



Exercise 3: Differentiation

Topic 3.1.1 : Derivative using first principle

1. By using differentiation from first principle, find the derivatives of the following functions.

a) $y = 4x^2$

c) $y = x + x^2$

e) $y = \frac{x}{x-1}$

b) $y = (x-1)^2 + 1$

d) $y = \sqrt{2x-1}$

f) $y = x - \frac{1}{x^2}$

$$\left[\text{(a) } 8x \quad \text{(b) } 2(x-1) \quad \text{(c) } 1+2x \quad \text{(d) } \frac{1}{\sqrt{2x-1}} \quad \text{(e) } -\frac{1}{(1-x)^2} \quad \text{(f) } 1 + \frac{2}{x^3} \right]$$

Topic 3.1.2 : Derivative using table

2. Find the derivative of the following functions

a) $y = 3$

b) $y = \frac{100}{3}$

c) $y = \pi$ ✓

d) $y = x^{56}$

e) $y = \frac{1}{\sqrt{x}}$ ✓

f) $y = 8\sqrt[4]{x^3}$

$$\left[\text{(a) } 0 \quad \text{(b) } 0 \quad \text{(c) } 0 \quad \text{(d) } 56x^{55} \quad \text{(e) } -\frac{1}{2x^{\frac{3}{2}}} \quad \text{(f) } \frac{6}{x^{\frac{1}{4}}} \right]$$

3. Differentiate $f(x) = 2x^4 + 3\cos x + \sin a$ with respect to x (a constant)

4. Find y' of the following functions

a) $y = \frac{1}{3}x^5$

b) $y = -\frac{2}{3}\sin x - e^x$

c) $y = \frac{1}{2}t^3 + \frac{3}{t^2}$

d) $y = a^3 + 2\tan a$ ✓

e) $y = \frac{1}{2}x^3 - \frac{3}{4}x^2 + \ln x$

f) $y = (x-1)(x^2-2x)x^{-4}$ ✓

g) $y = \frac{3x^2 - 2x + 1}{\sqrt{x}}$ ✓

h) $y = \frac{2\sin x - 3\cos x}{\cos x}$ ✓

i) $y = \frac{e^{2x} - 1}{e^x - 1}$

$$\left[\text{(a) } \frac{5}{3}x^4 \quad \text{(b) } -\frac{2}{3}\cos x - e^x \quad \text{(c) } \frac{3}{2}t^2 - \frac{6}{t^3} \right]$$
$$\left[\text{(d) } 3a^2 + 2\sec^2 a \quad \text{(e) } \frac{3}{2}x^2 - \frac{3}{2}x + \frac{1}{x} \quad \text{(f) } \frac{-x^2 + 6x - 6}{x^4} \right]$$

$$(g) \frac{9}{2}\sqrt{x} - \frac{1}{\sqrt{x}} - \frac{1}{2x\sqrt{x}} \quad (h) 2\sec^2 x \quad (i) e^x]$$

5. Differentiate

$$\begin{array}{lll} a) y = \frac{1}{4}(x^7 - 5) & b) y = (1 + \cos x) \tan x & c) y = (2 - x^3)(x^3 + x - 1) \\ d) y = (x^2 + 1)\left(x - 5 - \frac{1}{x}\right) \checkmark & e) y = \frac{3}{x^6} + \frac{1}{x^5} - \frac{7}{x^2} & f) y = (3x^2 + 1)^2 \checkmark \\ g) y = \frac{8}{\sqrt[4]{x}} - \frac{3}{\sqrt{x^3}} & h) y = \left(\frac{3x+2}{x}\right)(x^{-5} + 1) \checkmark & i) y = -10x^{-1/2} + 8x^{-3/2} \end{array}$$

$$[(a) \frac{7}{4}x^6 \quad (b) \sec^2 x + \cos x \quad (c) -6x^5 - 4x^3 + 9x^2 + 2 \quad (d) 3x^2 - 10x + \frac{1}{x^2} \quad (e) -\frac{18}{x^7} - \frac{5}{x^6} + \frac{14}{x^3}$$

$$(f) 12x(3x^2 + 1) \quad (g) \frac{-2}{x^4} + \frac{9}{2x^2} \quad (h) -\frac{15}{x^6} - \frac{12}{x^7} - \frac{2}{x^2} \quad (i) 5x^{-3/2} - 12x^{-5/2}]$$

Topic 3.2 : Chain Rule

6. Find y' of the following function by using chain rule.

$$\begin{array}{lll} a) y = (x + 2\sin x)^5 & b) y = \ln(x^2 - 1) & c) y = e^{\cos x - 3} \checkmark \\ d) y = \tan(e^x - 1) & e) y = \sqrt{x(1-x)} \checkmark & f) y = 3\sqrt{2x} - 4\cos(x) \\ g) y = \frac{2}{\sqrt{x+3}} & h) y = e^{x^3-2x} + \frac{5}{\sqrt{x+1}} & i) y = -3\ln(x^2 + \sin x) \checkmark \end{array}$$

$$[(a) 5(x + 2\sin x)^4(1 + 2\cos x) \quad (b) \frac{2x}{x^2 - 1} \quad (c) -e^{\cos x - 3} \sin x$$

$$(d) e^x \sec^2(e^x - 1) \quad (e) \frac{1}{2} \left(\frac{1-2x}{\sqrt{x-x^2}} \right) \quad (f) \frac{3}{\sqrt{2x}} + 4\sin x$$

$$(g) -\frac{1}{(\sqrt{x+3})^2 \sqrt{x}} \quad (h) (3x^2 - 2)e^{x^3-2x} - \frac{5}{2(x+1)^{3/2}} \quad (i) -\frac{3(2x + \cos x)}{x^2 + \sin x}]$$

