Universiti Malaysia PAHANG

Engineering • Technology • Creativity

FACULTY OF INDUSTRIAL SCIENCES & TECHNOLOGY
FINAL EXAMINATION

COURSE	**:**	**NUMERICAL METHODS/ ENGINEERING MATHEMATICS 3**
COURSE CODE	**:**	**BUM2313/BMM2112**
LECTURER	**:**	**NORHAYATI BINTI ROSLI NADIRAH BINTI MOHD NASIR MOHD ZUKI BIN SALLEH NAWWARAH BINTI SUHAIMY**
DATE	**:**	**2 JANUARY 2014**
DURATION	**:**	**3 HOURS**
SESSION/SEMESTER	**:**	**SESSION 2013/2014 SEMESTER I**
PROGRAMME CODE	**:**	**BEE/BEP/BEC/BMM/BMF/BMA/BMI/BMB/ BAA/BAE/BFF/BFM**

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of **SIX (6)** questions. Answer **ALL** questions.
2. Use **FOUR (4)** decimal places in all calculations, **EXCEPT** for **Question 6** use **SEVEN (7)** decimal places.
3. All answers to a new question should start on new page.
4. All the calculations and assumptions must be clearly stated.
5. Candidates are not allowed to bring any material other than those allowed by the invigilator into the examination room.

EXAMINATION REQUIREMENT

1. Scientific calculator.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **TEN (10)** printed pages including front page.

QUESTION 1

Given the following linear equations

$$2x_1 - x_2 + x_3 = -1$$
$$x_1 - x_2 + 2x_3 = -3$$
$$x_1 + 2x_2 - x_3 = 6$$

(a) If necessary, rearrange the equations to make a system diagonally dominant

(b) Derive Gauss Seidel’s formula from the linear equations obtained in (a)

(c) Determine the solution of the linear equations using **THREE** iterations of the Gauss-Seidel method. Compute the approximate percent relative error, ε_a after each iteration. Let $x^{(0)} = (0,0,0)$.

(16 Marks)

QUESTION 2

Concentrations of CO_2 in the Earth's atmosphere (parts per million) derived from remediation air measurements at the Mauna Loa Observatory, Hawaii: Latitude 19.5Â°N Longitude 155.6Â°W Elevation 3397 m from year of 2008 to 2013 for the first six months are listed in Table 1. Use **third-order Newton interpolation** to estimate the concentrations of CO_2 in the Earth's atmosphere (parts per million) in **June 2012**.

[***Hint*** *: Choose the sequence of the points for your estimates to attain the best possible accuracy*]

Table 1: Concentrations of CO_2 in the Earth's atmosphere (parts per million) derived from remediation air measurements at the Mauna Loa Observatory, Hawaii.

Year	**Concentrations of CO_2 in the Earth's atmosphere (parts per million)**					
	January	**February**	**March**	**April**	**May**	**June**
2008	385.07	385.85	385.81	386.77	388.51	388.06
2009	386.65	387.13	388.52	389.57	390.16	389.62
2010	388.55	390.06	391.02	392.38	393.22	392.24
2011	391.30	391.94	392.45	393.37	394.29	393.69
2013	395.66	396.89	397.27	398.35	399.89	398.78

(Source: http://co2now.org/Current-CO2/CO2-Now/noaa-mauna-loa-co2-data.html)

(13 Marks)

QUESTION 3

A certain species of microbial has concentration $P(t)$ at time t whose growth rate is affected by seasonal variations in the food supply. Suppose the microbial concentration (g/L) can be modelled as follows

$$P(t) = \int_1^t k(P(t_0) + A)\left|\cos u\right| du$$

where $P(t_0)$ is the initial concentration, k and A are physical constants. If initially there is 0.0315 (g/L) microbial concentration, $k = 0.5$ and $A = 15$, find the concentration of the microbial for time interval $1 \le t \le 12$ by using

(a) single Trapezoidal rule

(b) composite Trapezoidal rule with a step size of 1

(c) Simpson's rule with a step size of 1.

[***Hint** : Use radian mode*]

(15 Marks)

QUESTION 4

Evaporating raindrop describe about raindrop falls when it evaporates while retaining its spherical shape. If we make the further assumptions that the rate at which the raindrop evaporates is proportional to its surface area and that air resistance is negligible, then a model for the velocity $v(t)$ at any time t in second of the raindrop is

$$\frac{dv}{dt} = g - \frac{3\left(\frac{k}{\rho}\right)}{\left(\frac{k}{\rho}\right)t + r_0} v \quad \text{with } v(0) = 0.$$

In this equation, ρ is the density of water, r_0 is the radius of the raindrop at $t = 0$, k is the constant of proportionality and g is the gravity. The radius of the raindrop at time t is given by

$$r(t) = \left(\frac{k}{\rho}\right)t + r_0 \qquad (1)$$

Suppose that $r_0 = 0.01\,ft$, $r = 0.007\,ft$ after 10 seconds the rain falls from a cloud and $g = 32\ {}^{ft}\!/_{s^2}$. Find the velocity of the raindrop at $t = 10$ seconds by using the fourth order Runge-Kutta method. Use step size $h = 10$ seconds.

