Calculus
Applications of Differentiation

By

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Description

Aims

This chapter is aimed to :

1. introduce the concept of integration
2. evaluate the definite and indefinite integral
3. explain the basic properties of integral
4. compute the integral using different techniques of integration

Expected Outcomes

1. Students should be able to describe the concept of antiderivatives
2. Students should be able to explain about indefinite integral and definite integral
3. Students should be able to know the basic properties of definite integrals
4. Student should be able to determine the appropriate techniques to solve difficult integral.

References

1. Abdul Wahid Md Raji, Hamisan Rahmat, Ismail Kamis, Mohd Nor Mohamad, Ong Chee Tiong***. The First Course of Calculus for Science & Engineering Students***, Second Edition, UTM 2016.

Content

1. Maximum and Minimum
   - First Derivative Test
   - Second Derivative Test
   - Inflection Point(s)

Applications of Differentiation

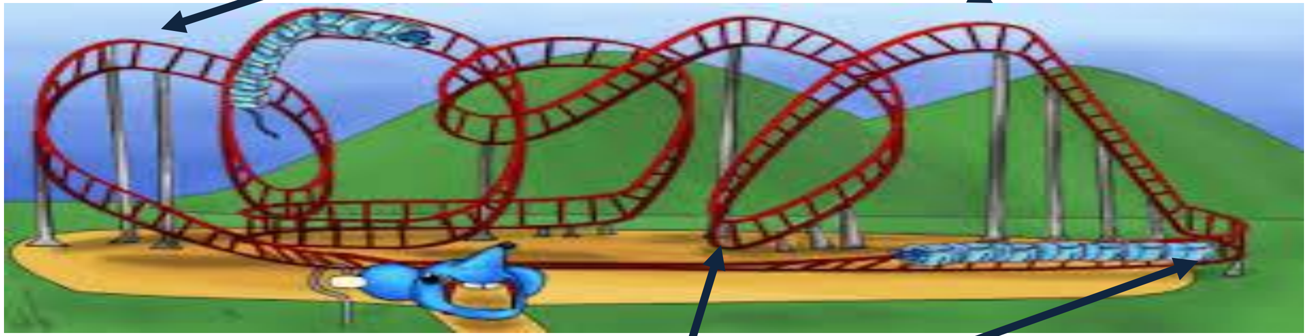
1 Curve/ Slope

2 Related Rates

3 Max & Min Points

Minimum and Maximum Point

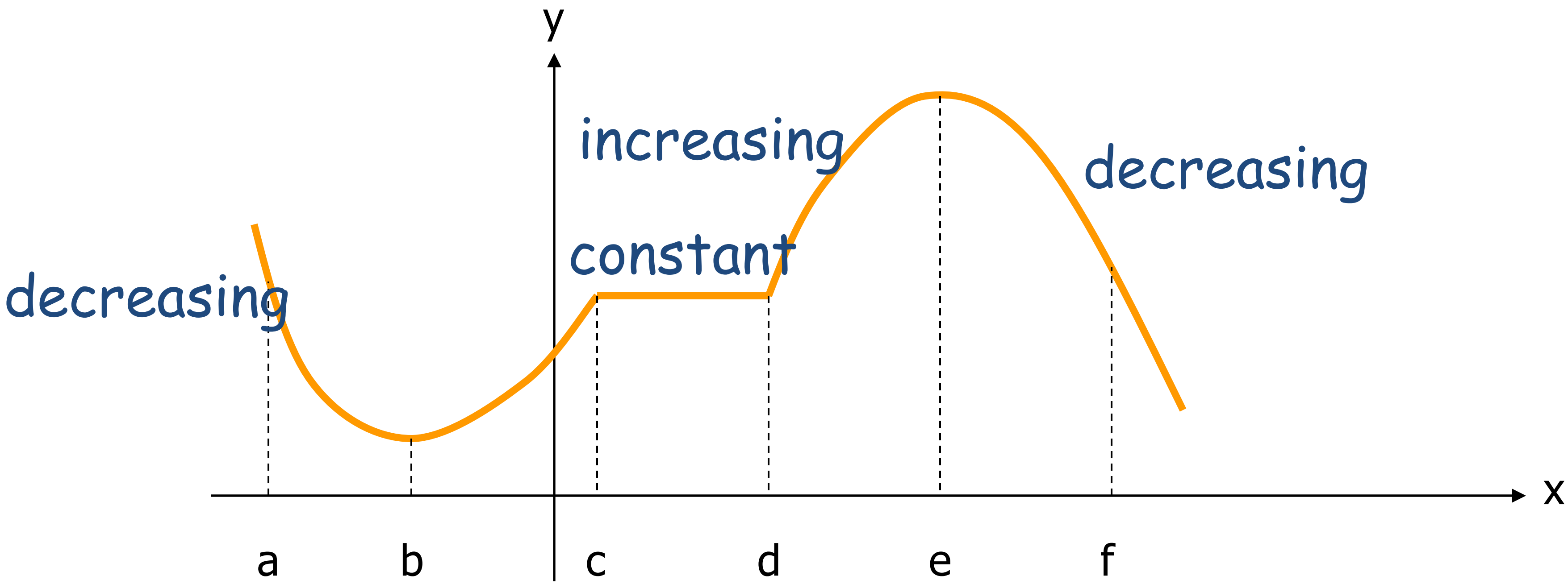
highest point



lowest point

Increasing or Decreasing?

The shape of a graph is related to its defining equation. By analyzing the equation we are able to determine the properties of the graph.

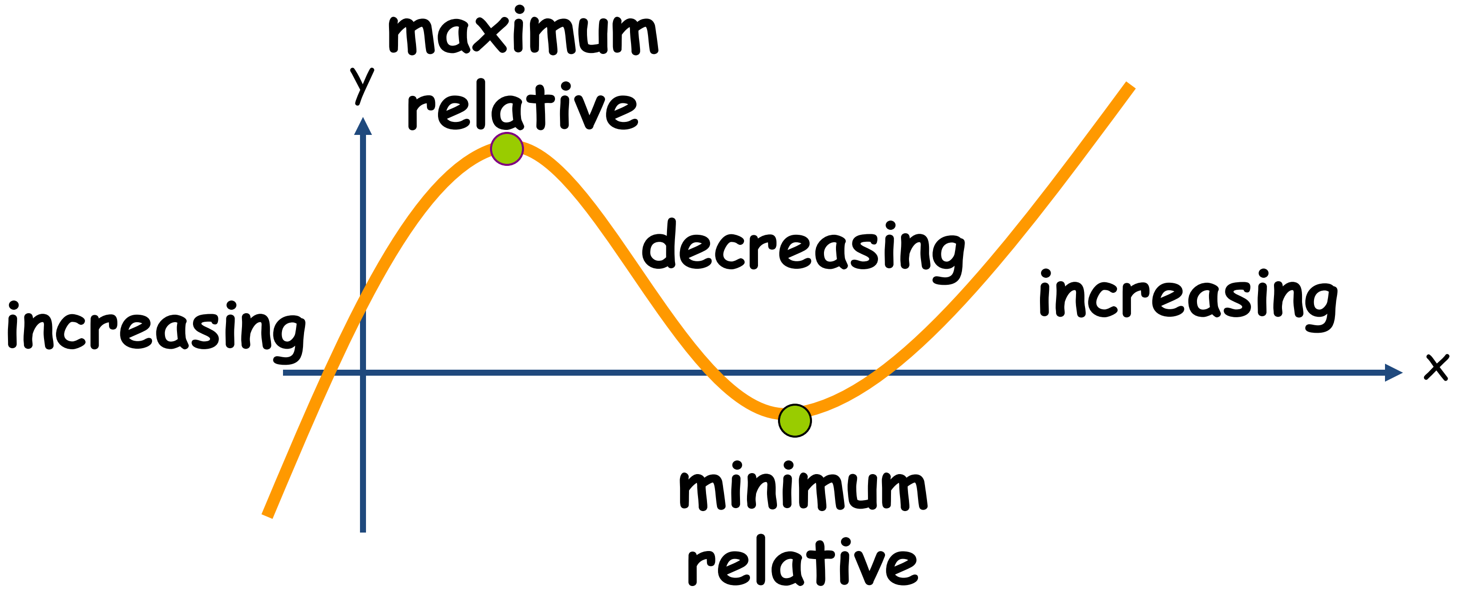


Increasing/Decreasing

Theorem: Let $f(x)$ be a continuous function on the interval $[a,b]$

1. $f(x)$ is increasing if $f(x) > 0$
2. $f(x)$ is decreasing if $f(x) < 0$
3. $f(x)$ is constant if $f(x) = 0$

