Ordinary Differential Equations

Chapter 4A: Fourier Series

by
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Chapter Description

- Expected Outcomes
  - Find the analytical descriptions from graphs of periodic functions and vice versa

- References
  - Abdullah, S., Nasir, N.M., Jusoh, R., Aziz, L.A. & Yusoff, W.N.S.W., *Ordinary Differential Equations for Engineering Students*. 2016. Universiti Malaysia Pahang.

Content

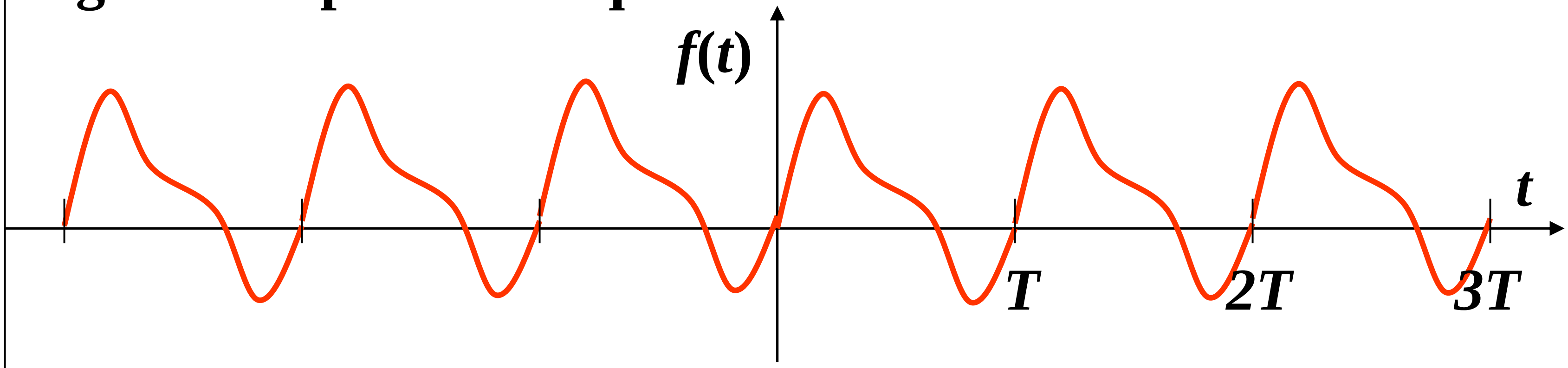
4.0 Introduction

- The ability to analyse waveforms of various types is an important engineering skill.
- Fourier analysis provides a set of mathematical tools which enable the engineer to break down a wave into its various frequency components.
- It is then possible to predict the effect a particular waveform may have from knowledge of the effects of its individual frequency components.
- Often an engineer finds it useful to think of a signal in terms of its frequency components rather than in terms of its time domain representation.
- But, in this subject, we cover up the essential properties.

4.1 Periodic Function

- Periodic function is a function values repeat at regular intervals, called **period** of the independent variable.

Figure 1: A periodic sequence



- From the Figure 1, the regular interval between repetitions is the period of the oscillations (**waveform**).
- Half distance between the minimum and maximum values is **amplitude A**.