[***Hint*** *: Term* $\frac{k}{\rho}$ *can be obtained from equation* (1)]

(12 Marks)

QUESTION 5

Use the shooting method to solve

$$5\frac{d^2y}{dx^2}-2\frac{dy}{dx}-y+x=0$$

with the boundary conditions $y(0)=5$ and $y(10)=15$. Perform Euler's method throughout your calculation with step size, $h=5$. Use initial guess with $z(0)=-10$ and $z(0)=10$.

(22 Marks)

QUESTION 6

The differential equation of the elastic curve for a uniformly loaded beam is formulated as

$$Mp\frac{d^2y}{dx^2}-\frac{wLx}{10}+\frac{wx^2}{8}=0, \qquad y(0)=0, \qquad y(12)=180$$

where M and p represent the modulus of elasticity and moment of inertia respectively.

(a) If $M=30000\ {}^{lbf}\!/_{ft^2}$, $p=66\ ft$, $w=1\ {}^{kip}\!/_{ft}$ and $L=10\ ft$, use a Finite Difference method with a time step, $\Delta x=3$ to reduce the above boundary value problem to a tridiagonal system

(b) Solve the tridiagonal system in (a) by using Thomas algorithm method for $\Delta x=3$.

[***Hint*** *: Use seven decimal places*]

(22 Marks)

END OF QUESTION PAPER

APPENDICES

Chapter 1: Errors	
True Error $E_t =$ true value - approximation value	**True percent relative error** $\varepsilon_t = \left\lvert \dfrac{\text{true value} - \text{approximation value}}{\text{true value}} \right\rvert \times 100$
Approximate percent relative error $\varepsilon_a = \left\lvert \dfrac{\text{present approximation} - \text{previous approximation}}{\text{true value}} \right\rvert \times 100$	**Stopping criterion** Terminate computation when $\lvert \varepsilon_a \rvert < \varepsilon_s$
Chapter 2: Roots of Equations	
Bisection method $x_r = \dfrac{(x_l + x_u)}{2}$	**False-position method** $x_r = x_u - \dfrac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$
Secant method $x_{i+1} = x_i - \dfrac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$	**Newton-Raphson method** $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$

Chapter 3: Linear Algebraic Equations and Matrices

System of linear algebraic equations

$[A]\{X\} = \{B\}$. Decomposition $[A] = [L][U]$ with $[L]$ and $[U]$ can be obtained as follows:

Using Doolittle decomposition

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}; [U] = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Using Cholesky method

$$[A] = [U]^T[U] \; ; \quad [U] = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix},$$

$$u_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} u_{ki}^2}$$

$$u_{ij} = \frac{a_{ij} - \sum_{k=1}^{i-1} u_{ki} u_{kj}}{u_{ii}} \quad \text{for } j = i+1, \ldots, n$$

Using Crout's method

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ l_{31} & l_{32} & l_{33} & 0 \\ l_{41} & l_{42} & l_{43} & l_{44} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} & u_{14} \\ 0 & 1 & u_{23} & u_{24} \\ 0 & 0 & 1 & u_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Jacobi method

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left[b_i - \left(\sum_{\substack{j=1 \\ j\neq i}}^{j=n} a_{ij} x_j^{(k)} \right) \right], i = 1, 2, ..., n.$$

Gauss seidel method

$$x_i^{(k+1)} = \frac{b_i}{a_{ii}} - \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{(k+1)} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{(k)}$$

where

$k = 1, 2, \ldots$

$i = 1, 2, \ldots, n$

Power method

$$v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}$$

$k = 0,1,2,....$

Nonlinear system: Newton-Raphson method

$$x_{1,i+1} = x_{1,i} - \frac{f_{1,i}\frac{\partial f_{2,i}}{\partial x_2} - f_{2,i}\frac{\partial f_{1,i}}{\partial x_2}}{|J|}$$

$$x_{2,i+1} = x_{2,i} - \frac{f_{2,i}\frac{\partial f_{1,i}}{\partial x_1} - f_{1,i}\frac{\partial f_{2,i}}{\partial x_1}}{|J|}$$

$$[J] = \begin{bmatrix} \frac{\partial f_{1,i}}{\partial x_1} & \frac{\partial f_{1,i}}{\partial x_2} \\ \frac{\partial f_{2,i}}{\partial x_1} & \frac{\partial f_{2,i}}{\partial x_2} \end{bmatrix}$$