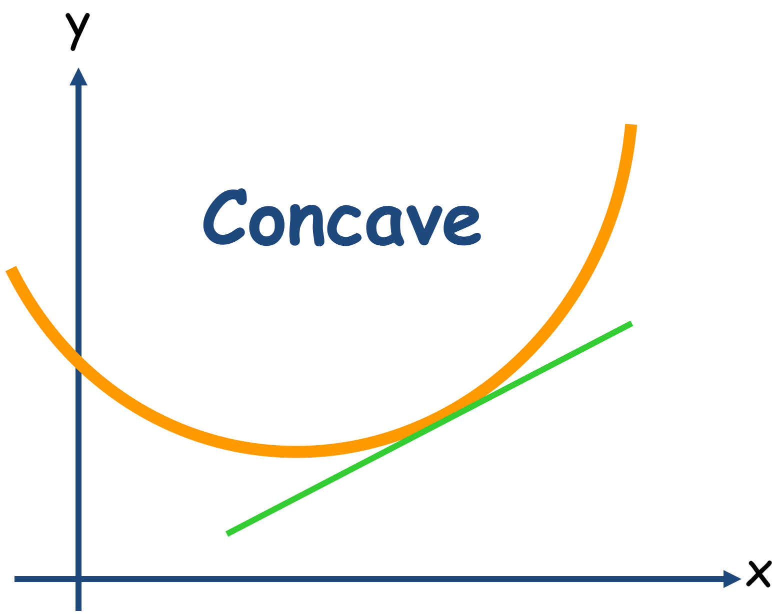
Extremum Relative

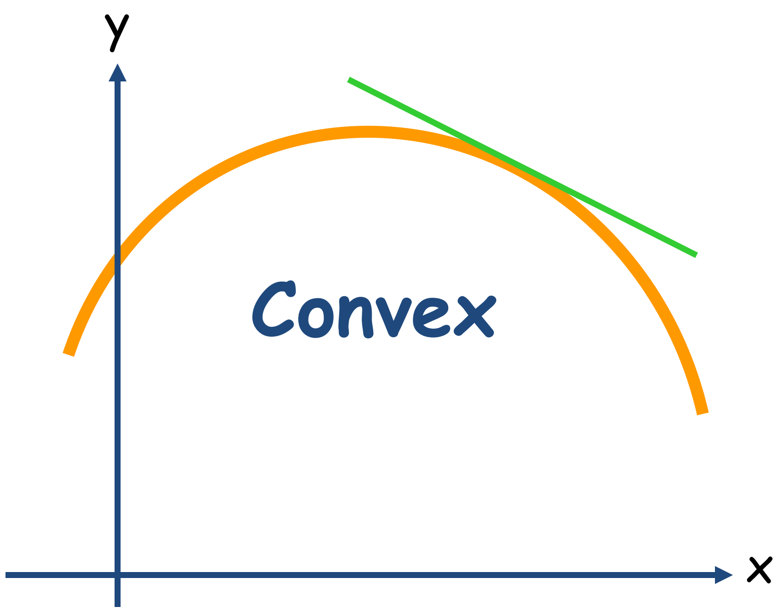


- Refer to the maximum relative and minimum relative point
- Also known as local maximum and local minimum point
- Only at critical points or stationary points
- Not all critical points are extremum relative

