Calculus Differentiation

By

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Description

Aims

This chapter is aimed to :

1. introduce the concept of integration
2. evaluate the definite and indefinite integral
3. explain the basic properties of integral
4. compute the integral using different techniques of integration

Expected Outcomes

1. Students should be able to describe the concept of antiderivatives
2. Students should be able to explain about indefinite integral and definite integral
3. Students should be able to know the basic properties of definite integrals
4. Student should be able to determine the appropriate techniques to solve difficult integral.

References

1. Abdul Wahid Md Raji, Hamisan Rahmat, Ismail Kamis, Mohd Nor Mohamad, Ong Chee Tiong***. The First Course of Calculus for Science & Engineering Students***, Second Edition, UTM 2016.

Content

1 Introduction to Differentiation

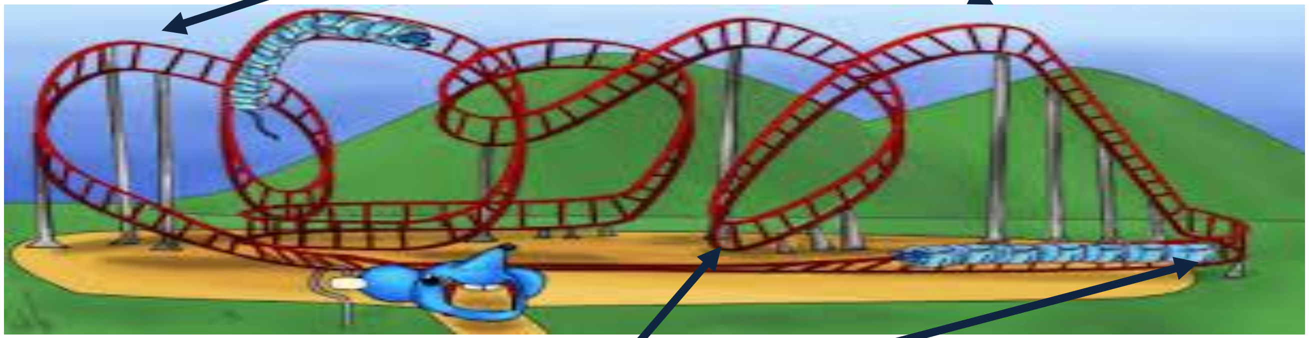
2 Derivative of a Function

3 Techniques of Differentiation

Applications

Do you know that we can use differentiation to find the highest point and the lowest point of the roller coaster track?

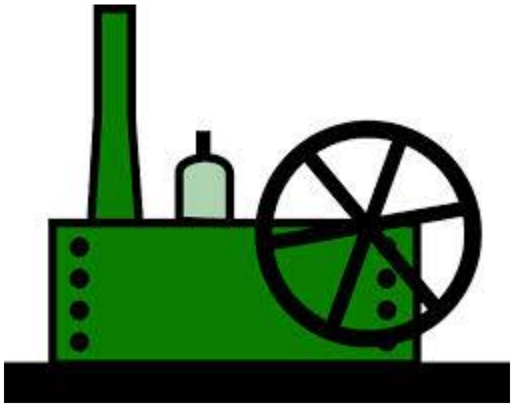
highest point



lowest point

http://tutorial.math.lamar.edu/Classes/CalcI/Tangents_Rates.aspx

- We use the **derivative** to determine the **maximum and minimum values** of particular functions
  - cost, strength,
  - amount of material used in a building
  - profit & loss



Introduction to Differentiation

- Differentiation is all about finding rates of change of one quantity compared to another.
- The process of finding a derivative is called differentiation
- The most common notations for the derivative are

$$y' \qquad \frac{dy}{dx} \qquad f'(x)$$

Derivative of a Function

Definition – First Principle: The derivative of the function $f(x)$ with respect at a point (x) if the function where

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The process of finding derivative using the above definition is called the first principle.

