given $f(x)=5-9x+6x^2-x^3$
   (i) find the critical point and determine their nature
   (ii) find the point of inflection
   (iii) hence, sketch the graph

$\left[(3,5)\text{max point},\ (1,1)\ \text{min point},\ (2,3)\ \text{point of inflection}\right]$

32. Find if exist, the point of inflection , the minimun and maximum points for the curve $f(x)=x^3-x^2-x$

$: \left[\left(-\frac{1}{3},\frac{5}{27}\right)\text{max point},\ (1,-1)\ \text{min point}\right]$

33. Sketch the graph of $f(x)=x^4-16x^2+1$, indicating max and min points and points of inflection

$\left[\left(\pm 2\sqrt{2},-63\right)\ \text{min point},\ (0,1)\ \text{max point},\ \left(\pm 0.94,-12.43\right)\ \text{point of inflection}\right]$

34. Given $f(x)=x^3(4-x)$, find if exist, the point of inflection, the max and min points.

$\left[(3,27)\text{max point},\ (0,0)\ \text{point of inflection}\right]$