FACULTY OF INDUSTRIAL SCIENCES & TECHNOLOGY
FINAL EXAMINATION

COURSE	**:**	**NUMERICAL METHODS**
COURSE CODE	**:**	**BUM2313**
LECTURER	**:**	**NORHAYATI BINTI ROSLI** **NADIRAH BINTI MOHD NASIR** **NAWWARAH BINTI SUHAIMY**
DATE	**:**	**11 JUNE 2014**
DURATION	**:**	**3 HOURS**
SESSION/SEMESTER	**:**	**SESSION 2013/2014 SEMESTER II**
PROGRAMME CODE	**:**	**BEE/BEP/BEC/BMM/BMF/BMA/BMI/BMB/** **BAA/BAE/BFF/BFM**

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of **SIX (6)** questions. Answer **ALL** questions.
2. Use **FOUR (4)** decimal places in all calculation, **EXCEPT** for **Question 2** use **SIX (6)** decimal places.
3. All answers to a new question should start on new page.
4. All the calculations and assumptions must be clearly stated.
5. Candidates are not allowed to bring any material other than those allowed by the invigilator into the examination room.

EXAMINATION REQUIREMENT

1. Scientific calculator.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **TEN (10)** printed pages including front page.

QUESTION 1

Solve the system of nonlinear equations

$$0.5e^{xy} + 3x^2 - y = -5$$
$$\sin x + \cos y = 5$$

using Newton-Raphson method for **FIRST** iteration only. Given that the initial guesses, $x = 1.5$ and $y = 0.5$.
(*Hint : Use radian mode*)

(12 Marks)

QUESTION 2

A solid steel shaft at room temperature of $27\ ^{\circ}\mathrm{C}$ is needed to be contracted so that it can be shrunk-fit into a hollow hub. It is placed in a refrigerated chamber that is maintained at $-33\ ^{\circ}\mathrm{C}$. The rate of change of temperature for the solid shaft θ is given by

$$M\,\frac{d\theta}{dt} = -5.33\times10^{-6}\left(-3.69\times10^{-6}\theta^4 + 2.33\times10^{-5}\theta^3 + 1.35\times10^{-3}\theta^2\right)(\theta+33)$$

$$\theta(0) = 27\ ^{\circ}\mathrm{C}$$

where a physical constant $M = 3.25$. Find the temperature of the steel shaft after 86400 seconds by using second order Runge-Kutta of Heun's method. Use a step size of $h = 43200$ seconds.

(13 Marks)

QUESTION 3

In physics and thermodynamics, the Redlich–Kwong equation of state is an empirical, algebraic equation that relates temperature, pressure, and volume of gases. It is generally more accurate than the van der Waals equation and the ideal gas equation at temperatures above the critical temperature. The properties of specific volume, v, and pressure, ρ, as a function of temperature, T, for three gasses based on the Redlich–Kwong equation of state are given in Table 1.

Table 1

		Air Properties	Oxygen(O_2) Properties	Carbon Dioxide(CO_2) Properties
T (K)	v $(m^3/kmol)$	ρ (N/m^2)	ρ (N/m^2)	ρ (N/m^2)
350	0.28	10430330	10188750	7649998
400	0.32	10565630	10371810	8573591
450	0.36	10638510	10477620	9159231
500	0.40	10677250	10540230	9547238
550	0.44	10696520	10577460	9813341
600	0.48	10704340	10599220	10000960
650	0.52	10705300	10611290	10136240
700	0.56	10702130	10617160	10235580
750	0.60	10696500	10619010	10309620

Use third-order Newton interpolating polynomial to estimate pressure of the oxygen temperature at $733\,K$.

(12 Marks)

QUESTION 4

Solve the system

$$\frac{dy}{dx} = \sin(x) + \cos(y) + \sin(z)$$
$$\frac{dz}{dx} = \cos(x) + \sin(z)$$

with initial condition $y(0) = 2.5689$ and $z(0) = 1.5689$ over the interval $0 \le x \le 2$ using fourth order Runge-Kutta method with the step size h = 2.

(Hint : *Use radian mode*)

(17 Marks)

QUESTION 5

The following differential equation

$$\frac{d^2y}{dx^2} - \mu\frac{dy}{dx} + \mu\left(1 - x^2\right)y = \sin(\mu x) - x\,, \qquad y(0) = 0, \qquad y(2.5) = 4$$

governs the flow of current in a vacuum tube with four internal elements.

(a) If the parameter $\mu = \frac{1}{2}$, use a Finite Difference method with a time step, $\Delta x = 0.5$ to reduce the boundary value problem to a tridiagonal system.