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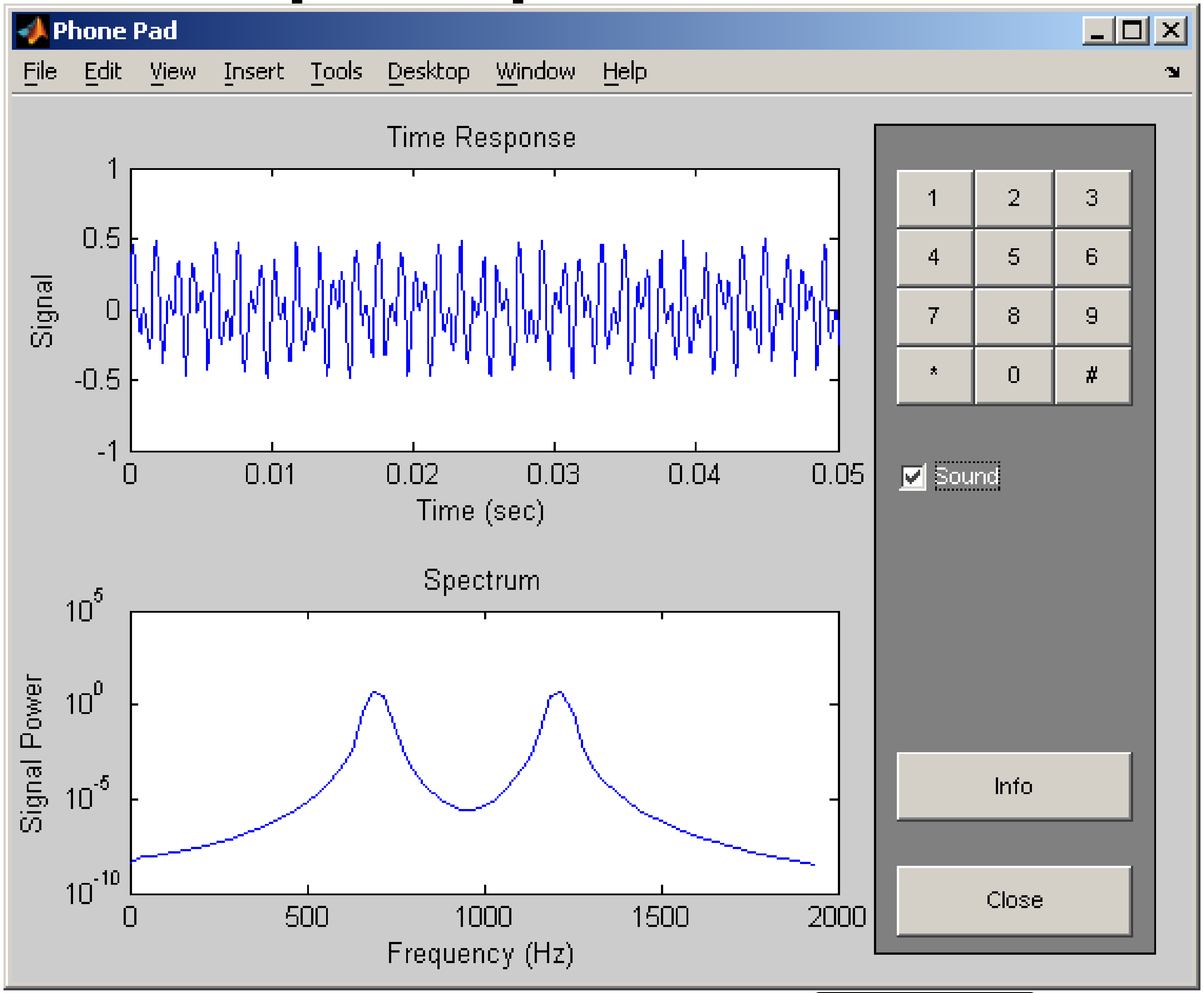
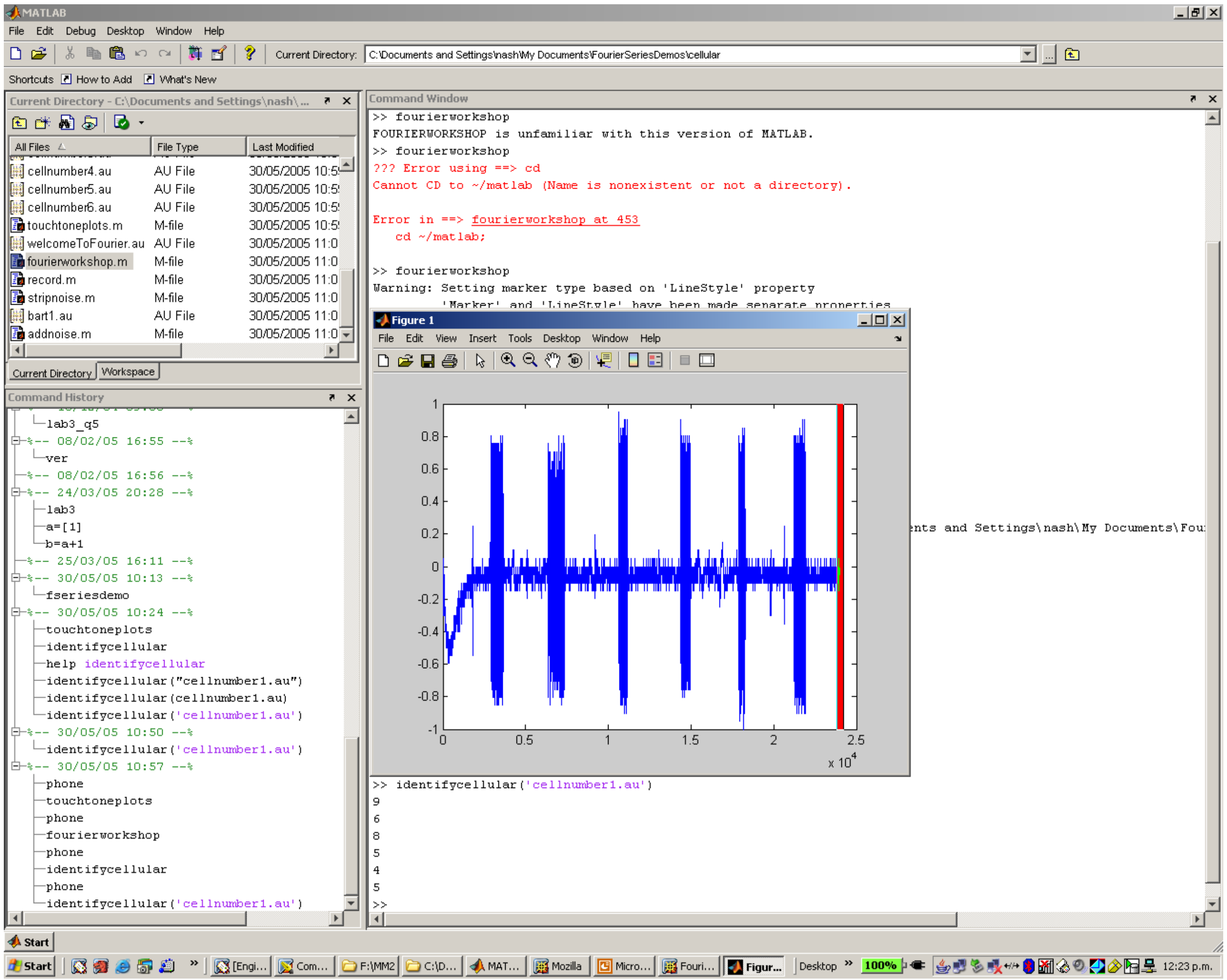
Example of periodic function