Concavity and Convex

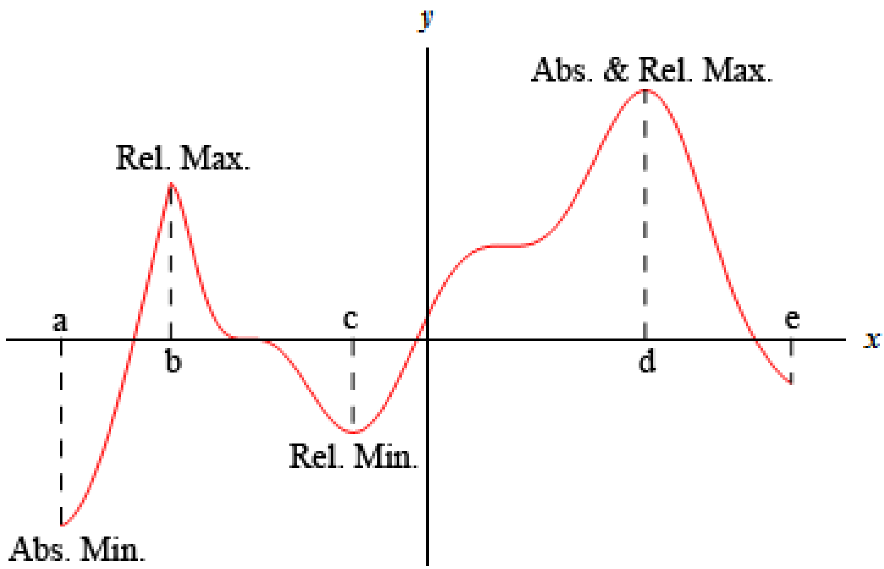
There is another useful way to describe a curve that is convexity and concavity. A curve is said to be concave at the point P if, at a neighbourhood of point P, the curve lies above the tangent line at the point P. Otherwise, it is convex.





Critical Values

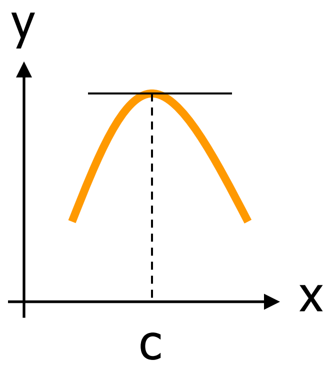
A point $(c, f(c))$ of the function $f(x)$ is a critical values if $f'(c) = 0$ or if does not exist

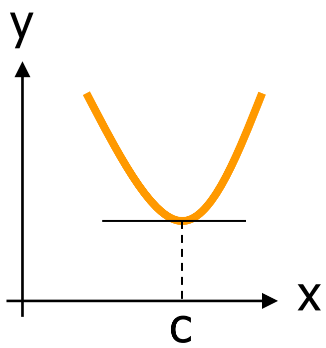


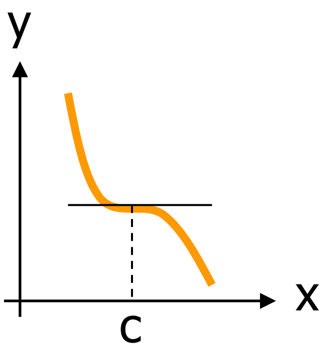
If $f'(x)$ is a quadratic function or a function of higher order, then there is possibility of having more than one critical values.

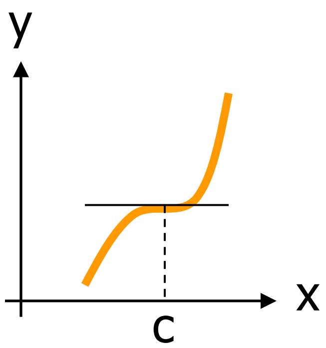
Graphs

Tangent line (horizontal) at x=c → f′(c) = 0;









Example

Find the critical points of the curve

$$f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1$$

$$\text{Answer : The critical points are } \left(1, \frac{7}{3}\right) \text{ and } (3,1)$$

First Derivative Test

MAXIMUM AND MINIMUM VALUE

First derivative test

Suppose c is the critical point.

