Exercise 1: Limit & Continuity

Topic 1.1 : Limit of A Function

1. For each of the following, state the value or values of k for which the limit does not exist.

   (a) $\lim\limits_{x\to k}\dfrac{5x+1}{\sqrt{x-4}}$ (b) $\lim\limits_{x\to k}\dfrac{2}{\sqrt{x^2-4}}$

   [(a) 4 (b) $2,-2$]

2. Given that $\lim\limits_{x\to c} f(x)=\mathrm{P}$, $\lim\limits_{x\to c}\ \mathrm{g}(x)=Q$, and $\lim\limits_{x\to c}\ \mathrm{h}(x)=R$. Determine the following limits, in term of P, Q and/or R.

   (a) $\lim\limits_{x\to c}\left[f(x)-3g(x)\right]$ (b) $\lim\limits_{x\to c}\left[\dfrac{g(x)}{h(x)+2f(x)}\right]$ (c) $\lim\limits_{x\to c}\dfrac{f(x)\mathrm{h}(x)}{\left[\mathrm{g}(x)\right]^2+f(x)}$

   $\left[(a)\ P-3Q\ \ (b)\ \dfrac{\mathrm{Q}}{\mathrm{R}+2\mathrm{P}}\ \ (c)\ \dfrac{\mathrm{PR}}{\mathrm{Q}^2+P}\right]$

3. If $\lim\limits_{x\to a} f(x)=-2$, and $\lim\limits_{x\to a} g(x)=\sqrt{10}$, evaluate

   a) $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}$ b) $\lim\limits_{x\to a}\left\{\dfrac{\left[f(x)\right]^2}{g(x)}\right\}$ c) $\lim\limits_{x\to a}\sqrt{\left[f(x)\right]^2+g(x)}$

   $\left[(a)\ -\dfrac{\sqrt{10}}{5}\ \ (b)\ \dfrac{2\sqrt{10}}{5}\ \ (c)\ \left(4+\sqrt{10}\right)^{\frac{1}{2}}\right]$

Topic 1.2 : Finding Limit using Numerical Method

4. Complete the table and use the result to estimate the limit numerically.

   a) $\lim\limits_{x\to 1}\left(2x^2+x-4\right)$

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$							

b) $\lim_{x\to -1}\dfrac{x+1}{x^2-x-2}$

x	-1.1	-1.01	-1.001	-1	-0.999	-0.99	-0.9
$f(x)$							

c) $\lim_{x\to 0}\dfrac{\sin 2x}{x}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$							

d) $\lim_{x\to 0}\dfrac{\tan x}{2x}$ ✓

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$							

e) $\lim_{x\to 1}\dfrac{\ln x}{x-1}$ ✓

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$							

5. Evaluate each of the following limits numerically

a) $\lim_{x\to -1}\dfrac{x^2-1}{x+1}$

b) $\lim_{x\to 2}\dfrac{4-x^2}{x+1}$ ✓

c) $\lim_{x\to 1}\dfrac{x-1}{x^2+2x-3}$

d) $\lim_{x\to -2}\dfrac{x+2}{x^2+5x+6}$

e) $\lim_{x\to 0}\dfrac{\sqrt{x+5}-\sqrt{5}}{x}$ ✓

f) $\lim_{x\to -3}\dfrac{\sqrt{1-x}-2}{x+3}$

g) $\lim_{x\to -4}\dfrac{\dfrac{x}{x+2}-2}{x+4}$

h) $\lim_{x\to 2}\dfrac{\dfrac{1}{x+2}-\dfrac{1}{4}}{x-2}$

i) $\lim_{x\to 0}\dfrac{\sin^2 x}{x}$ ✓

j) $\lim_{x\to 0}\dfrac{2x}{\tan 4x}$

k) $\lim_{x\to 0}\dfrac{e^{2x}-1}{2x}$

l) $\lim_{x\to 0}\dfrac{1-e^{-4x}}{x}$ ✓

m) $\lim_{x\to 2}\dfrac{\ln(2x-3)}{x-2}$

n) $\lim_{x\to 1}\dfrac{\ln(x^2)}{x-1}$ ✓

6. Evaluate each of the following limits

a) $\lim_{x\to 4^+}\sqrt{x^2-16}$ b) $\lim_{x\to -2^-}\left(x^2-4x\right)$ c) $\lim_{x\to 1^+}\frac{\sqrt{x^2-1}}{x}$ d) $\lim_{x\to 5^+}\sqrt{5-x}$