(b) Solve the tridiagonal system in (a) by using Thomas algorithm method for $\Delta x = 0.5$.

(Hint : *Use radian mode*)

(25 Marks)

QUESTION 6

A simply supported beam at $x = 0$ and $x = L$ with a uniform load q and the vertical deflection $v(x)$ is illustrated in Figure 1.

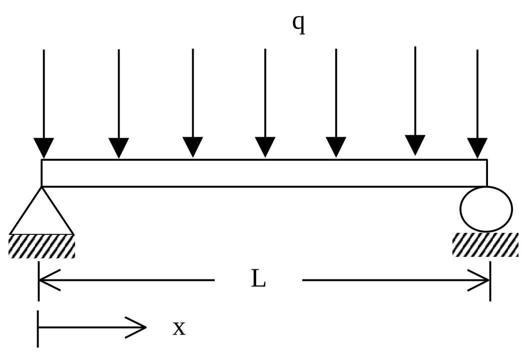

Figure 1

The relationship of the vertical deflection at $x = 0$ and $x = L$ with a uniform load q is described by the boundary value ordinary differential equation as

$$\frac{d^2v}{dx^2} = \left(\frac{qx\left(x-L\right)}{2EI}\right)\frac{dv}{dx}, \quad 0 \le x \le L,$$

where E represents Young's modulus of elasticity of beam and I is a second moment of area. Given that the boundary condition $v(0) = 5$ and $v(L) = 100$ for $L = 14$, $E = 30000$, $q = 10$ and $I = 12$.

(a) Convert the above boundary value problem into equivalent initial value problem.

(b) By using shooting method with a step size of $h = 7$, solve this equation for position vertical deflection. Use initial guesses of the first guess $z(0) = 10$ and second guess $z(0) = 5$.

(21 Marks)

END OF QUESTION PAPER

APPENDICES

Chapter 1: Errors	
True Error $E_t =$ true value - approximation	**True percent relative error** $\varepsilon_t = \left\| \frac{true \quad value - approximation}{true \quad value} \right\| 100$
Approximate percent relative error $\varepsilon_a = \left\| \frac{present \quad approximation - previous \quad approximation}{present \quad approximation} \right\| 100$	**Stopping criterion** Terminate computation when $\left\|\varepsilon_a\right\| < \varepsilon_s$
Chapter 2: Roots of Equations	
Bisection method $x_r = \frac{(x_l + x_u)}{2}$	**False-position method** $x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$
Secant method $x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$	**Newton-Raphson method** $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

Chapter 3: Linear Algebraic Equations and Matrices

System of linear algebraic equations

$[A]\{X\} = \{B\}$. Decomposition $[A] = [L][U]$ with $[L]$ and $[U]$ can be obtained as follows:

Using Gauss elimination method

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}; [U] = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Using Cholesky method

$$[A] = [U]^T [U] \; ; \quad [U] = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix},$$

$$u_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} u_{ki}^2}$$

$$u_{ij} = \frac{a_{ij} - \sum_{k=1}^{i-1} u_{ki} u_{kj}}{u_{ii}} \quad \text{for } j = i+1, \ldots, n$$

Crout's method

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ l_{31} & l_{32} & l_{33} & 0 \\ l_{41} & l_{42} & l_{43} & l_{44} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} & u_{14} \\ 0 & 1 & u_{23} & u_{24} \\ 0 & 0 & 1 & u_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Jacobi method

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left[b_i - \left(\sum_{\substack{j=1 \\ j\neq i}}^{j=n} a_{ij} x_j^{(k)} \right) \right], i = 1, 2, ..., n.$$

Gauss seidel method

$$x_i^{(k+1)} = \frac{b_i}{a_{ii}} - \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{(k+1)} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{(k)}$$

where

$k = 1, 2, \ldots$

$i = 1, 2, \ldots, n$

Power Method

$$v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}$$

$k = 0,1,2,....$

Nonlinear system: Newton-Raphson method

$$x_{1,i+1} = x_{1,i} - \frac{f_{1,i}\frac{\partial f_{2,i}}{\partial x_2} - f_{2,i}\frac{\partial f_{1,i}}{\partial x_2}}{|J|}$$

$$x_{2,i+1} = x_{2,i} - \frac{f_{2,i}\frac{\partial f_{1,i}}{\partial x_1} - f_{1,i}\frac{\partial f_{2,i}}{\partial x_1}}{|J|}$$

$$[J] = \begin{bmatrix} \frac{\partial f_{1,i}}{\partial x_1} & \frac{\partial f_{1,i}}{\partial x_2} \\ \frac{\partial f_{2,i}}{\partial x_1} & \frac{\partial f_{2,i}}{\partial x_2} \end{bmatrix}$$