a. If f' changes positive to negative at c, then f has a **relative maxima** at c

x	x < c	x > c
f'(x)	+	-
Slope		

b. If f' changes negative to positive at c, then f has a **relative minima** at c

x	x < c	x > c
f'(x)	-	+
Slope		

c. If f' does not change in sign at c, then f has **no relative maxima or minima** at c

x	x < c	x > c
f'(x)	+	+
Slope		

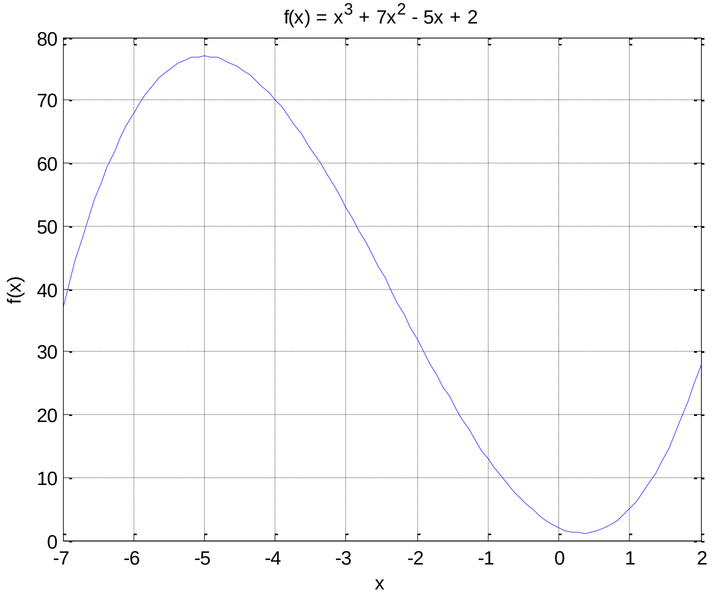
Example

Find all the critical points of the curve and determine their nature using by using first derivative test

(a) $f(x) = x^2 - 4x - 1$

(b) $f(x) = x^3 - 12x + 3$

(c) $f(x) = x^3 - 2x^2 + x$



Second Derivative Test

Assuming that $y = f(x)$ has a critical point at $x = x_0$

1. If $f''(x) < 0$ the graph is convex and $f(x)$ has a maximum point

2. If $f''(x) > 0$ the graph is concave and $f(x)$ has a minimum point

3. If $f''(x) = 0$ or does not exist, the second derivate test fails. We have to use first derivative to determine the property of the extreme points

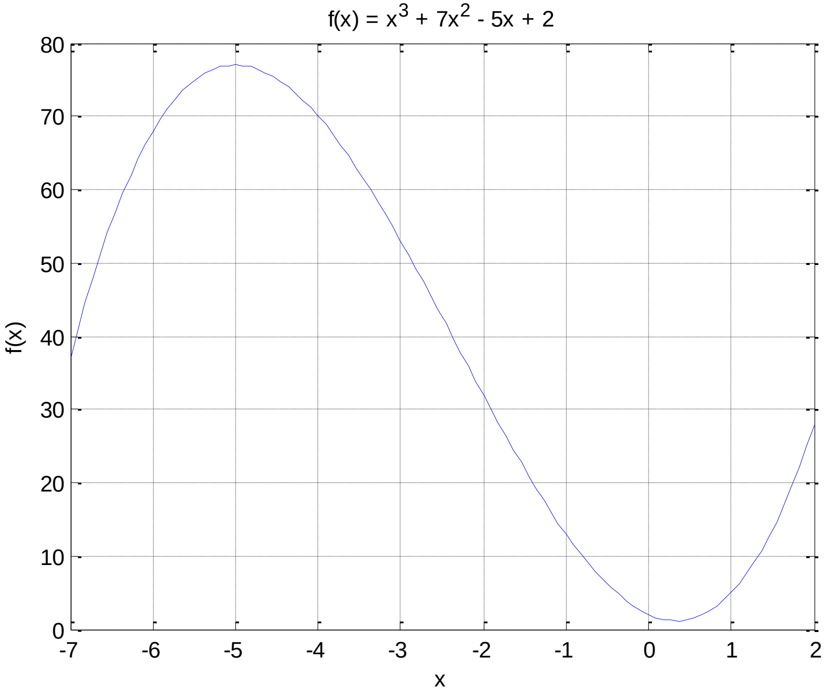
Example

Find all the critical points of the curve and determine their nature using by using second derivative test