Identifying a telephone number

http://math.arizona.edu/~rims/workshops/softwarematlab/



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- Periodic function is defined as

$$f(t) = f(t+T),$$
$$t = \text{time}$$
$$T = \text{period}$$

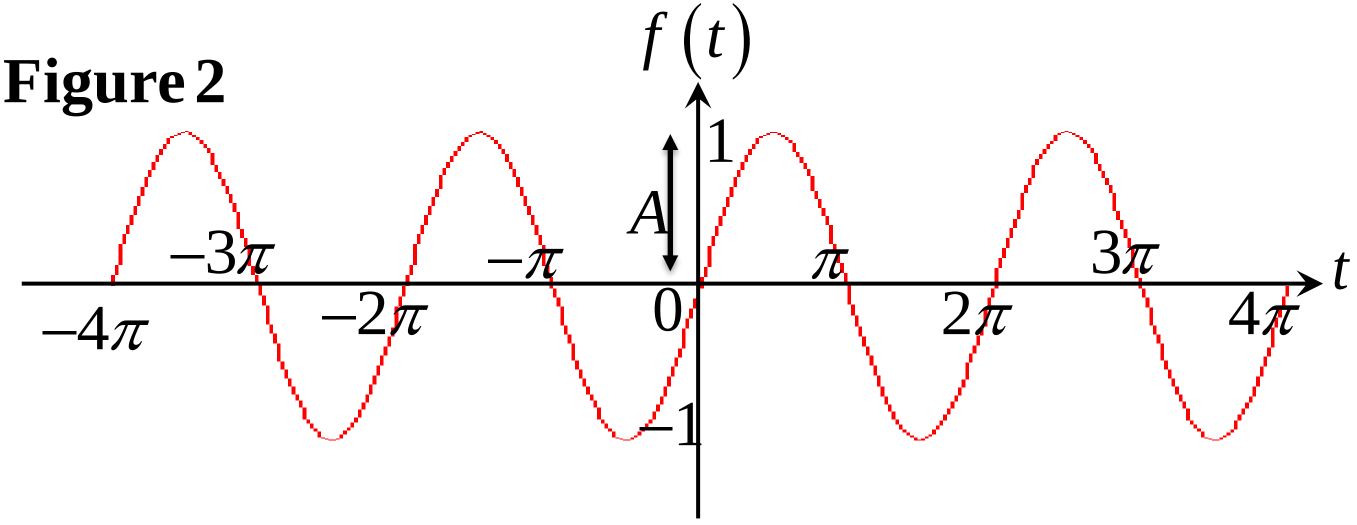
- If T in second, f in hertz (oscillation per second), the periodic function is

