

# Ordinary Differential Equations

## Chapter 4A: Fourier Series

by

Nor Aida Zuraimi binti Md Noar, Wan Nur Syahidah Wan Yusoff,  
Zulhibri Ismail@Mustofa, Samsudin Abdullah, Nadirah Mohd Nasir,  
Rahimah Jusoh@Awang, Laila Amera Aziz

Faculty of Industrial Sciences & Technology



Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

Communitising Technology

# Chapter Description

- Expected Outcomes
  - Find the analytical descriptions from graphs of periodic functions and vice versa
- References
  - Abdullah, S., Nasir, N.M., Jusoh, R., Aziz, L.A. & Yusoff, W.N.S.W., *Ordinary Differential Equations for Engineering Students*. 2016. Universiti Malaysia Pahang.



Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

Communitising Technology

# Content

- 4.0 Introduction
- 4.1 Periodic Function
- 4.2 Even and Odd Function



Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

Communitising Technology

# 4.0 Introduction

- The ability to analyse waveforms of various types is an important engineering skill.
- Fourier analysis provides a set of mathematical tools which enable the engineer to break down a wave into its various frequency components.
- It is then possible to predict the effect a particular waveform may have from knowledge of the effects of its individual frequency components.
- Often an engineer finds it useful to think of a signal in terms of its frequency components rather than in terms of its time domain representation.
- But, in this subject, we cover up the essential properties.



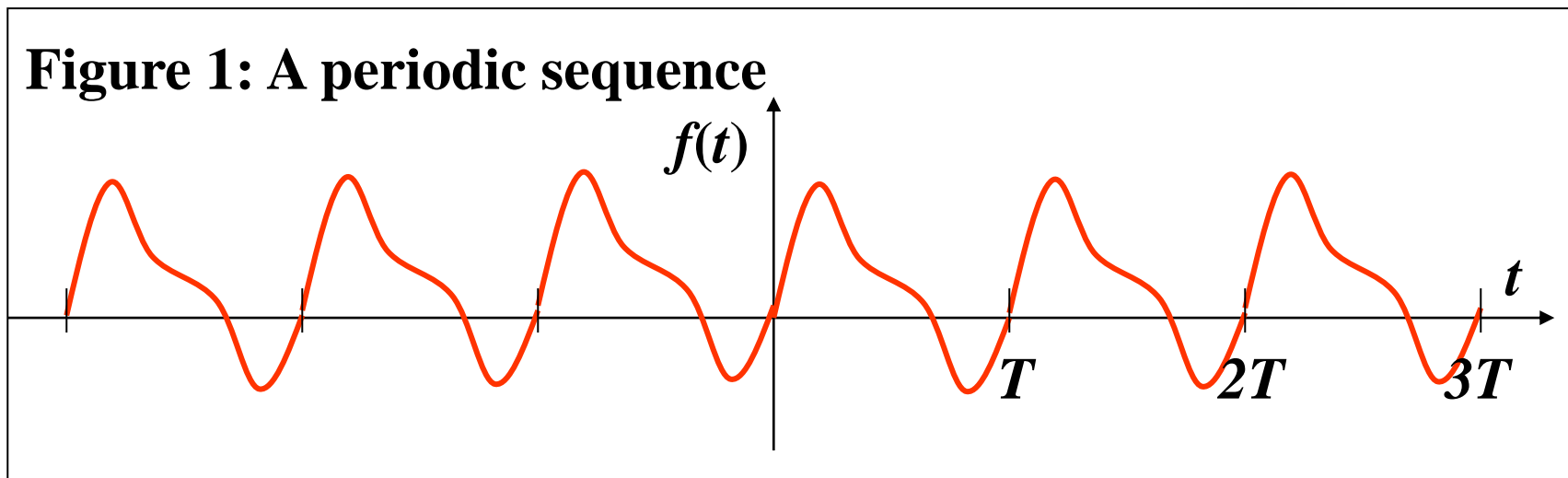
Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

Communitising Technology

# 4.1 Periodic Function

- Periodic function is a function values repeat at regular intervals, called **period** of the independent variable.