(a) $f(x) = x^2 - 4x - 1$

(b) $f(x) = |x|$

(c) $f(x) = x^3 - 2x^2 + x$



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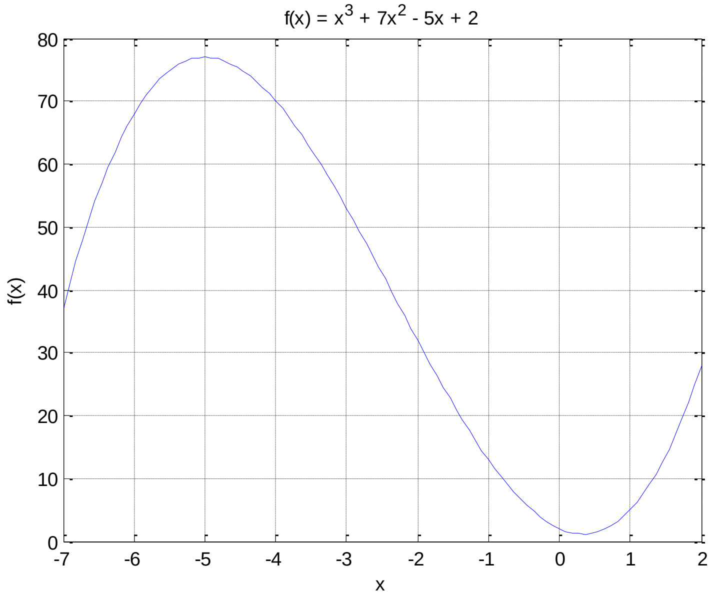
Example

Given $y = x^3 + 7x^2 - 5x + 2$. Find all the critical points of the curve and determine their nature using:

(a) First derivative test

(b) Second derivative test



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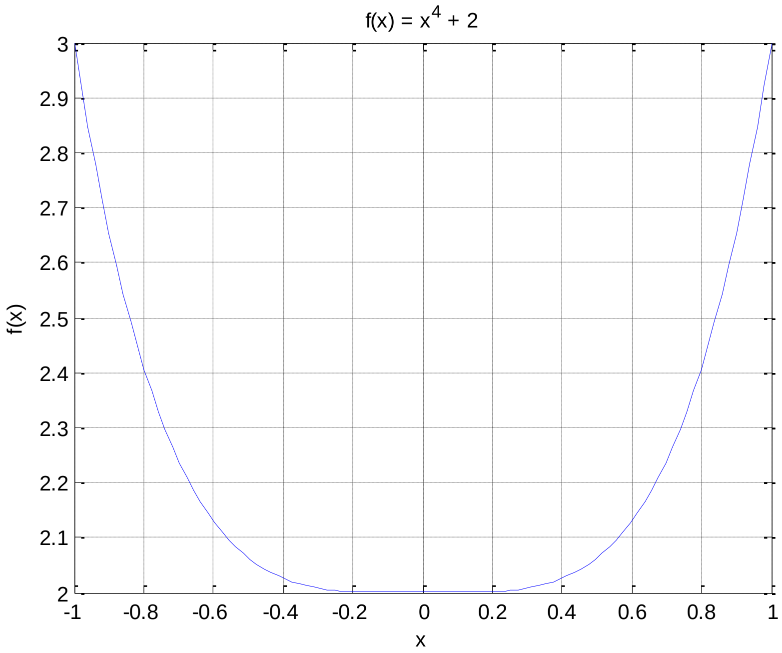
Example

Find the critical points of $y = 2x^3 - 3x^2 - 36x + 14$ and state their nature.

Example

Find the stationary point on the curve $y = x^4 + 2$ and determine the nature of that point.