Differentiation using first principle

1. Write an expression for $f(x)$
2. Write an expression for $f(x+h)$
3. Substitute $f(x)$ and $f(x+h)$ into the formula
4. Simplify the expression
5. Evaluate limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example

By using differentiation from the first principle, find the derivatives of the following functions

(a) -4 (b) $2x+5$ (c) $\frac{1}{x}$ (d) $\sqrt{x+2}$

(a) Given $f(x)=4$, then $f(x+h)=4$

Substitute $f(x)$ and $f(x+h)$ into first principle formula,

$$f'(x)=\lim_{h\to 0}\frac{4-4}{h}$$

So,

$$\therefore f'(x)=0$$

Example

(b) $f(x)=2x+5$

$f(x)=2x+5$ — 1. Write an expression for $f(x)$

$f(x+h)=2(x+h)+5$ — 2. Write an expression for $f(x+h)$

$$f'(x)=\lim_{h\to 0}\frac{2(x+h)+5-2x+5}{h}$$ — 3. Substitute $f(x)$ and $f(x+h)$

$$\therefore f'(x)=\lim_{h\to 0}\frac{2\cancel{h}}{\cancel{h}}=\lim_{h\to 0}2=2$$ — 5. Evaluate the limit

4. Simplify the expression

Example

(c) $f(x)=\frac{1}{x}$

$$f(x)=\frac{1}{x}$$

1. Write an expression for $f(x)$

$$f(x+h)=\frac{1}{x+h}$$

2. Write an expression for $f(x+h)$

$$f'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

3. Substitute $f(x)$ and $f(x+h)$

$$f'(x)=\lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h}=\lim_{h\to 0}\frac{-\cancel{h}}{x\cancel{h}(x+h)}$$

4. Simplify the expression *

$$\therefore\ f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}=-\frac{1}{x^2}$$

5. Evaluate the limit

(d) $f(x)=\sqrt{x+2}$

$f(x+h)=\sqrt{x+h+2}$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\\
&= \lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\cdot\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}\\
&= \lim_{h\to 0}\frac{(x+h+2)-(x+2)}{h(\sqrt{x+h+2}+\sqrt{x+2})}\\
&= \frac{1}{\sqrt{x+2}+\sqrt{x+2}}\\
&= \frac{1}{2\sqrt{x+2}}
\end{aligned}
$$

Multiply the numerator and denominator with conjugate

Table of Differentiation

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Common Functions	Functions	Derivatives
Constant	$y = c$	$\frac{dy}{dx} = 0$
Power	$y = x^n$	$\frac{dy}{dx} = nx^{n-1}$
Exponential	$y = e^x$	$\frac{dy}{dx} = e^x$
	$y = a^x$	$\frac{dy}{dx} = a^x(\ln a)$
Logarithms	$y = \ln x$	$\frac{dy}{dx} = \frac{1}{x}$
	$y = \log_a x$	$\frac{dy}{dx} = \frac{1}{x \ln a}$
Trigonometry (x is in rad)	$y = \sin x$	$\frac{dy}{dx} = \cos x$
	$y = \cos x$	$\frac{dy}{dx} = -\sin x$
	$y = \tan x$	$\frac{dy}{dx} = \sec^2 x$
	$y = \sec x$	$\frac{dy}{dx} = \sec x \tan x$
Hyperbolic	$y = \sinh x$	$\frac{dy}{dx} = \cosh x$
	$y = \cosh x$	$\frac{dy}{dx} = -\sinh x$
	$y = \tanh x$	$\frac{dy}{dx} = \text{sech}^2 x$
Inverse Trigonometry	$y = \sin^{-1} x$	$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$
	$y = \cos^{-1} x$	$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$
	$y = \tan^{-1} x$	$\frac{dy}{dx} = \frac{1}{1+x^2}$

Techniques of Differentiation

Definition – Chain Rule: If $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

The process of finding derivative using the above definition is called the chain rule.

Basic function such as $\sin t, e^t, \ln t$ can be solve using table of differentiation. However, for case $\sin t^2, e^{3t-2}, \ln 5t$ we need to use chain rule to obtain the solution.