[(a) 0 (b) 12 (c) 0 (d) does not exist]

7. Evaluate each of the following limits

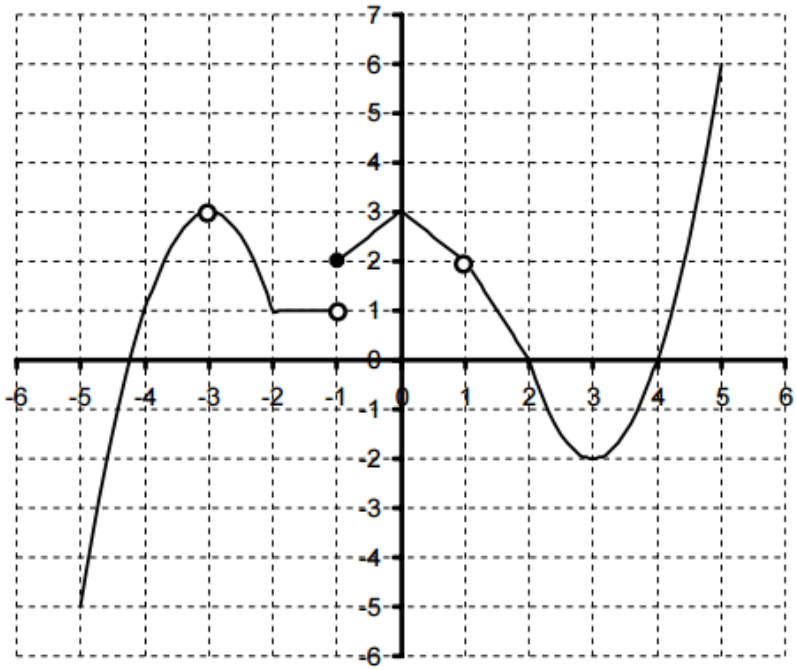
(a) $\lim_{x\to 4^+}\frac{3}{x-4}$ (b) $\lim_{x\to -0.5^-}\frac{1+x}{2x+1}$ (c) $\lim_{x\to 3}\frac{-3}{(x-3)^2}$

(d) $\lim_{x\to -2^+}\frac{x^2+x+1}{x+2}$ (e) $\lim_{x\to 1^-}\frac{3x}{\sqrt{1-x^2}}$ (f) $\lim_{x\to 4^+}\frac{x^2+1}{\sqrt{x}-2}$

[(a) ∞ (b) $-\infty$ (c) $-\infty$ (d) ∞ (e) ∞ (f) ∞]

Topic 1.3 : Finding Limit using Graphical Method

8. Use the graph of $f(x)$ below to find limit ✓



(a) $\lim_{x\to -3} f(x)$ (b) $\lim_{x\to 0} f(x)$ (c) $\lim_{x\to -2} f(x)$ (d) $\lim_{x\to 1} f(x)$ (e) $\lim_{x\to -1} f(x)$

[(a) 3 (b) 3 (c)1 (d) 2 (e) does not exist]

9. Estimated the limit graphically

$$\lim_{x\to 1}\frac{x^3-x^2+x-1}{x-1}$$

[2]