$$T = \frac{1}{f}$$

- If angular frequency, ω in radians per second is defined by $\omega = 2\pi f$, then

$$\omega = \frac{2\pi}{T}$$

Figure 2



- Figure 2 show a waveform with an amplitude A = 1, period $T = 2\pi$ and the angular frequency, $\omega = 1$. This waveform represented analytically by

$$f(t) = \sin t, \quad -\pi < t < \pi$$

$$f(t) = f(t + 2\pi)$$

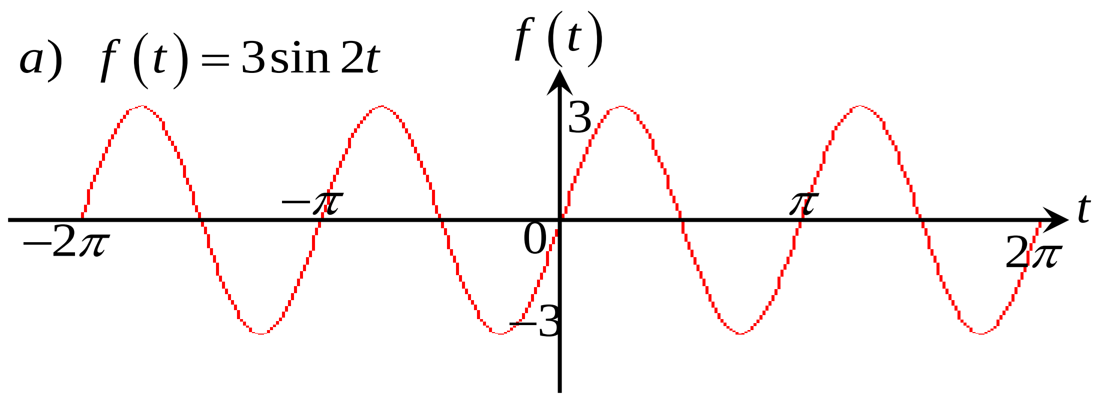
- The pair of above equations is called the **analytical description** of the periodic function.

Example 1

From the graph, find the period T, the angular frequency ω, and the amplitude A, for each of the periodic functions.