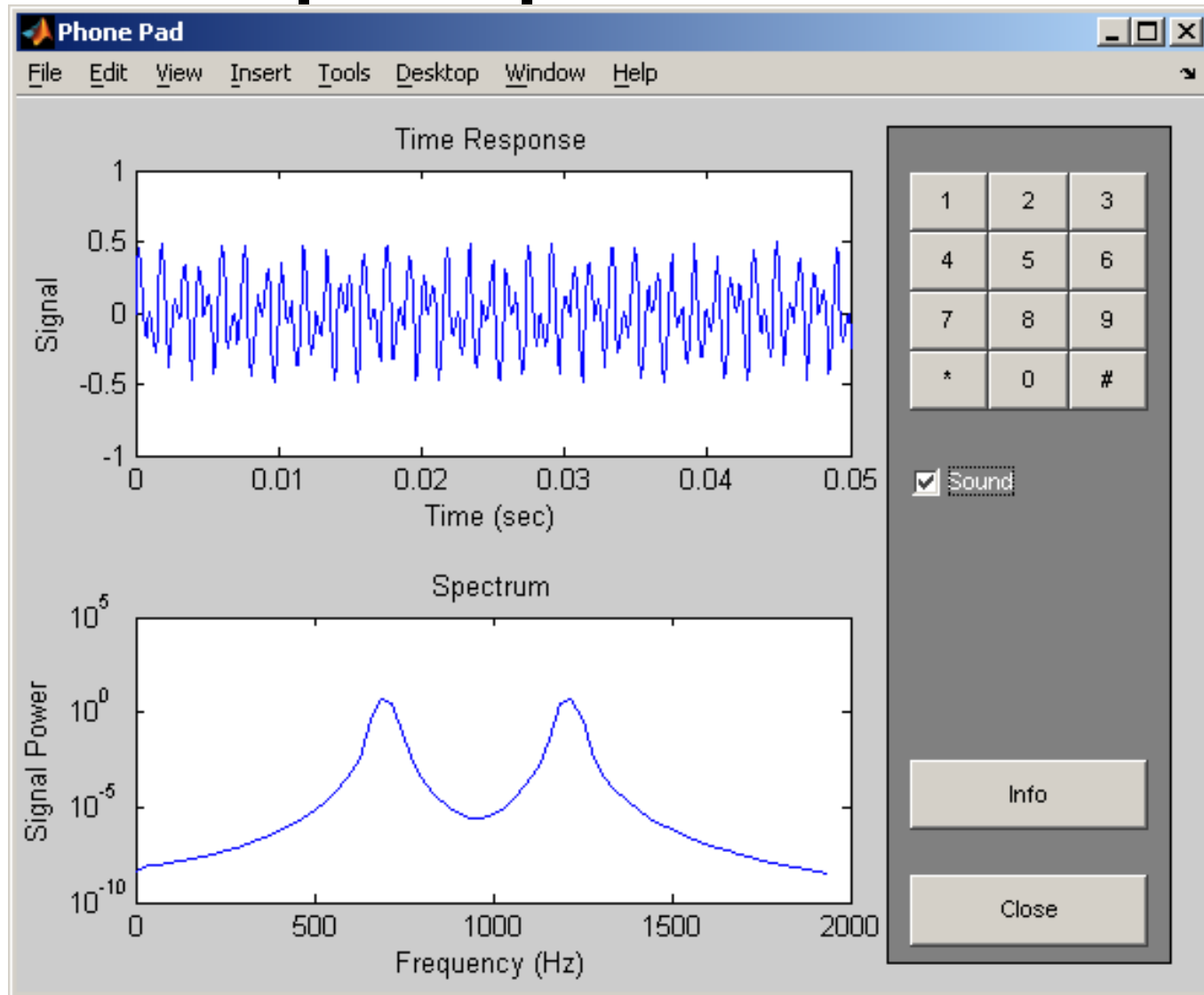
- From the Figure 1, the regular interval between repetitions is the period of the oscillations (**waveform**).
- Half distance between the minimum and maximum values is **amplitude A**.



Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

# Example of periodic function

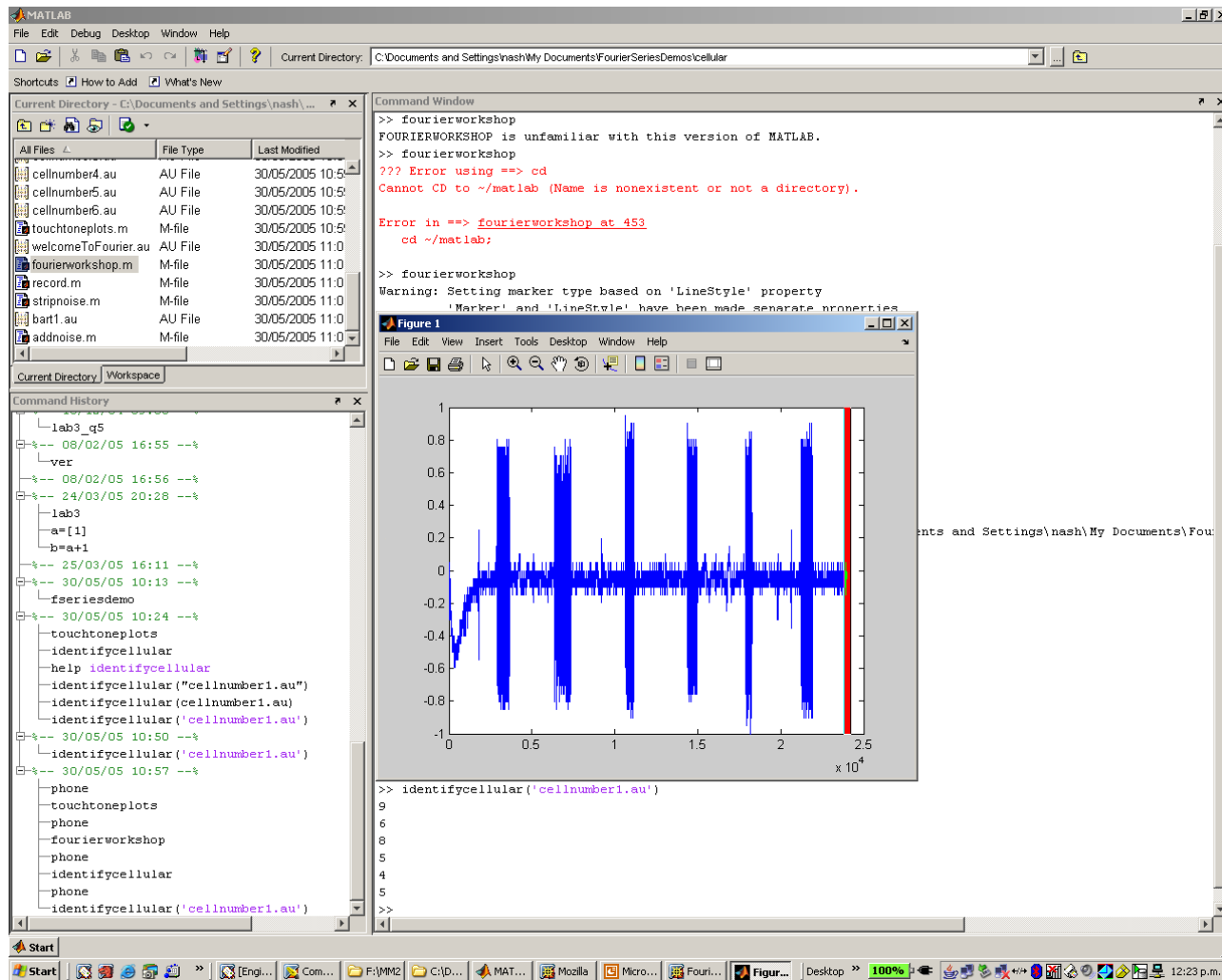


Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

# Identifying a telephone number

<http://math.arizona.edu/~rims/workshops/softwarematlab/>



The screenshot shows the MATLAB environment. The Command Window contains the following text:

```
>> fourierworkshop
FOURIERWORKSHOP is unfamiliar with this version of MATLAB.
>> fourierworkshop
??? Error using ==> cd
Cannot CD to ~/matlab (Name is nonexistent or not a directory).

Error in ==> fourierworkshop at 453
    cd ~/matlab;

>> fourierworkshop
Warning: Setting marker type based on 'LineStyle' property
'Marker' and 'LineStyle' have been made separate properties.
```

The Figure window displays a plot of a signal with a blue line and a red vertical line at the end. The x-axis is labeled  $x 10^4$  and ranges from 0 to 2.5. The y-axis ranges from -1 to 1. The signal shows a complex waveform with several sharp peaks and troughs.

The Command History window shows the following commands:

```
lab3_q5
-- 08/02/05 16:55 --%
ver
-- 08/02/05 16:56 --%
-- 24/03/05 20:28 --%
lab3
a=[1]
b=a+1
-- 25/03/05 16:11 --%
-- 30/05/05 10:13 --%
fseriesdemo
-- 30/05/05 10:24 --%
touchtoneplots
identifycellular
help identifycellular
identifycellular('cellnumber1.au')
identifycellular(cellnumber1.au)
identifycellular('cellnumber1.au')
-- 30/05/05 10:50 --%
identifycellular('cellnumber1.au')
-- 30/05/05 10:57 --%
phone
touchtoneplots
phone
fourierworkshop
phone
identifycellular
phone
identifycellular('cellnumber1.au')
```



Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

Communitising Technology

- Periodic function is defined as

$$f(t) = f(t + T),$$

$t$  = time

$T$  = period

- If  $T$  in second,  $f$  in hertz (oscillation per second), the periodic function is

$$T = \frac{1}{f}$$

- If angular frequency,  $\omega$  in radians per second is defined by

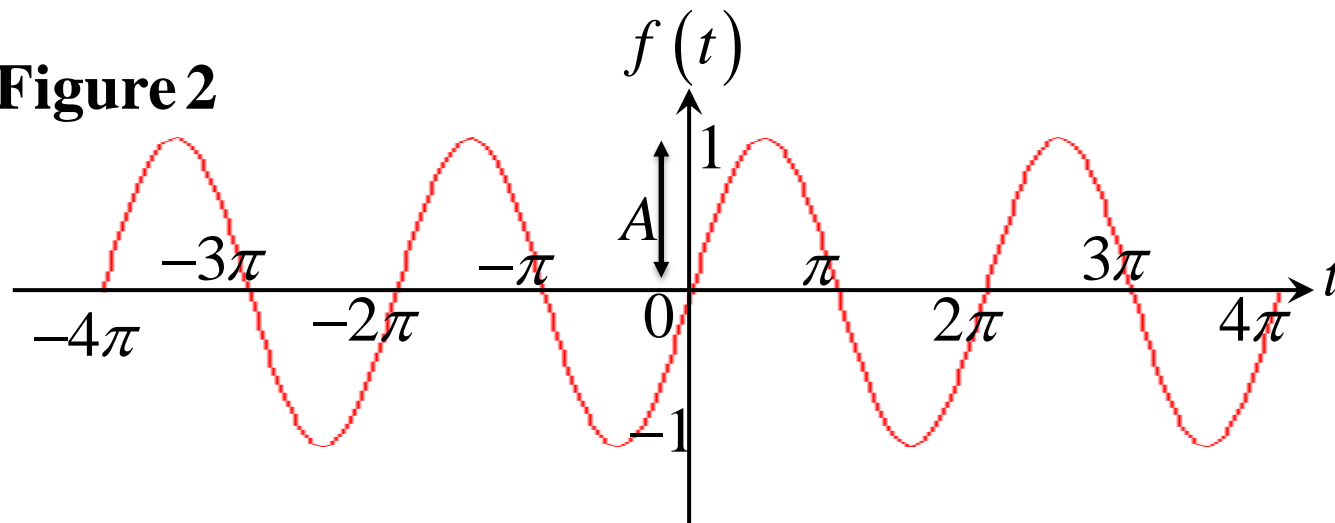
$\omega = 2\pi f$ , then

$$\omega = \frac{2\pi}{T}$$





**Figure 2**



- Figure 2 show a waveform with an amplitude  $A = 1$ , period  $T = 2\pi$  and the angular frequency,  $\omega = 1$ . This waveform represented analytically by

$$f(t) = \sin t, \quad -\pi < t < \pi$$

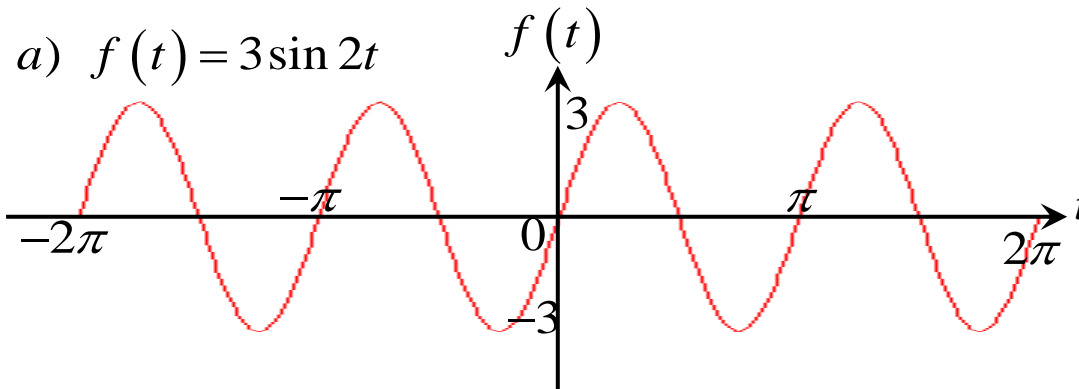
$$f(t) = f(t + 2\pi)$$

- The pair of above equations is called the **analytical description** of the periodic function.

## Example 1

From the graph, find the period  $T$ , the angular frequency  $\omega$ , and the amplitude  $A$ , for each of the periodic functions.

