Calculus
Differentiation

By

Norhafizah Md Sarif
Faculty of Industrial Science & Technology
norhafizah@ump.edu.my

Description

Aims

This chapter is aimed to :

1. introduce the concept of integration
2. evaluate the definite and indefinite integral
3. explain the basic properties of integral
4. compute the integral using different techniques of integration

Expected Outcomes

1. Students should be able to describe the concept of antiderivatives
2. Students should be able to explain about indefinite integral and definite integral
3. Students should be able to know the basic properties of definite integrals
4. Student should be able to determine the appropriate techniques to solve difficult integral.

References

1. Abdul Wahid Md Raji, Hamisan Rahmat, Ismail Kamis, Mohd Nor Mohamad, Ong Chee Tiong***. The First Course of Calculus for Science & Engineering Students***, Second Edition, UTM 2016.

Content

1. Derivative of Composite Function
2. Higher Derivative
3. Implicit Derivatives

Implicit Differentiation

Explicit

- The variable y appears alone one side of the equation.
- E.g: $y = 5x^3 - 4$

Implicit

- The variable y is not alone on one side of the equation.
- E.g: $\sin x + 3y = 5x^3 - 4y^2$

How to differentiate implicit functions?:

1. Differentiate both sides of the equation
2. Collect terms dy/dx on one side of the equation
3. Factor out dy/dx
4. Solve for dy/dx

Example

Find $\frac{dy}{dx}$ for $x^3 + y^2 + 1 = 2y$.

$$\frac{d}{dx}\left[x^3 + y^2 + 1\right] = \frac{d}{dx}[2y]$$

1. Differentiate both sides of the equation

$$3x^2\frac{dx}{dx} + 2y\frac{dy}{dx} + 0 = 2\frac{dy}{dx}$$

$$2y\frac{dy}{dx} - 2\frac{dy}{dx} = -3x^2$$

2. Collect terms dy/dx on one side of the equation

$$(2y-2)\frac{dy}{dx} = -3x^2$$

3. Factor out

$$\frac{dy}{dx} = \frac{-3x^2}{2y-2}$$

4. Solve for dy/dx

Example

Find $\frac{dx}{dt}$ for $x^2 + t^2 - 2x = \sin 3t$

Example

Find $\frac{dy}{dx}$ for $2y^2 - 3x^3 = x + y$.

$$\frac{d}{dx}\left[2y^2 - 3x^3\right] = \frac{d}{dx}\left[x + y\right]$$

$$\frac{d}{dx}(2y^2) - \frac{d}{dx}(3x^3) = \frac{d}{dx}\left[x + y\right]$$

$$4y\frac{dy}{dx} - 9x^2 = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1 + 9x^2}{4y - 1}$$

Example

Find $\frac{dy}{dx}$ for $x^2y^3 - e^y = e^{2x}$.

$$\frac{d}{dx}\left[x^2y^3 - e^y\right] = \frac{d}{dx}\left[e^{2x}\right]$$

$$\frac{d}{dx}(x^2y^3) - \frac{d}{dx}(e^y) = \frac{d}{dx}\left[e^{2x}\right]$$

$$x^2\frac{d}{dx}(y^3) + y^3\frac{d}{dx}(x^2) - \frac{d}{dx}(e^y) = 2e^{2x}$$

$$3x^2y^2\frac{dy}{dx} + 2y^3x - e^y\frac{dy}{dx} = 2e^{2x}$$

$$\frac{dy}{dx} = \frac{2(e^{2x} - y^3x)}{3x^2y^2 - e^y}$$

Example

If $x^2 + y^2 - 2x - 6y + 5 = 0$, find $\frac{dy}{dx}$ at $x = 3,\ y = 2$.

$$\frac{d}{dx}\left[x^2 + y^2 - 2x - 6y + 5\right] = \frac{d}{dx}[0]$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) - \frac{d}{dx}(2x) - \frac{d}{dx}(6y) + \frac{d}{dx}(5) = \frac{d}{dx}[0]$$

$$2x + 2y\frac{dy}{dx} - 2 - 6\frac{dy}{dx} + 0 = 0$$

$$\frac{dy}{dx} = \frac{1 - x}{y - 3}$$

At $x = 3, y = 2$

$$\frac{dy}{dx} = \frac{1-3}{2-3} = 2$$

Parametric Equation

- Consider the parametric equation

$$y = f(t) \quad \text{and} \quad x = g(t)$$

Solve first derivative by using chain rule

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Example

Parametric equation of a curve is given by $x = \frac{t-3}{t}$ and $y = \frac{t^2+4}{t}$

(i) Find $\frac{dy}{dx}$ in terms of u.