$a)\quad f(t) = 3\sin 2t \qquad b)\quad f(t) = 5\cos 2t$

$a)\quad f(t) = 3\sin 2t$



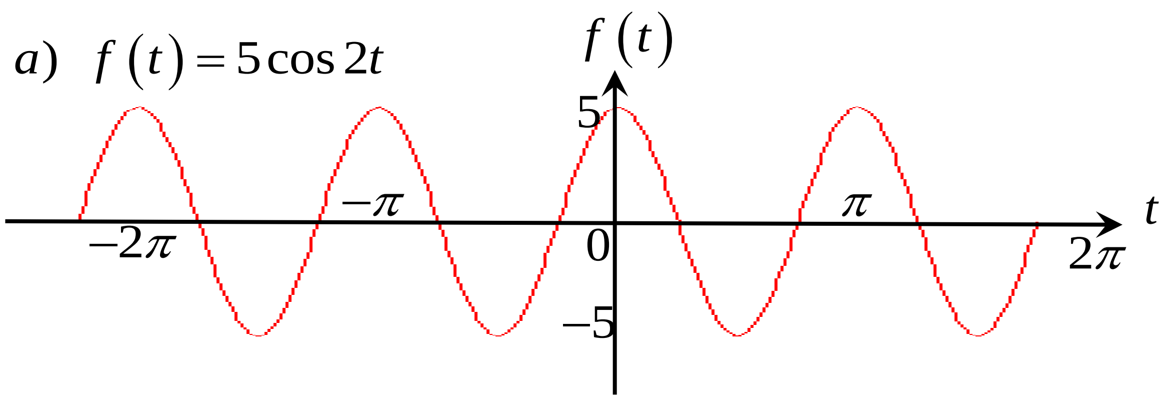
answer

$A = 3$

$T = \pi$

$\omega = \dfrac{2\pi}{\pi} = 2$

$a)\quad f(t) = 5\cos 2t$



answer

$A = 5$

$T = \pi$

$\omega = \dfrac{2\pi}{\pi} = 2$

Example 2

From each of the following waveform, find the analytical description.

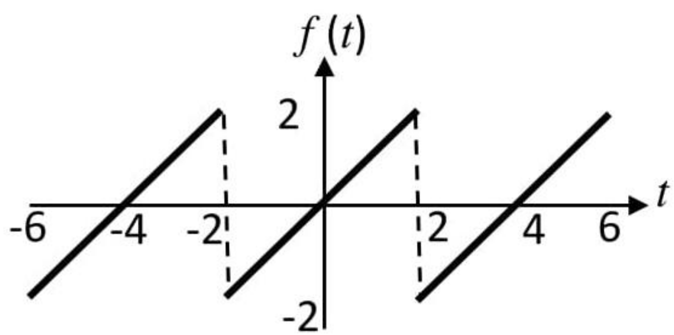
a)

answer

$$f(t) = \begin{cases} 4, & 0 \le t < 5 \\ 0, & 5 \le t < 7 \end{cases}$$

$$f(t) = f(t+7)$$

b)