7. Use chain rule to compute the derivative and write your answer in terms of x

$$a) y = u^2 + 1; u = 3x - 2 \quad b) y = \frac{2}{u^2}; u = x^2 - 9 \quad c) y = u^3 - 3u^2 + 1; u = x^2 + 2 \checkmark$$

[(a) $6(3x-2)$ (b) $\frac{-8x}{(x^2-9)^3}$ (c) $6x^5+12x^3$]

8. Differentiate $y = (3x+2)^2$ ✓

- (a) By expansion
 (b) By the product rule
 (c) By the chain rule

[$6(3x+2)$]

9. Find the derivatives of each function.

a) $y = \sqrt[3]{5x^6 - 12}$

b) $y = \frac{2}{\sqrt{4x^2 + 1}}$ ✓

c) $y = \sqrt{1 + \frac{1}{4x^3}}$

d) $y = \frac{3}{2(5x^4 + 1)}$

e) $y = \frac{5}{3} \cos(2\pi - 5x^2)$

f) $y = \sin^3(4x - 1)$ ✓

g) $y = \sqrt{\cos \sqrt{x}}$

h) $y = \tan^2 2x^2$

i) $y = \ln(x^3 - x)$

j) $y = 3(\ln 2x)^3$

k) $y = \ln(x+1)(2x-1)$ ✓

l) $y = -6 \ln \sqrt{2x+3}$

m) $y = \sqrt{\frac{2}{e^{2x}}}$ ✓

n) $y = \frac{1}{3\sqrt{e^x}}$

o) $y = e^{2x} - e^{-3x}$

[(a) $\frac{10x^2}{\sqrt[3]{(5x^6 - 12)^2}}$

(b) $\frac{-8x}{\sqrt{(4x^2 + 1)^3}}$

(c) $\frac{3}{8x^4 \left(\sqrt{1 + \frac{1}{4x^3}} \right)}$

(d) $\frac{-30x^3}{(5x^4 + 1)^2}$

(e) $\frac{50}{3} x \sin(2\pi - 5x^2)$

(f) $12(\sin^2(4x - 1))(\cos(4x - 1))$

(g) $\frac{-\sin \sqrt{x}}{4\sqrt{x} \cos \sqrt{x}}$

(h) $-(\sin x \tan x) + (1 + \cos x) \sec^2 x$

(i) $\frac{3x^2 - 1}{x^3 - x}$

(j) $\frac{3(\ln 2x)^2}{2x}$

(k) $\frac{4x + 1}{(x + 1)(2x - 1)}$

(l) $\frac{-6}{2x + 3}$

(m) $-\frac{\sqrt{2}}{e^x}$

(n) $\frac{-1}{6\sqrt{e^x}}$

(o) $2e^{2x} + 3e^{-3x}$]

Topic 3.2 : Product Rule

10. Find the derivatives of each function defined as follow

a) $y = (4x^2 + 2)(3x^3 - 1)$ ✓

b) $y = 4x^2(x^2 + 1)^{5/4}$

c) $y = (2x - 1)^3(x + 1)^5$

d) $y = 5x \sin x$ e) $y = x\sqrt{\sin(3-x^2)}$ ✓ f) $y = \sin x \tan x$
g) $y = \ln(x \sin x)$ ✓ h) $y = x^2 e^{-x}$ i) $y = x e^{\sin x}$ ✓
j) $y = e^x \ln x$ k) $y = x \csc 2x$ l) $y = \sin^2 x \tan^3 3x$

[(a) $60x^4 + 18x^2 - 8x$ (b) $2x(x^2 + 1)^{1/4} (9x^2 + 4)$ (c) $(16x + 1)(2x - 1)^2 (x + 1)^4$
(d) $5x \cos x + 5 \sin x$ (e) $\frac{\sin(3-x^2) - x \cos(3-x^2)}{\sqrt{\sin(3-x^2)}}$ (f) $\tan x (\sec x + \cos x)$
(g) $\frac{1}{x} + \cot x$ (h) $x(2-x)e^{-x}$ (i) $(x \cos x + 1)e^{\sin x}$ (j) $\left(\ln x + \frac{1}{x}\right)e^x$
(k) $\csc 2x(1 - 2x \cot 2x)$ (l) $\sin x \tan^2 3x(9 \sin x \sec^2 3x + 2 \tan 3x \cos x)$]

Topic 3.2 : Quotient Rule

11. Find the derivatives of each function defined as follow

a) $y = \frac{x}{x+1}$ b) $y = \frac{1}{x^3 - 2x + 5}$ ✓ c) $y = \frac{2x+5}{3x+2}$
d) $y = \frac{x}{\cos x}$ e) $y = \frac{\sin x}{2 + \sin 2x}$ f) $y = \cot\left(\frac{1}{1+x^2}\right)$
g) $y = \ln\left(\frac{\sqrt{1-2x}}{3x}\right)$ ✓ h) $y = \frac{e^{3x}}{(x+1)^2}$ ✓ i) $y = \frac{e^{-x}}{1+x^2}$

[(a) $\frac{1}{(x+1)^2}$ (b) $\frac{2-3x^2}{(x^3-2x+5)^2}$ (c) $-\frac{11}{(3x+2)^2}$ (d) $\frac{\cos x + x \sin x}{\cos^2 x}$
(e) $\frac{(2 + \sin 2x) \cos x - 2 \sin x \cos 2x}{(2 + \sin 2x)^2}$ (f) $\frac{2x}{(1+x^2)^2} \csc^2\left(\frac{1}{1+x^2}\right)$ (g) $\frac{x-1}{x(1-2x)}$
(h) $\frac{(3x+1)}{(x+1)^3} e^{3x}$ (i) $\frac{-(1+x)^2}{(1+x^2)^2} e^{-x}$]