Example

Given $y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + 5,$ find if exist, the inflection point, maximum and minimum points using

(a) first derivative test

(b) second derivative test

Point of Inflection

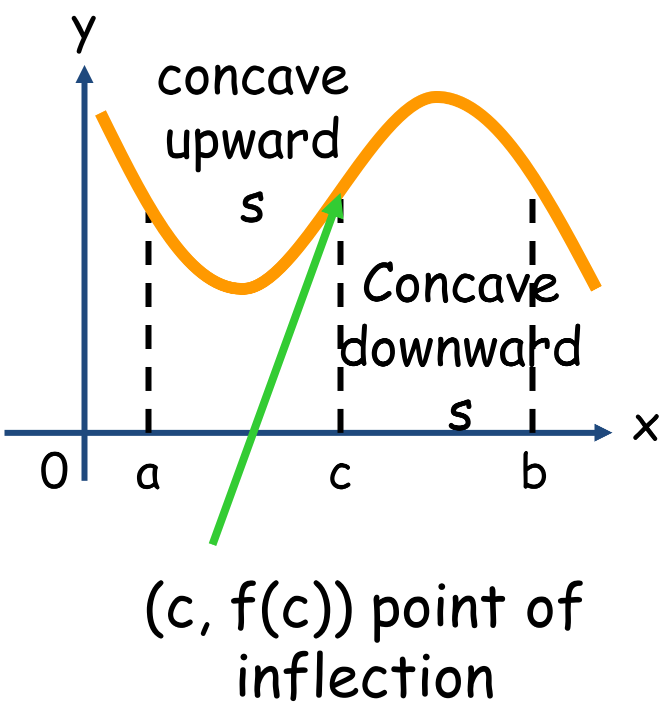
- A point which separates a convex and a concave sections of a continuous function is called a point of inflection.
- If $f''(c)=0$ or if $f''(c)$ does not exist, and if the value of $f''(c)$ changes sign then $\left(c, f(c)\right)$ on the curve is a point of inflection.

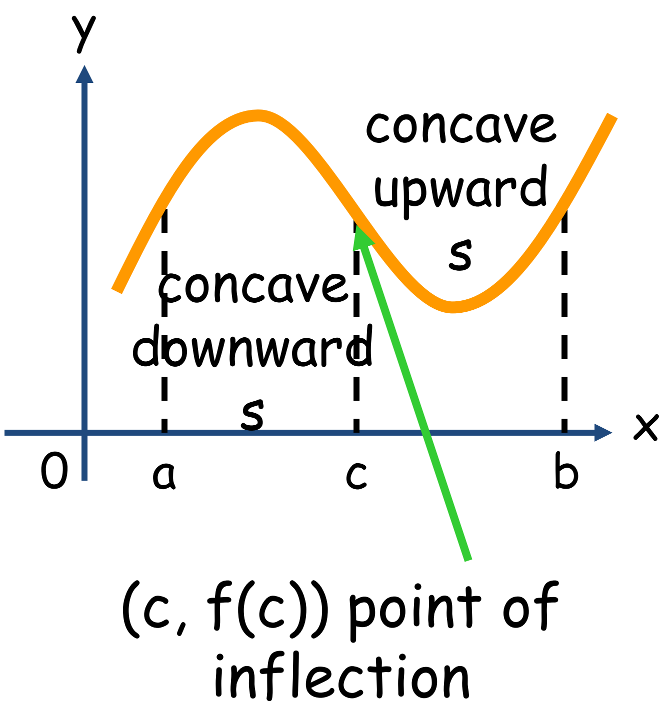
* For point of inflection, not all f'(c) = 0, but **f''(c) = 0**

Let f be any function and (c,f(c)) is a point of inflection if:

a) $f''(x)>0$ for (a, c) and $f''(x)<0$ for (c, b) OR

b) $f''(x)<0$ for (a, c) and $f''(x)>0$ for (c, b)





Example

Given $y = 2\sin x + \cos 2x$ find if exist, the inflection point, maximum and minimum points using second derivative test

Example

Find the inflection point of the curve

$$f(x) = x^3 - 3x^2 + 5$$

$$\text{Answer}: (1,3)\ \text{is an inflection point}$$

Example

Given $y = 3x^4 - 4x^3,$ find if exist, the inflection point, maximum and minimum points using

(a) first derivative test

(b) second derivative test

Conclusion

- In integration by substitution, making appropriate choices for *u* will come with experience.
- Selecting u for by part techniques should follow the LATE guideline.
- If the power of denominator is less than the power of numerator, then the fraction is called proper fraction

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