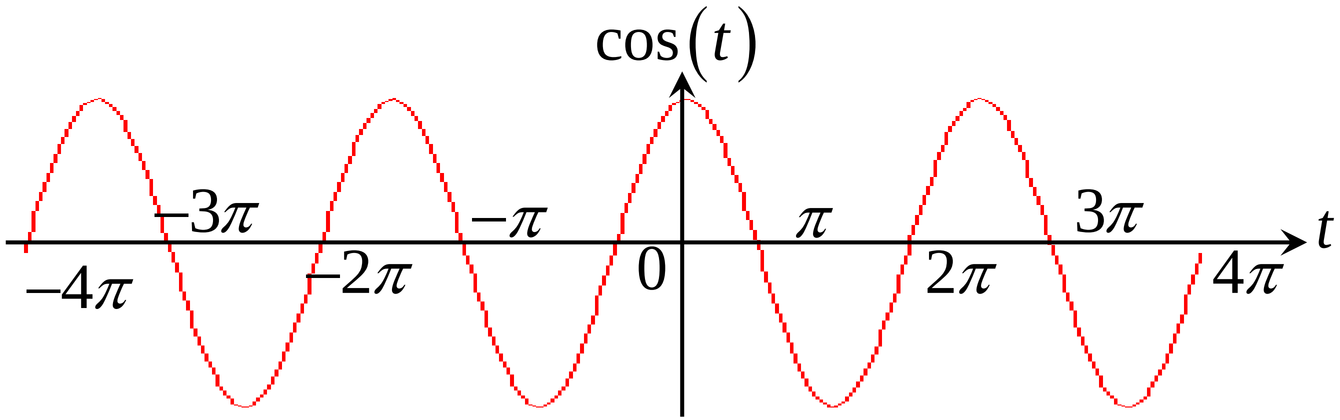
answer

$$f(t) = t, \quad -2 \le t < 2$$

$$f(t) = f(t+4)$$

4.2 Even and Odd Function

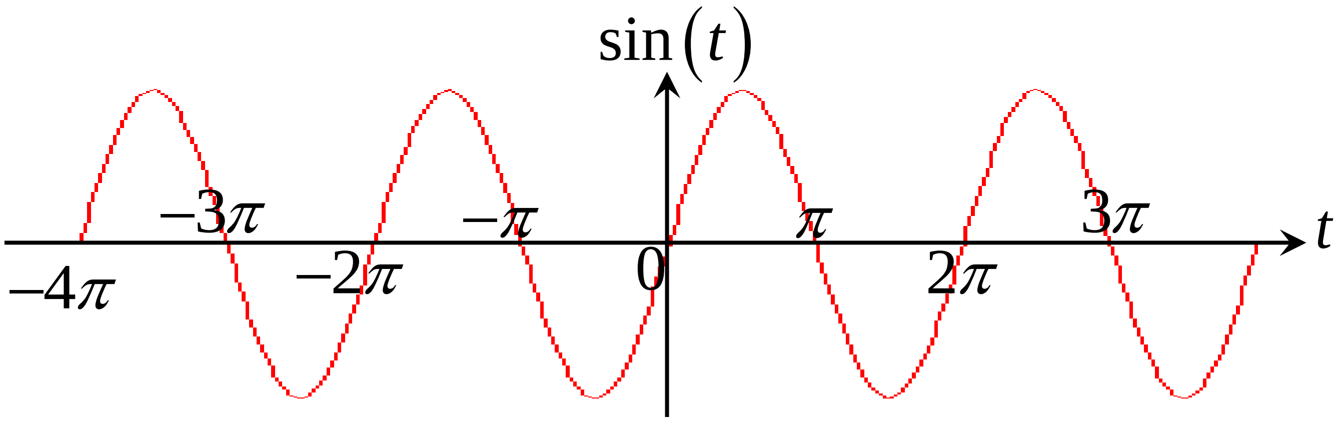
Even function



For even function, the function is inverted on the other side of the $y-axis$. That is say :

$f(t) = f(-t)$ for all $t \in \mathbb{R}$.

Odd function



For odd function, the function is symmetric about the *origin*. That is say :

$f(-t) = -f(t)$ for all $t \in \mathbb{R}$.

Example 3

Sketch the graph of each of these periodic functions and determine whether its is even, odd or neither.

$$a)\quad f(t)=\begin{cases} 2, & -\pi \le t < 0 \\ -2, & 0 < t < \pi \end{cases}$$

$$f(t)=f(t+2\pi)$$

$$b)\quad f(t)=\begin{cases} 0, & -\pi \le t < -\dfrac{\pi}{2} \\ 3, & -\dfrac{\pi}{2} \le t < \dfrac{\pi}{2} \\ 0 & \dfrac{\pi}{2} \le t < \pi \end{cases}$$

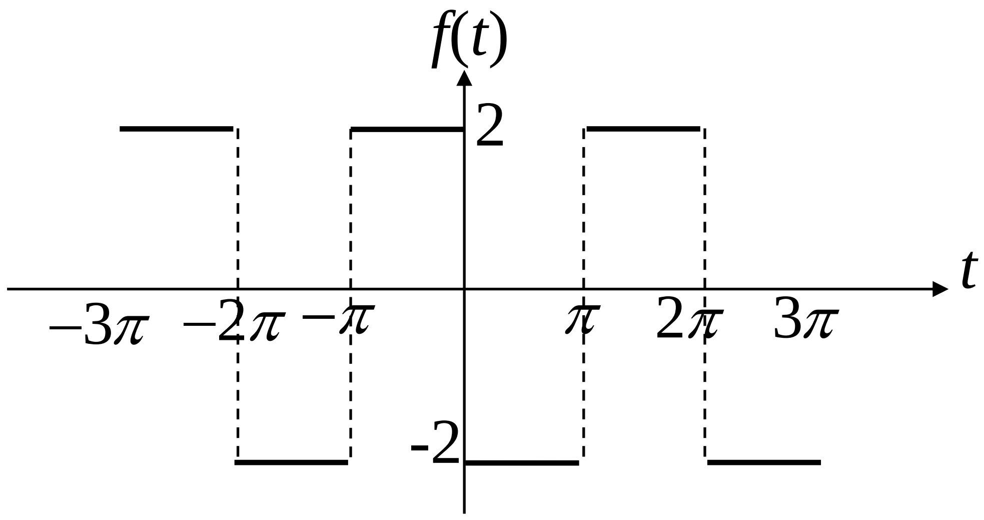
$$f(t)=f(t+2\pi)$$

$$c)\quad f(t)=\begin{cases} \dfrac{4t}{\pi}+3, & -\pi \le t < 0 \\ 3, & 0 < t < \pi \end{cases}$$