a)  $f(t) = 3 \sin 2t$       b)  $f(t) = 5 \cos 2t$

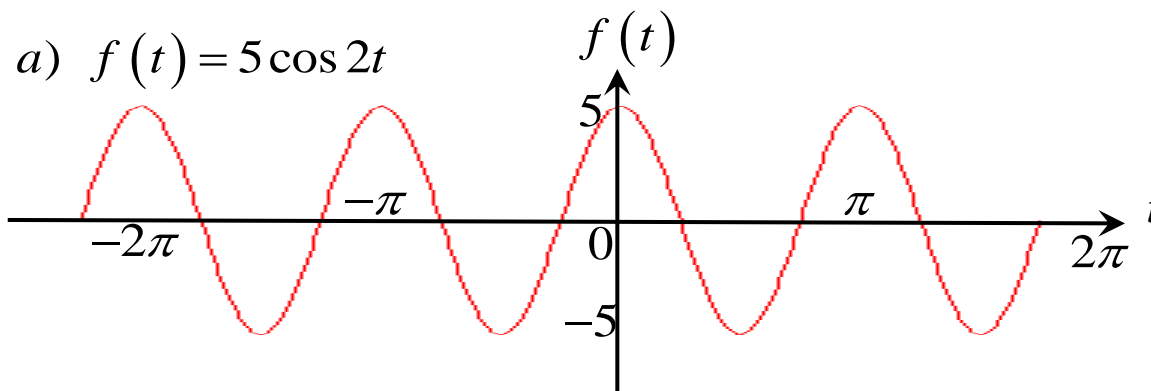


answer

$$A = 3$$

$$T = \pi$$

$$\omega = \frac{2\pi}{\pi} = 2$$



answer

$$A = 5$$

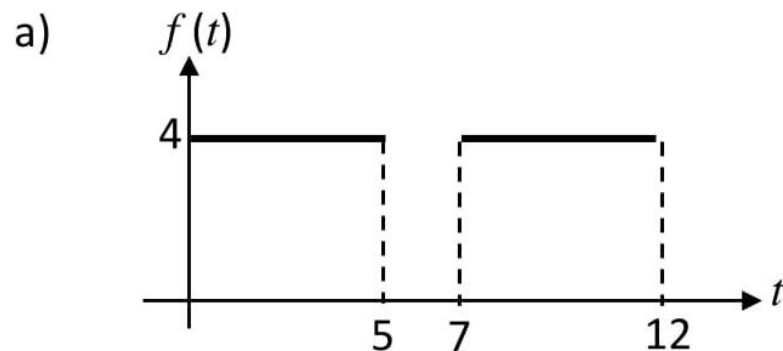
$$T = \pi$$

$$\omega = \frac{2\pi}{\pi} = 2$$



## Example 2

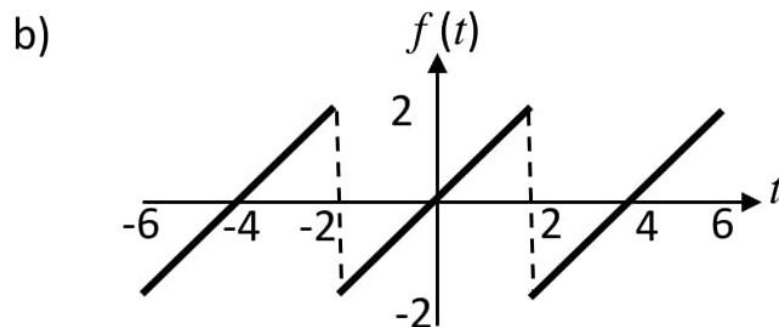
From each of the following waveform, find the analytical description.



answer

$$f(t) = \begin{cases} 4, & 0 \leq t < 5 \\ 0, & 5 \leq t < 7 \end{cases}$$

$$f(t) = f(t+7)$$



answer

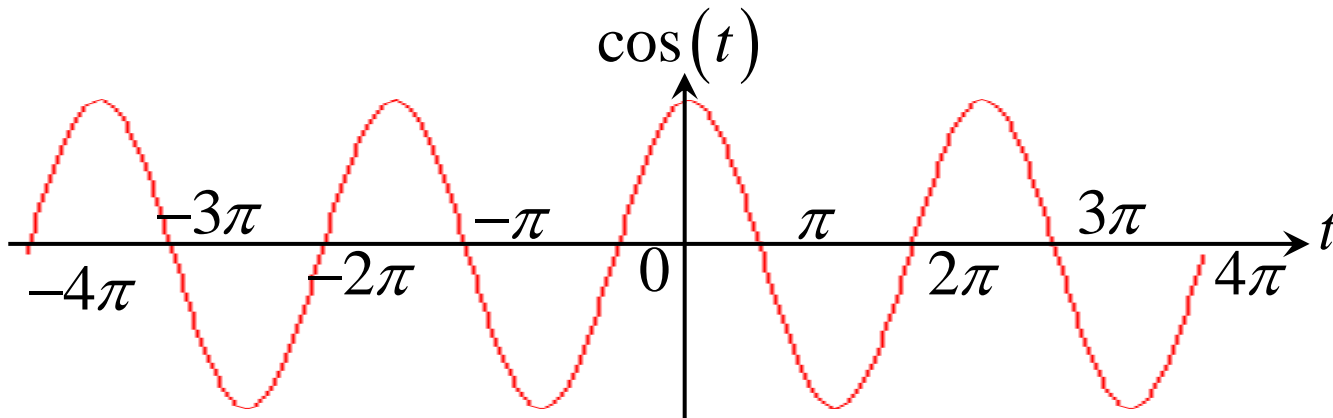
$$f(t) = t, \quad -2 \leq t < 2$$

$$f(t) = f(t+4)$$



## 4.2 Even and Odd Function

### Even function



For **even** function, the function is inverted on the other side of the  $y$ -axis. That is say :

