5. Solving Linear Algebraic of Equations– Iterative Methods

Mohd Zuki Salleh,
Applied & Industrial Mathematics Research Group,
Faculty of Industrial Sciences & Technology (FIST),
Universiti Malaysia Pahang,
Lebuhraya Tun Razak,
26300 Gambang, Kuantan,
Pahang, Malaysia
E-mail: zuki@ump.edu.my

5.1 Exercises

Exercises: Jacobi's Method

Exercise 5.1 Solve the following systems of linear equations by using Jacobi's method. Iterate two iterations with initial guesses of $x_i^{(0)} = (0,0,0)^T$. Use four decimal places in your calculation.

i.

$$\begin{aligned} 7x_1 + 4x_2 - 4x_3 &= 2 \\ -x_1 + 2x_2 + 3x_3 &= 6 \\ x_1 - 3x_2 + 2x_3 &= 5 \end{aligned}$$

ii.

$$\begin{aligned} 5x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 + 3x_2 + x_3 &= 5 \\ x_1 - 2x_2 + 3x_3 &= 6 \end{aligned}$$

iii.

$$\begin{aligned} 4x+7y+3z &= 14 \\ 5x-2y-2z &= -14 \\ x+2y-5z &= -15 \end{aligned}$$

■

Exercise 5.2 Amar who is a businessman, run an international textile business in Malaysia, made phone calls to his business partner at Australia, German and Qatar regularly. The international call rates per minute charge by one of the telecommunication service provider in Malaysia vary for the different countries. The total phone calls charge that he spent for three months in year 2015 is described as in the following table.

Month	Duration call to Australia (min)	Duration call to German (min)	Duration call to Qatar (min)	Charges (RM)
September	90	120	220	1247.50
October	170	100	65	628.10
November	50	105	50	460.05

i. Based on the above information, setup a system of linear equations of $\mathbf{Ax} = \mathbf{b}$ where x_i represent the call rate per minute charge for each country.
ii. Rearrange the equations so that the iterative process will converge.
iii. Find the call rate per minute charge for each country using two iterations of Jacobi's method with initial vectors, $(x_0, y_0, z_0)^T = (0,0,0)^T$.

■

Exercise 5.3 Given the system of linear equations

$$\begin{aligned} 12x_1+4x_2-2x_3+x_4 &= 46.8 \\ 3x_1+x_2-6x_3+2x_4 &= 62.3 \\ x_1-6x_2+4x_3 &= 76 \\ 5x_1+2x_2+x_3+9x_4 &= 88.6 \end{aligned}$$

i. Transform the above system of linear equations in matrix form of $\mathbf{Ax} = \mathbf{b}$.
ii. Rearrange the equations so that the iterative process will converge.
iii. Solve the above system using Jacobi's method with initial guesses, $x_i^0 = (1,-1,0,1)^T$. Compute until two iterations.

■

Exercise 5.4 Determine the solution of the linear equations

$$\begin{aligned} x_1+2x_2-10x_3 &= 42 \\ 7x_1+2x_2+x_3 &= 24 \\ 2x_1+5x_2-x_3 &= -12 \end{aligned}$$

By using Jacobi's method with two iterations. Compute the estimated errors after each iteration. If necessary, rearrange the equations to achieve the convergence criterion. Let $x_i^{(0)} = (1,0,1)^T$.
■

Exercises: Gauss–Seidel Method

Exercise 5.5 Solve the following systems of linear equations by using Gauss–Seidel method. Iterate two iterations with initial guesses of $x_i^{(0)} = (0,0,0)^T$. Use four decimal places in your calculation.

i.

$$\begin{aligned} 7x_1 + 4x_2 - 4x_3 &= 2 \\ -x_1 + 2x_2 + 3x_3 &= 6 \\ x_1 - 3x_2 + 2x_3 &= 5 \end{aligned}$$

ii.

$$\begin{aligned} 5x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 + 3x_2 + x_3 &= 5 \\ x_1 - 2x_2 + 3x_3 &= 6 \end{aligned}$$

iii.

$$\begin{aligned} 4x + 7y + 3z &= 14 \\ 5x - 2y - 2z &= -14 \\ x + 2y - 5z &= -15 \end{aligned}$$

Exercise 5.6 Given the following linear equations

$$\begin{aligned} 2x_1 - x_2 + x_3 &= -1 \\ x_1 - x_2 + 2x_3 &= -3 \\ x_1 + 2x_2 - x_3 &= 6 \end{aligned}$$

i. If necessary, rearrange the equations to make a system diagonally dominant

ii. Derive Gauss–Seidel formula from the linear equations obtained in i.

iii. Determine the solution of the linear equations using THREE iterations of the Gauss–Seidel method. Compute the approximate percent relative error, ε_a after each iteration. Let $x^{(0)} = (0,0,0)^T$.

References 1. Chapra, C. S. & Canale, R. P. Numerical Methods for Engineers, Sixth Edition, McGraw–Hill, 2010.