$$f(t)=f(t+2\pi)$$

Solution

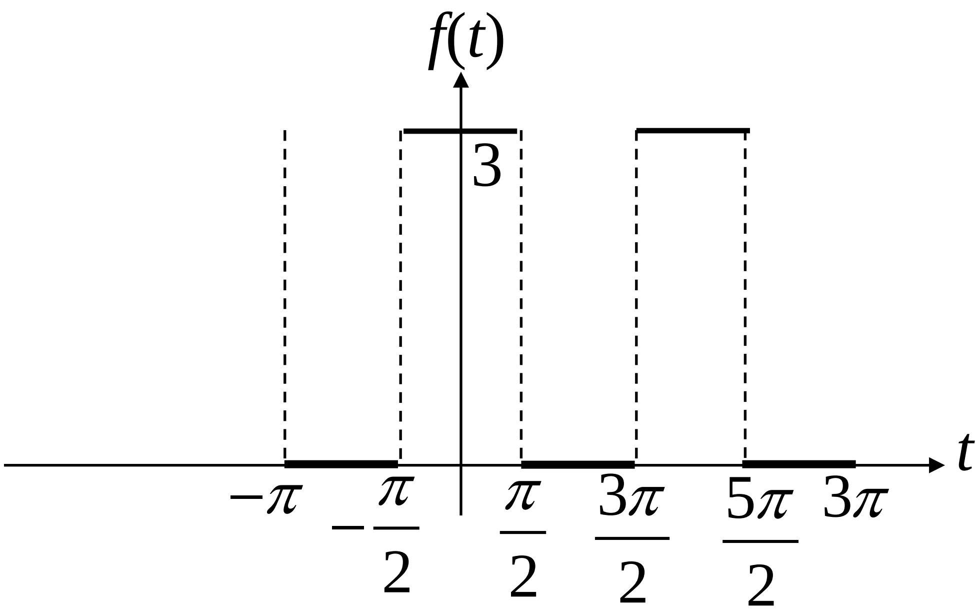
a)



The graph is symmetric about the origin. Hence, the periodic function is odd.

Solution

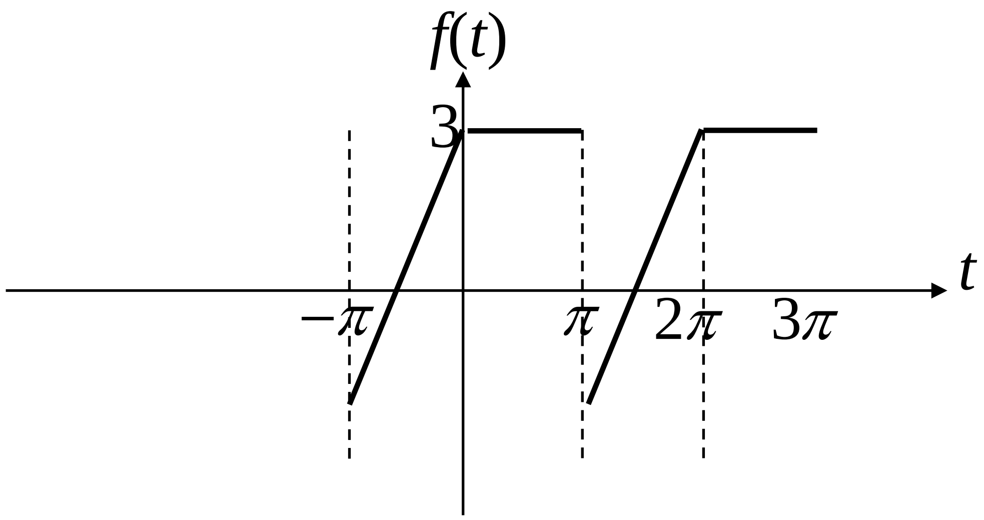
b)



The periodic function is even since the graph is symmetric about the vertical axis.

Solution

c)



The periodic function is neither even nor odd function since the graph is not symmetric about the both the origin and the vertical axis.

Products of even and odd functions

- For $f(t)$ and $g(t)$ are both even, then $f(t)g(t)$ is even.
- For $f(t)$ and $g(t)$ are both odd, then $f(t)g(t)$ is even.
- For $f(t)$ is even and $g(t)$ is odd or vice versa, then $f(t)g(t)$ is odd.