$$f(t) = f(-t) \text{ for all } t \in \mathbb{R}.$$

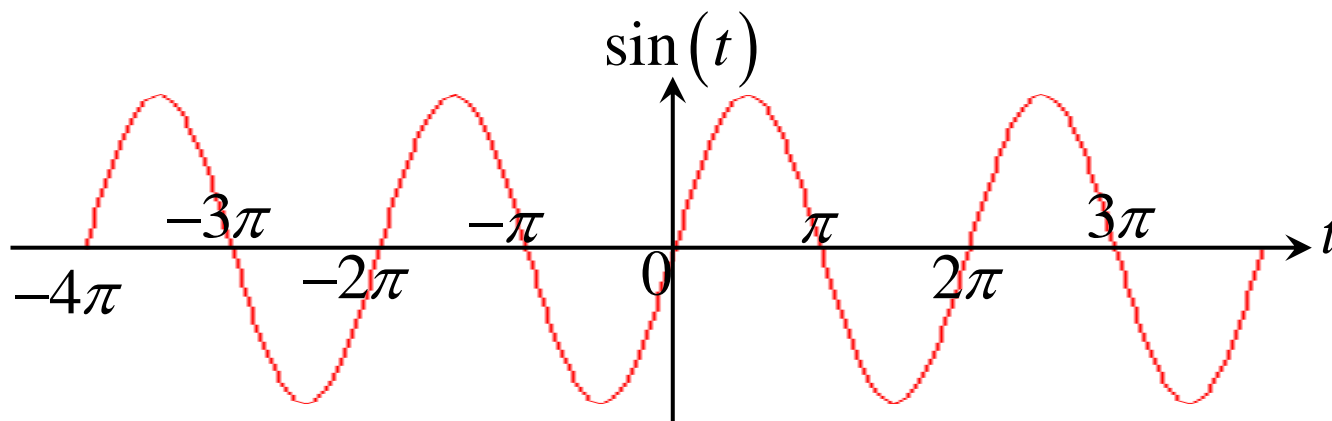


Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

Communitising Technology

# Odd function



For **odd** function, the function is symmetric about the *origin*. That is say :

$$f(-t) = -f(t) \text{ for all } t \in \mathbb{R}.$$



### Example 3

Sketch the graph of each of these periodic functions and determine whether its is even, odd or neither.

$$a) f(t) = \begin{cases} 2, & -\pi \leq t < 0 \\ -2, & 0 < t < \pi \end{cases}$$

$$f(t) = f(t + 2\pi)$$

$$b) f(t) = \begin{cases} 0, & -\pi \leq t < -\frac{\pi}{2} \\ 3, & -\frac{\pi}{2} \leq t < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \leq t < \pi \end{cases}$$

$$f(t) = f(t + 2\pi)$$

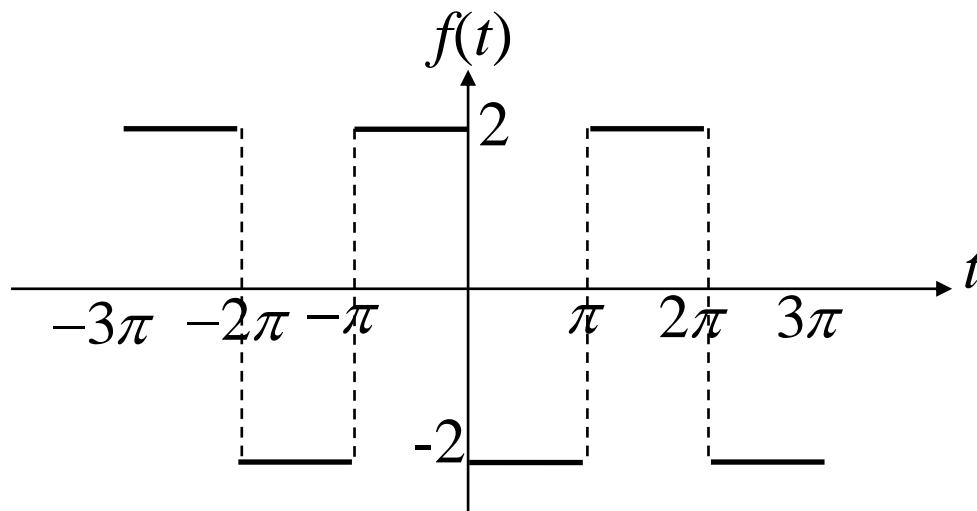
$$c) f(t) = \begin{cases} \frac{4t}{\pi} + 3, & -\pi \leq t < 0 \\ 3, & 0 < t < \pi \end{cases}$$