10. By using graph, find the limit (if it exists) as x approaches 2.

a) $f(x)=\begin{cases}3, & x\neq 2\\ 1, & x=2\end{cases}$

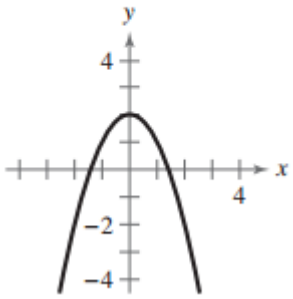
b) $f(x)=\begin{cases}2x+1, & x<2\\ x+3, & x\geq 2\end{cases}$

c) $f(x)=\begin{cases}-2x, & x\leq 2\\ x^2-4x+1, & x>2\end{cases}$

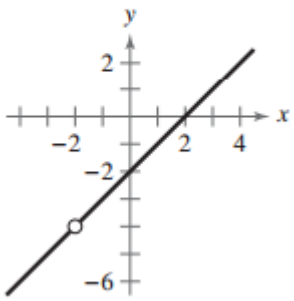
d) $f(x)=\begin{cases}x, & x\neq 2\\ -4, & x=2\end{cases}$

11. Use the graph to find the limit (if it exists). If the limit does not exist, explain why.

a) $\lim_{x\to 0}(2-x^2)$

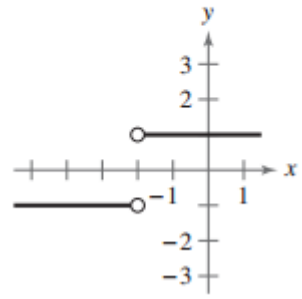


b) $\lim_{x\to -2}\frac{x^2-4}{x+2}$



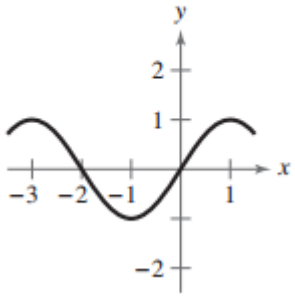
c)

$$\lim_{x\to -2}\frac{|x+2|}{x+2}$$



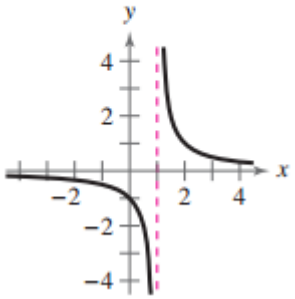
d)

$$\lim_{x\to -1}\sin\frac{\pi x}{2}$$



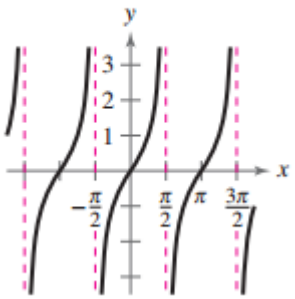
e)

$$\lim_{x\to 1}\frac{1}{x-1}$$



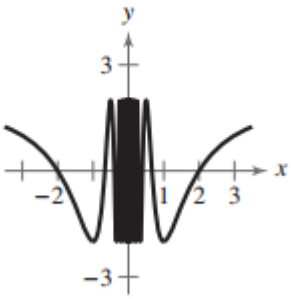
f)

$$\lim_{x\to \pi/2}\tan x$$



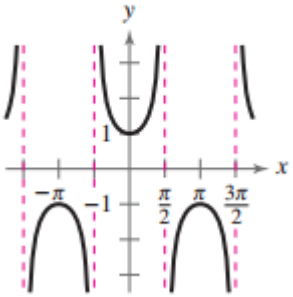
g)

$$\lim_{x\to 0}2\cos\frac{\pi}{x}$$



h)

$$\lim_{x\to \pi/2}\sec x$$



Topic 1.4 : Finding Limit using Analytical Method

12. Using the analytical method, evaluate the following limits

(b) $\lim_{x\to 3}(5x-2)$ (b) $\lim_{x\to -4}(2-7x^2)$ (c) $\lim_{x\to 1}\frac{1}{x+1}$ (d) $\lim_{x\to 0}\frac{3}{x-2}$