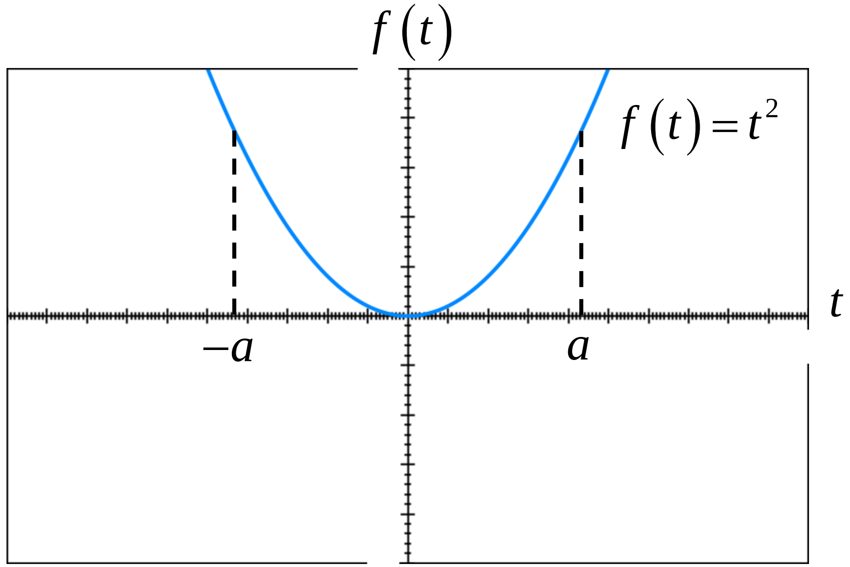
Example 4

a) If $f(t) = 2t^2$ (even) and $g(t) = \cos 3t$ (even), then $f(t)g(t) = 2t^2 \cos 3t$ is even.

b) If $f(t) = t^7$ (odd) and $g(t) = \sin 2t$ (odd), then $f(t)g(t) = t^7 \sin 2t$ is even.

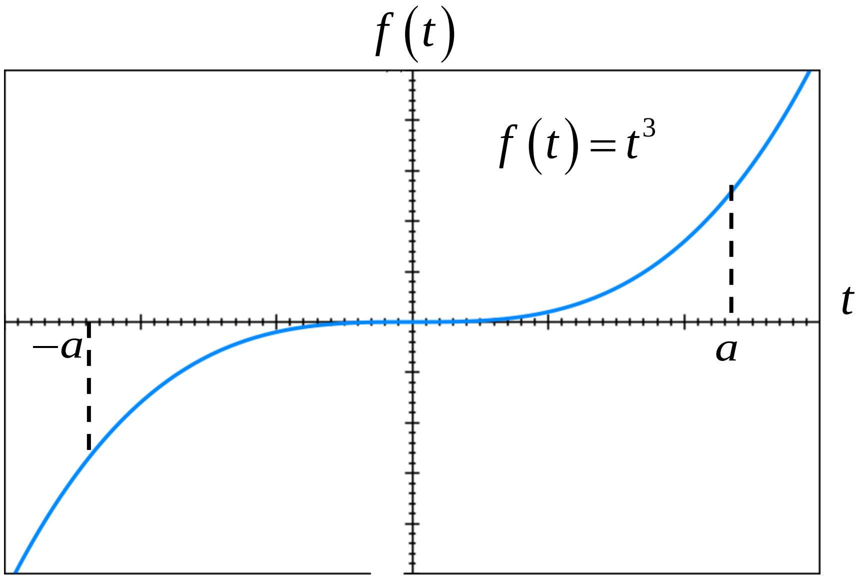
c) If $f(t) = t^4$ (even) and $g(t) = \sin 4t$ (odd), then $f(t)g(t) = t^4 \sin 4t$ is odd.

Integral of even and odd function



$f(t)=t^2$ is an even function, hence

$$\int_{-a}^{a} t^2 dt = 2\int_{-a}^{a} t^2 dt$$



$f(t) = t^3$ is an odd function, hence

$$\int_{-a}^{a} t^3 dt = \left[\frac{t^4}{4}\right]_{-a}^{a} = 0$$

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