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Numerical Methods
Nonlinear System

by

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Description

AIMS

This chapter is aimed to solve nonlinear system by using **Newton Raphson method**.

EXPECTED OUTCOMES

1. Students should be able to find the Jacobian matrix of the nonlinear system.
2. Students should be able to solve nonlinear system by using Newton Raphson method.

REFERENCES

1. Norhayati Rosli, Nadirah Mohd Nasir, Mohd Zuki Salleh, Rozieana Khairuddin, Nurfatihah Mohamad Hanafi, Noraziah Adzhar. *Numerical Methods,* Second Edition, UMP, 2017 (Internal use)
2. Chapra, C. S. & Canale, R. P. *Numerical Methods for Engineers*, Sixth Edition, McGraw–Hill, 2010.

Content

1. Introduction
2. Newton Rapshon Method

INTRODUCTION

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1

Nonlinear system of equations refer to the equations which cannot be written as a linear combination of the unknown variables or functions .

Nonlinear System

2

Nonlinear systems with multiple variables can be solved by using Newton Raphson's method.

NEWTON RAPHSON METHOD

Newton Raphson Method Formula

Suppose we have two nonlinear equations with two variables, $f_1(x, y)$ and $f_2(x, y)$, the Newton Raphson formula is

$$x_{i+1} = x_i - \left[\frac{f_{1,i} \dfrac{\partial f_{2,i}}{\partial y} - f_{2,i} \dfrac{\partial f_{1,i}}{\partial y}}{\dfrac{\partial f_{1,i}}{\partial x} \dfrac{\partial f_{2,i}}{\partial y} - \dfrac{\partial f_{1,i}}{\partial y} \dfrac{\partial f_{2,i}}{\partial x}} \right] \qquad y_{i+1} = y_i - \left[\frac{f_{2,i} \dfrac{\partial f_{1,i}}{\partial x} - f_{1,i} \dfrac{\partial f_{2,i}}{\partial x}}{\dfrac{\partial f_{1,i}}{\partial x} \dfrac{\partial f_{2,i}}{\partial y} - \dfrac{\partial f_{1,i}}{\partial y} \dfrac{\partial f_{2,i}}{\partial x}} \right]$$

where the denominator can be simplified by write it as the determinant of the Jacobian, $|\mathbf{J}|$

NEWTON RAPHSON METHOD (Cont.)

Newton Raphson Method Formula (Cont.)

The Jacobian matrix, **J** is given by

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x} & \dfrac{\partial f_1}{\partial y} \\ \dfrac{\partial f_2}{\partial x} & \dfrac{\partial f_2}{\partial y} \end{bmatrix}$$

The determinant of Jacobian matrix, |**J**| is

$$\mathbf{J} = \begin{vmatrix} \dfrac{\partial f_1}{\partial x} & \dfrac{\partial f_1}{\partial y} \\ \dfrac{\partial f_2}{\partial x} & \dfrac{\partial f_2}{\partial y} \end{vmatrix}$$

NEWTON RAPHSON METHOD (Cont.)

Newton Raphson Method Procedures

Step 1

Rearrange the nonlinear systems so that the equation becomes

$$f_1(x, y) = 0$$
$$f_2(x, y) = 0$$

Step 2

Find the Jacobian matrix, $\mathbf{J}$

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x} & \dfrac{\partial f_1}{\partial y} \\ \dfrac{\partial f_2}{\partial x} & \dfrac{\partial f_2}{\partial y} \end{bmatrix}$$

NEWTON RAPHSON METHOD (Cont.)

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Newton Raphson Method Procedures (Cont.)

Step 3

Find the determinant of Jacobian matrix, $|\mathbf{J}|$

$$\mathbf{J} = \begin{vmatrix} \dfrac{\partial f_1}{\partial x} & \dfrac{\partial f_1}{\partial y} \\ \dfrac{\partial f_2}{\partial x} & \dfrac{\partial f_2}{\partial y} \end{vmatrix}$$

Step 4

The calculation process starts by iterating the next values of x and y.

NEWTON RAPHSON METHOD (Cont.)

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Example 1

Use the multiple equations Newton–Raphson's method to determine the roots of the following nonlinear equations

$$z + 2xy = 15$$

$$xy + 3x^2 y = 10$$

Initiate two iterations with initial guesses of $x = 1.5$ and $y = 2.5$.

Solution

Rearrange the nonlinear systems

$$f_1 = x + 2xy - 15$$

$$f_2 = xy + 3x^2 y - 10$$

Find the Jacobian matrix, $\mathbf{J}$

$$\mathbf{J} = \begin{bmatrix} 1 + 2y & 2x \\ y + 6xy & x + 3x^2 \end{bmatrix}$$

NEWTON RAPHSON METHOD (Cont.)

Solution (Cont.)

Find the determinant of the Jacobian

$$\mathbf{J} = \begin{vmatrix} 1+2y & 2x \\ y+6xy & x+3x^2 \end{vmatrix} = (1+2y)(x+3x^2) - (2x)(y+6xy)$$

NEWTON RAPHSON METHOD (Cont.)

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Solution (Cont.)

Do iteration to calculate the values of x and y

First iteration: $i = 0, x_0 = 1.5, y_0 = 2.5$

$$f_1 = 1.5 + 2(1.5)(2.5) - 15 = -6$$

$$f_2 = (1.5)(2.5) + 3(1.5)^2(2.5) - 10 = 10.625$$

$$\mathbf{J} = \begin{bmatrix} 1 + 2(2.5) & 2(1.5) \\ 2.5 + 6(1.5)(2.5) & 1.5 + 3(1.5)^2 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 25 & 8.25 \end{bmatrix}$$

$$|\,\mathbf{J}\,| = (6)(8.25) - 3(25) = -25.5$$

NEWTON RAPHSON METHOD (Cont.)

Solution (Cont.)

First iteration: $i = 0, x_0 = 1.5, y_0 = 2.5$ (Cont.)

$$x_1 = 1.5 - \left[\frac{(-6)(8.25) - 10.625(3)}{-25.5} \right] = -1.6912$$

$$y_1 = 2.5 - \left[\frac{(10.625)(6) - (-6)(25)}{-25.5} \right] = 10.8824$$

NEWTON RAPHSON METHOD (Cont.)

Solution (Cont.)

Second iteration: $i = 1, x_1 = -1.6912, y_1 = 10.8824$

$$f_1 = -53.4998$$

$$f_2 = 64.9718$$

$$\mathbf{J} = \begin{bmatrix} 22.7648 & -3.3824 \\ -99.5435 & -92.6542 \end{bmatrix}$$

$$|\mathbf{J}| = -2445.9503$$

$$x_2 = -2.5128$$

$$y_2 = 9.3098$$

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