$$f(t) = f(t + 2\pi)$$



## Solution

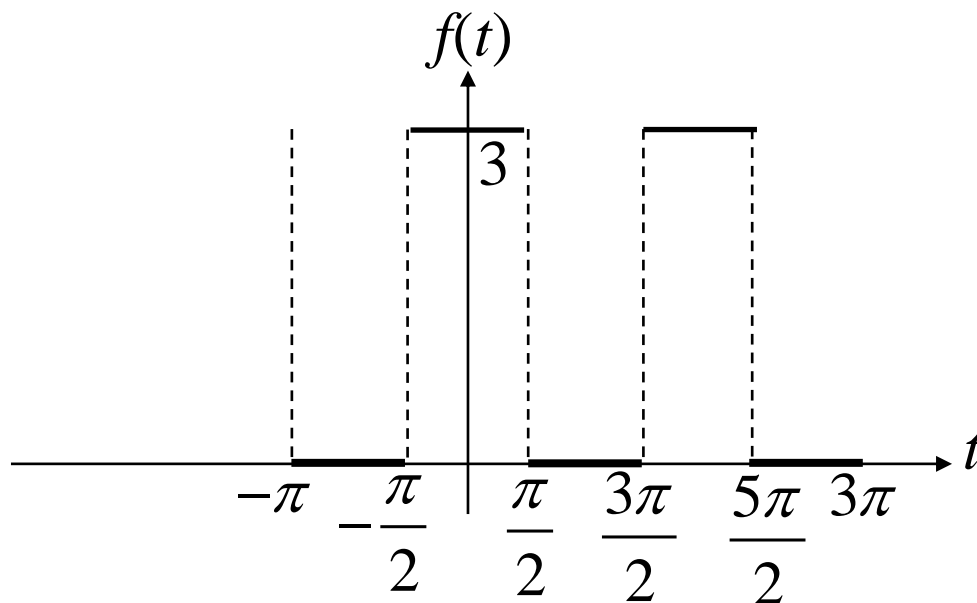
a)



The graph is symmetric about the origin. Hence, the periodic function is odd.

## Solution

b)



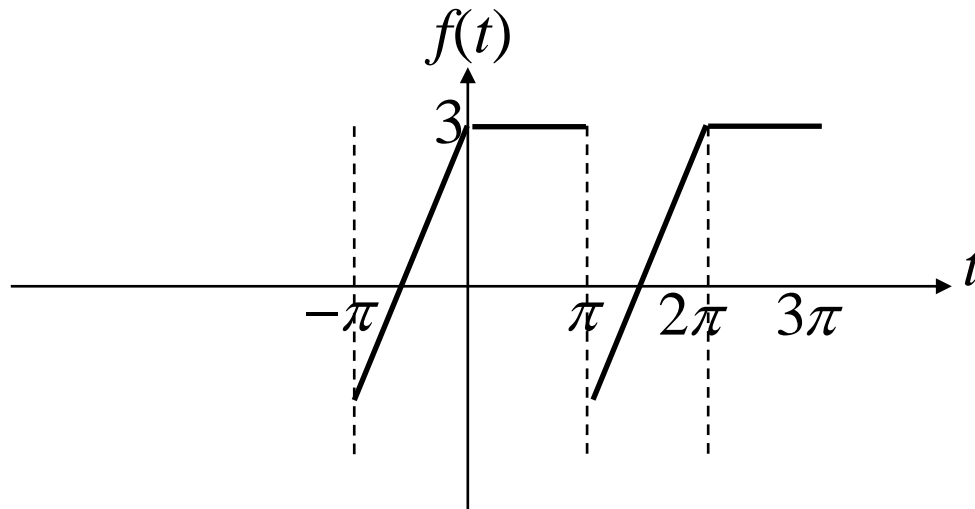
The periodic function is even since the graph is symmetric about the vertical axis.





## Solution

c)



The periodic function is neither even nor odd function since the graph is not symmetric about both the origin and the vertical axis.



Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

Communitising Technology

# Products of even and odd functions

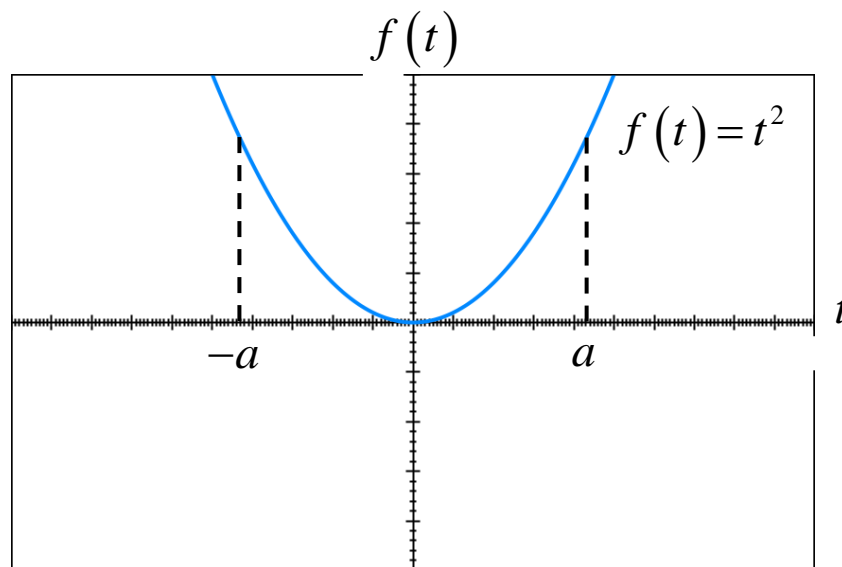
- For  $f(t)$  and  $g(t)$  are both even, then  $f(t)g(t)$  is even.
- For  $f(t)$  and  $g(t)$  are both odd, then  $f(t)g(t)$  is even.
- For  $f(t)$  is even and  $g(t)$  is odd or vice versa, then  $f(t)g(t)$  is odd.

## Example 4

- If  $f(t) = 2t^2$  (even) and  $g(t) = \cos 3t$  (even), then  $f(t)g(t) = 2t^2 \cos 3t$  is even.
- If  $f(t) = t^7$  (odd) and  $g(t) = \sin 2t$  (odd), then  $f(t)g(t) = t^7 \sin 2t$  is even.
- If  $f(t) = t^4$  (even) and  $g(t) = \sin 4t$  (odd), then  $f(t)g(t) = t^4 \sin 4t$  is odd.



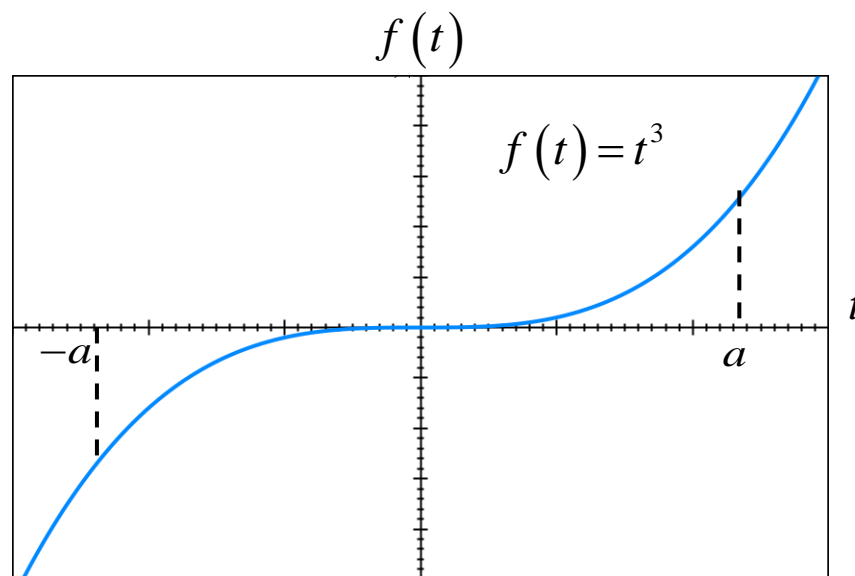
# Integral of even and odd function



$f(t) = t^2$  is an even function, hence

$$\int_{-a}^a t^2 dt = 2 \int_{-a}^a t^2 dt$$





$f(t) = t^3$  is an odd function, hence

$$\int_{-a}^a t^3 dt = \left[ \frac{t^4}{4} \right]_{-a}^a = 0$$





Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

Communitising Technology

## Author Information

Nor Aida Zuraimi binti Md Noar  
[aidaz@ump.edu.my](mailto:aidaz@ump.edu.my)

Wan Nur Syahidah binti Wan Yusoff  
[wnsyahidah@ump.edu.my](mailto:wnsyahidah@ump.edu.my)

Zulhibri Ismail@Mustofa  
[zulhibri@ump.edu.my](mailto:zulhibri@ump.edu.my)

Nadirah Mohd Nasir  
[nadirah@ump.edu.my](mailto:nadirah@ump.edu.my)

Rahimah Jusoh@Awang  
[rahimahj@ump.edu.my](mailto:rahimahj@ump.edu.my)

Laila Amera Aziz  
[laila@ump.edu.my](mailto:laila@ump.edu.my)

Samsudin Abdullah  
[samsudin382@gmail.com](mailto:samsudin382@gmail.com)



Ordinary Differential Equations  
by Nor Aida Zuraimi bt Md Noar

<http://ocw.ump.edu.my/course/view.php?id=446>

Communitising Technology