Chapter 4: Curve Fitting

Newton interpolation polynomial

$$\begin{aligned} f_n(x) = {} & f(x_0) + b_1(x - x_0) \\ & + b_2(x - x_0)(x - x_1) \\ & + \cdots + b_n(x - x_0)(x - x_1)\cdots(x - x_{n-1}) \end{aligned}$$

where

$$b_n = \frac{f[x_n, x_{n-1}, \ldots, x_1] - f[x_{n-1}, x_{n-2}, \ldots, x_0]}{x_n - x_0}$$

Lagrange interpolation polynomial

$$f_n(x) = \sum_{i=1}^{n+1} L_i(x) f(x_i) \text{ where } L_i(x) = \prod_{\substack{j=1 \\ j\neq i}}^{n+1} \frac{x - x_j}{x_i - x_j}$$

$n -$ order of interpolation

Inverse Newton interpolation polynomial

$$P_n(f) = x_0 + b_1(f - f_0) + b_2(f - f_0)(f - f_1) + b_3(f - f_0)(f - f_1)(f - f_2) + \cdots + b_n(f - f_0)(f - f_1)\cdots(f - f_{n-1})$$

$n -$ order of interpolation

Inverse Lagrange interpolation polynomial

$$P_n(f) = \sum_{i=1}^{n+1} L_i(f)x_i \quad \text{where} \quad L_i(f) = \prod_{\substack{j=1 \\ j\neq i}}^{n+1} \frac{(f - f_j)}{(f_i - f_j)}$$

$n -$ order of interpolation

Linear Splines

$$s_i(x) = f(x_i) + \frac{f_{i+1} - f_i}{x_{i+1} - x_i}(x - x_i) \qquad x_i \le x \le x_{i+1}$$

Quadratic Splines

$$s_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 \qquad x_i \le x \le x_{i+1}$$

For $i = 1, 2, \ldots, n-1,$ find

$h_i = x_{i+1} - x_i; \qquad f_i + b_i h_i + c_i h_i^2 = f_{i+1}$

$b_i + 2c_i h_i = b_{i+1};$

Also given,

$c_1 = 0$

$a_i = f_i$

Chapter 5: Numerical Integration

Trapezoidal rule

$$I \cong \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

where

$$h = \frac{x_n - x_0}{n}$$

Simpson's 1/3rd rule

$$I \cong \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n) \right]$$

where

$h = \dfrac{x_n - x_0}{n}$ and n must even segment

Simpson's 3/8 rule

$$I \cong \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3) \right], \text{ where } h = \frac{x_3 - x_0}{3}$$

Chapter 6: Ordinary Differential Equations (IVP)

Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$
$$x_{i+1} = x_i + h$$

2nd order Runge-Kutta: Heun method	**2nd order Runge-Kutta: Midpoint method**
$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)h$ $x_{i+1} = x_i + h$ where $k_1 = f(x_i, y_i)$ $k_2 = f(x_i + h, y_i + k_1 h)$	$y_{i+1} = y_i + k_2 h$ $x_{i+1} = x_i + h$ where $k_1 = f(x_i, y_i)$ $k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$

2nd order Runge-Kutta: Ralston's method

$$y_{i+1} = y_i + \frac{1}{3}(k_1 + 2k_2)h$$

$$x_{i+1} = x_i + h$$

where

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{3}{4}h, y_i + \frac{3}{4}k_1 h)$$

Fourth order Runge-Kutta method

$$y_{i+1} = y_i + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$x_{i+1} = x_i + h$$

where

$$k_1 = f\left(x_i, y_i\right)$$

$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$$

$$k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right)$$

$$k_4 = f\left(x_i + h, y_i + k_3 h\right)$$

Chapter 6: Ordinary Differential Equations (BVP)

Shooting method

Extrapolate estimate for initial slope

$$z(0) = G1 + \frac{G2 - G1}{R2 - R1}(D - R1)$$

where

G1 = First guess at initial slope

G2 = Second guess at initial slope

R1 = Final result at endpoint (using G1)

R2 = Second result at endpoint (using G2)

D = the desired value at the endpoint

Finite Difference method

$$\frac{d^2y}{dx^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2}$$

$$\frac{dy}{dx} = \frac{y_{i+1} - y_{i-1}}{2\Delta x}$$

$$\left(1 - \frac{\Delta x}{2} p_i\right) y_{i-1} - \left(2 - \Delta x^2 q_i\right) y_i + \left(1 + \frac{\Delta x}{2} p_i\right) y_{i+1} = \Delta x^2 r_i$$

Thomas Algorithm

$$\alpha_i = d_i - c_i \beta_{i-1} \quad , \quad \alpha_1 = d_1$$

$$\beta_i = \frac{e_i}{\alpha_i}$$

$$w_i = \frac{b_i - c_i w_{i-1}}{\alpha_i} \quad , \quad w_1 = \frac{b_1}{\alpha_1}$$

$$y_i = w_i - \beta_i y_{i+1} \quad , \quad y_3 = w_3$$