Chapter 4: Curve Fitting

Newton interpolation polynomial

$$f_n(x) = f(x_0) + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \cdots + b_n(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

where

$$b_n = \frac{f[x_n, x_{n-1}, \ldots, x_1] - f[x_{n-1}, x_{n-2}, \ldots, x_0]}{x_n - x_0}$$

Lagrange interpolating polynomial

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i) \text{ where } L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$n -$ order of interpolation

Inverse Newton interpolation Polynomial

$$P_n(f) = x_0 + b_1(f - f_0) + b_2(f - f_0)(f - f_1) + b_3(f - f_0)(f - f_1)(f - f_2)$$
$$+ \cdots + b_n(f - f_0)(f - f_1)\cdots(f - f_{n-1})$$

$n -$ order of interpolation

Inverse Lagrange interpolating polynomial

$$P_n(f) = \sum_{i=0}^{n} L_i(f)x_i \quad \text{where} \quad L_i(f) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(f - f_j)}{(f_i - f_j)}$$

$n -$ order of interpolation

Linear Splines

$$s_i(x) = f(x_i) + \frac{f_{i+1} - f_i}{x_{i+1} - x_i}(x - x_i) \qquad x_i \le x \le x_{i+1}$$

Quadratic Splines

$$s_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 \qquad x_i \le x \le x_{i+1}$$

For $i = 1, 2, \ldots, n-1$, find

$h_i = x_{i+1} - x_i; \qquad f_i + b_i h_i + c_i h_i^2 = f_{i+1}$

$b_i + 2c_i h_i = b_{i+1};$

Also given,

$c_1 = 0$

$a_i = f_i$

Chapter 5: Numerical Integration

Trapezoidal rule

$$I \cong \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

where

$$h = \frac{x_n - x_0}{n}$$

Simpson's 1/3 rule

$$I \cong \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n) \right]$$

where

$h = \dfrac{x_n - x_0}{n}$ and n must even segment

Simpson's 3/8 rule

$$I \cong \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3) \right], \text{ where } h = \frac{x_3 - x_0}{3}$$

Chapter 6: Ordinary Differential Equations (IVP)

Euler's Method

$$y_{i+1} = y_i + hf(x_i, y_i)$$
$$x_{i+1} = x_i + h$$

2nd order Runge-Kutta: Heun Method

$$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)h$$

$$x_{i+1} = x_i + h$$

where

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + h, y_i + k_1 h)$$

2nd order Runge-Kutta: Midpoint Method

$$y_{i+1} = y_i + k_2 h$$

$$x_{i+1} = x_i + h$$

where

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$$

2nd order Runge-Kutta: Ralston's Method

$$y_{i+1} = y_i + \frac{1}{3}(k_1 + 2k_2)h$$

$$x_{i+1} = x_i + h$$

where

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{3}{4}h, y_i + \frac{3}{4}k_1 h)$$

Fourth order Runge-Kutta Method

$$y_{i+1} = y_i + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$x_{i+1} = x_i + h$$

where

$$k_1 = f\left(x_i, y_i\right)$$

$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$$

$$k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right)$$

$$k_4 = f\left(x_i + h, y_i + k_3 h\right)$$

Chapter 7: Ordinary Differential Equations (BVP)

Shooting method

Extrapolate estimate for initial slope

$$z(0) = G1 + \frac{G2 - G1}{R2 - R1}(D - R1)$$

where

G1 = First guess at initial slope
G2 = Second guess at initial slope
R1 = Final result at endpoint (using G1)
R2 = Second result at endpoint (Using G2)
D = the desired value at the endpoint

Finite Difference method

$$\frac{d^2y}{dx^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2}$$

$$\frac{dy}{dx} = \frac{y_{i+1} - y_{i-1}}{2\Delta x}$$

$$\left(1 - \frac{\Delta x}{2} p_i\right) y_{i-1} - \left(2 - \Delta x^2 q_i\right) y_i + \left(1 + \frac{\Delta x}{2} p_i\right) y_{i+1} = \Delta x^2 r_i$$

Thomas Algorithm

$$\alpha_i = d_i - c_i \beta_{i-1} \quad , \quad \alpha_1 = d_1$$

$$\beta_i = \frac{e_i}{\alpha_i}$$

$$w_i = \frac{b_i - c_i w_{i-1}}{\alpha_i} \quad , \quad w_1 = \frac{b_1}{\alpha_1}$$

$$y_i = w_i - \beta_i y_{i+1} \quad , \quad y_{n-1} = w_{n-1}$$