[(a) 13 (b) -110 (c) 0.5 (d) -1.5]

13. Using the rules of limits, evaluate the following limits.

a) $\lim_{x\to 1}\left(x^2-5\right)$

b) $\lim_{x\to \frac{1}{2}}\left(2x+7\right)$

c) $\lim_{x\to -2}\left(x^3-8x^2+4x-10\right)$

d) $\lim_{x\to 2}\left[5x^2(x-4)\right]$

e) $\lim_{x\to -1}\dfrac{2x^2-1}{x-3}$

f) $\lim_{x\to \pi}\dfrac{x^2-4}{x+2}$

$\left[\text{(a) } -4 \text{ (b) } 8 \text{ (c) } -58 \text{ (d) } -40 \text{ (e) } -\frac{1}{4} \text{ (f) } \pi-2\right]$

14. Evaluate the limit by direct substitution

g) $\lim_{x\to 5}\left(10-x^2\right)$

h) $\lim_{x\to -2}\left(0.5x^3-5x\right)$

i) $\lim_{x\to -3}\left(2x^2+4x+1\right)$

j) $\lim_{x\to -2}\left(x^3-6x+5\right)$

k) $\lim_{x\to -3}\dfrac{3x}{x^2+1}$

l) $\lim_{x\to 4}\dfrac{x-1}{x^2+2x+3}$

m) $\lim_{x\to -2}\dfrac{5x+3}{2x-9}$

n) $\lim_{x\to 3}\dfrac{x^2+1}{x}$

o) $\lim_{x\to -1}\sqrt{x+2}$

p) $\lim_{x\to 3}\sqrt[3]{x^2-1}$

q) $\lim_{x\to 3}e^x$

r) $\lim_{x\to e}\ln x$

s) $\lim_{x\to \pi}\left(\sin 2x\right)$

t) $\lim_{x\to \pi}\left(\tan x\right)$

u) $\lim_{x\to 1/2}\left(arc\sin x\right)$

v) $\lim_{x\to 1}\left(\arccos\dfrac{x}{2}\right)$

15. Evaluate the following limits using the factorisation or conjugate multiplication method

(a) $\lim_{x\to 6}\dfrac{x^2-36}{x-6}$

(b) $\lim_{x\to -7}\dfrac{x^2-49}{x+7}$

(c) $\lim_{x\to 0}\dfrac{2x^2+5x}{x}$

(d) $\lim_{x\to -2}\dfrac{3x^2-12}{5x^2+10x}$

(e) $\lim_{x\to -1}\dfrac{x^2+x}{x^2-1}$

(f) $\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}$

(g) $\lim_{x\to 1}\dfrac{-\sqrt{x}+1}{x-1}$

(h) $\lim_{x\to 3}\dfrac{x^2-x-6}{x^2-9}$

(i) $\lim_{x\to 9}\dfrac{3-\sqrt{x}}{x-9}$

(j) $\lim_{x\to 2}\dfrac{\sqrt{x+7}-3}{x-2}$

(k) $\lim_{x\to -1}\dfrac{x+1}{\sqrt{3-x}-2}$

(l) $\lim_{x\to 2}\dfrac{x^2-4}{x^3-8}$

(m) $\lim_{x\to 2}\dfrac{\sqrt{x+7}-3}{x-2}$

(n) $\lim_{x\to 0}\dfrac{e^x-1}{x}$

(o) $\lim_{x\to 0}\dfrac{e^{2x}-1}{e^x-1}$

Hints: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$, (l) & (m): Solve using long division

[(a) 12 (b) -14 (c) 5 (d) $\frac{6}{5}$ (e) $\frac{1}{2}$ (f) 4 (g) $-\frac{1}{2}$ (h) $\frac{5}{6}$
(i) $-\frac{1}{6}$ (j) $\frac{1}{6}$ (k) -4 (l) $\frac{1}{3}$ (m) $-\frac{5}{2}$ (n) 1 (o) 2]