(ii) y in terms of x and hence find $\frac{dy}{dx}$

Example

Let $x = a\cos^3\theta$ and $y = a\sin^3\theta$. Find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$ rad.

Example

Given $x = t + \frac{1}{t}$ and $y = (t+1)^2$. Show that $\frac{dy}{dx} = \frac{2t^2}{t-1}$.

Example

Given $y = \dfrac{u-5}{1+u^2}$ and $u = \sqrt{x+7}$. Find $\dfrac{dy}{dx}$ when $u = 1$.

Higher Derivative

If $y = f(x)$, then

$\frac{dy}{dx} = f'(x)$ is the first derivative of y

$\frac{d^2y}{dx^2} = f''(x)$ is the second derivative of y

$\frac{d^3y}{dx^3} = f'''(x)$ is the third derivative of y

$\vdots$

$\frac{d^{(n)}y}{dx^{(n)}} = f^{n}(x)$ is the n^{th} derivative of y

Example

Find $\frac{d^2y}{dx^2}$ for each of the following

(a) $y = 8x^4 + 7x^3$

(b) $y = 5(3x+1)^6$

(c) $y = 2e^{-2x} + 3e^{3x}$

Example

If $y = 2x^2 + \frac{1}{x^3}$ prove that $x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 6y = 0$.

First derivative: $\frac{dy}{dx} = 4x - \frac{3}{x^4}$ (1)

Second derivative: $\frac{d^2y}{dx^2} = 4 + \frac{12}{x^5}$ (2)

Substitute (1), (2), and y into second order differential equation to prove the equation

$$x^2 \frac{d^2y}{dx} + 2x\frac{dy}{dx} - 6y = x^2\left(4 + \frac{12}{x^5}\right) + 2x\left(4x - \frac{3}{x^4}\right) - 6\left(2x^2 + \frac{1}{x^3}\right)$$

$$= 4x^2 + \frac{12}{x^3} + 8x^2 - \frac{6}{x^3} - 12x^2 - \frac{6}{x^3}$$

$$= 0 \text{ (proven)}$$

Example

If $y = xe^{3x}$ show that $6\frac{dy}{dx} - \frac{d^2y}{dx^2} - 9y = 0$.

First derivative: $\frac{dy}{dx} = 3xe^{3x} + e^{3x}$ (1)

Second derivative: $\frac{d^2y}{dx^2} = 6e^{3x} + 9xe^{3x}$ (2)

Substitute (1), (2), and y into second order differential equation to prove the equation

$$6\frac{dy}{dx} - \frac{d^2y}{dx} - 9y = 6\left(3xe^{3x} + e^{3x}\right) - \left(6e^{3x} + 9xe^{3x}\right) - 9\left(xe^{3x}\right)$$

$$= 18xe^{3x} + 6e^{3x} - 6e^{3x} - 9xe^{3x} - 9xe^{3x}$$

$$= 0 \text{ (shown)}$$

Example

Find $\frac{d^2y}{dx^2}$ for $y = 3t^4$ and $x = 2t^2 - 4$

Example

Given $x = t^2 + 2t$ for $y = \frac{1}{t}$ and $\frac{d^2y}{dx^2}$.

Conclusion #1

- Any indefinite integral will have +c at the end of the solution.
- There are two approach in getting the solution of definite integral: by changing the limit of x into u, or by changing function u into x and use the original limit.
- Product or quotient function cannot be integrate directly. Appropriate techniques should be used to solve this type of integral
- In integration by substitution, making appropriate choices for *u* will come with experience.
- Selecting u for by part techniques should follow the LATE guideline.

Author Information

Norhafizah Binti Md Sarif
Lecturer
Faculty of Industrial Sciences & Technology (website)
Universiti Malaysia Pahang
Email: norhafizah@ump.edu.my
Google Scholar: : Norhafizah Md Sarif
Scopus ID : 57190252369
UmpIR ID: 3479