Exercise 2: Limit & Continuity

Topic 2.1 : Limit at Infinity

1. Evaluate the following limits at infinity.

(a) $\lim_{x\to\infty}\dfrac{2x-1}{5x+6}$ (b) $\lim_{x\to\infty}\dfrac{4x}{x^2-1}$ (c) $\lim_{x\to+\infty}\dfrac{x^2-1}{x+2}$

(d) $\lim_{x\to\infty}\left(\dfrac{2}{x+1}+\dfrac{1-7x}{3x+4}\right)$ (e) $\lim_{x\to+\infty}\dfrac{2x}{\sqrt{x^2+1}}$ (f) $\lim_{x\to-\infty}\dfrac{2x+3}{\sqrt{9x^2-3x+2}}$

(g) $\lim_{x\to+\infty}\left(\sqrt{x^2+3x}-x\right)$ (h) $\lim_{x\to-\infty}\left[-3+\dfrac{\sqrt{x^2+9}}{x+3}\right]$ (i)

Hint : f) & g)- using numerical method

$\left[(a)\ \frac{2}{5}\ \ (b)\ 0\ \ (c)\ \infty\ \ (d)\ -\frac{7}{3}\ \ (e)\ 2\ \ (f)\ -\frac{2}{3}\ \ (g)\ \frac{3}{2}\ \ (h)\ -4\right]$

Topic 2.2 : Continuity

2. Determine the contuinity of $f(x)$ at $x=a$ for each of the following.

(a) $f(x)=\dfrac{2}{x^2-1}\quad x=2$ (b) $f(x)=e^{x+2}\quad x=0$ (c) $f(x)=\dfrac{x+1}{x-4}\quad x=4$

(d) $f(x)=\dfrac{x^2-6x-7}{x-7}\quad x=7$ (e) $f(x)=\sqrt{3-x}\quad x=3$ (f) $f(x)=\dfrac{2x-3}{|2x-3|}\quad x=\dfrac{3}{2}$

[(a) continuous (b) continous (c) discontinuous
(d) discontinous (e) discontinuous (f) discontinous]

3. For each of the following piecewise functions, determine whether $f(x)$ is continous at $x=2$.

(a) $f(x)=\begin{cases}x^2+1, & x<2\\ 3+x, & x>2\end{cases}$ (b) $f(x)=\begin{cases}\ln(x-1), & 1<x\le 2\\ x^2-4x+4, & x>2\end{cases}$

[(a) discontinuous (b) continous]

4. Let $f(x)$ be defined as

$$f(x)=\begin{cases}\dfrac{1}{x+2}, & x<2\\ k, & x=2\\ \dfrac{x-1}{4}, & x>2\end{cases}$$

Determine k so that f is continous at $x=2$

$$\left[k=\frac{1}{4}\right]$$