Example

Differentiate the following functions

(a) $y = (3x+5)^4$, (b) $y = \sqrt{1-t^2}$, (c) $y = 5\sin(2x^3)$,

(a)

$$y = \left(3x+5\right)^4 \quad \text{and} \quad u = 3x+5, \quad \text{hence} \;\; y = u^5$$

$$\text{Then,} \; \frac{dy}{du} = 5u^4, \quad \text{and} \quad \frac{du}{dx} = 3$$

$$\therefore \quad \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = (5u^4)(3) = 15u^4 = 15(3x+5)^4$$

(b)

$$y = \sqrt{1-t^2} \quad \text{and} \quad u = 1-t^2, \quad \text{hence} \;\; y = u^{1/2}$$

$$\text{Then,} \; \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}, \quad \text{and} \quad \frac{du}{dx} = -2t$$

$$\therefore \quad \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = (\frac{1}{2}u^{-\frac{1}{2}})(-2t) = -tu^{-\frac{1}{2}} = -\frac{t}{\sqrt{1-t^2}}$$

- How about these examples?

(a) $y=(4x+1)^2(3x)$

(b) $y=x\sin x$

(c) $y=t^2e^t$

(d) $y=e^{-2t}\cos(3t)$

(e) $y=\dfrac{x}{x+3}$

(f) $y=\dfrac{\sin x}{x}$

(g) $y=\dfrac{4e^{-2x}}{1+x^2}$

Techniques of Differentiation

- Product rule is a formula used to find the derivatives of product of two or more functions

Definition – Product Rule: If $u(x)$ and $v(x)$ are differentiable function and $y = u(x) \cdot v(x)$, then

$$\frac{dy}{dx} = v\frac{dy}{du} + u\frac{dv}{dx}$$

Example

Differentiate the following functions

(a) $y=(4x+1)(3x)$, (b) $y=e^{-2t}\cos(3t)$, (c) $y=x\sin x$

(a)
$$\frac{dy}{dx}=(4x+1)\frac{d}{dx}(3x)+\frac{d}{dx}(4x+1)\cdot(3x)$$
$$=(4x+1)\cdot(3)+(4)\cdot(3x)$$
$$=24x+3$$

(b)
$$\frac{dy}{dt}=e^{-2t}\frac{d}{dx}(\cos 3t)+\frac{d}{dx}(e^{-2t})\cdot\cos(3t)$$
$$=e^{-2t}\cdot(-3\sin 3t)+(-2e^{-2t})\cdot\cos(3t)$$
$$=-3e^{-2t}\sin 3t-2e^{-2t}\cos 3t$$

Techniques of Differentiation

- The quotient rule is a formal rule for differentiating problems where one function is divided by another

Definition – Quotient Rule: If $u(x)$ and $v(x)$ are differentiable function and $y = \dfrac{u(x)}{v(x)}$ where $v(x) \neq 0$, then

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

Example

Differentiate the following functions

$$\text{(a) } y=\frac{4x+1}{3x}, \quad \text{(b) } y=\frac{\sin x}{x}, \quad \text{(c) } y=\frac{4e^{-2x}}{1+x^2}$$

(a)

$$\frac{dy}{dx}=\frac{3x\cdot\frac{d}{dx}(4x+1)-\frac{d}{dx}(3x)\cdot(4x+1)}{(3x)^2}$$

$$=\frac{3x(4)-3\cdot(4x+1)}{(3x)^2}=-\frac{1}{3x^2}$$

(b)

$$\frac{dy}{dx}=\frac{x\cdot\frac{d}{dx}(\sin x)-\frac{d}{dx}(x)\cdot(\sin x)}{x^2}$$

$$=\frac{x(\cos x)-1\cdot\sin x}{x^2}=\frac{x\cos x-\sin x}{x^2}$$

Conclusion #1

- Any indefinite integral will have +c at the end of the solution.
- There are two approach in getting the solution of definite integral: by changing the limit of x into u, or by changing function u into x and use the original limit.
- Product or quotient function cannot be integrate directly. Appropriate techniques should be used to solve this type of integral
- In integration by substitution, making appropriate choices for *u* will come with experience.
- Selecting u for by part techniques should follow the LATE guideline.
- If the power of denominator is less than the power of numerator, then the fraction is called proper fraction

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