FACULTY OF INDUSTRIAL SCIENCES & TECHNOLOGY

FINAL EXAMINATION

COURSE	**:**	**DISCRETE MATHEMATICS & APPLICATIONS**
LECTURER	**:**	**MOHD SHAM MOHAMAD** **ADAM SHARIFF ADLI AMINUDDIN** **INTAN SABARIAH SABRI**

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of **FIVE(5)** questions. Answer **ALL** questions.
2. All answers to a new question should start on new page.
3. All the calculations and assumptions must be clearly stated.
4. Candidates are not allowed to bring any material other than those allowed by the invigilator into the examination room.

EXAMINATION REQUIREMENTS

1. Scientific calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **SEVEN (7)** printed pages including front page.

QUESTION 1

1. Table 1 below shows a Cayley Table for a group *G*.

$\cdot$	a	b	c	d
a	c	a	d	b
b	a	b	c	d
c	d	c	b	a
d	b	d	a	c

Table 1

(i) What is the identity element of *G*?

(ii) Find the inverse of element *c* in *G*.

(2 Marks)

2. Define the center, $Z(G)$ of a group *G*.

(2 Marks)

3. Find $\gcd(3a,9b)$ if $\gcd(a,b)=3$.

(1 Mark)

4. Let $\gcd(x,5x)=25$ and $\operatorname{lcm}(x,5x)=125$. Find a value of *x*.

(2 Marks)

5. Determine each of the following is a contrapositive, converse or inverse for the proposition

Special rate for ticket of Legoland Malaysia Resort is only available to Johorean.

(i) If special rate for ticket of Legoland Malaysia Resort is not available for you, then you are not Johorean.

(ii) If you get special rate for ticket of Legoland Malaysia Resort, then you are Johorean

(iii) If you are not Johorean, then you will not get special rate for ticket of Legoland Malaysia Resort

(3 Marks)

6. Refer **Table 2** and state the truth value for the given statement.

Table 2

p	q	$\sim q$	$(\sim q) \wedge p$	$\boldsymbol{x}$	$p \vee \sim(\sim q \wedge p)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T

(i) The possible proposition in $\boldsymbol{x}$ is $\sim((\sim q) \vee p)$.

(ii) $p \vee \sim(\sim q \wedge p)$ is tautology and equivalent with $\sim(\sim p \wedge (\sim q \wedge p))$

(2 Marks)

[20 Marks]

QUESTION 2

(a) Let $a,b \in \mathbb{Z}^{+}$ and $\gcd(a,b)=5$. Show that

$$25 | ab.$$

(4 Marks)

(b) Let $m = 12345 \text{ div } 6$ and $n = 12345 \bmod 678$. Then, find $\gcd(m,n)$ by using prime factorization method.

(6 Marks)

(c) Given a predicate $R(x,y): x^2 + y^2 \leq 2xy$. Find the truth value each of the following and give your justification

(i) $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ R(x,y)$,

(ii) $\exists x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ R(x,y)$.

(4 Marks)

(d) Determine whether $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are logically equivalent or not.

(6 Marks)

[20 Marks]

QUESTION 3

(a) Let x, y and z be even integers. By using direct method, prove that the product of x, y and z is an even integer $\forall x, y, z \in Z$

(5 Marks)

(b) Prove that the sum of two odd perfect squares is even.

(5 Marks)

(c) By using mathematical induction, show that for every integer $k \geq 1$,

$$1(1!) + 2(2!) + ... + (n)(n!) = (n+1)! - 1$$

(10 Marks)

[20 Marks]

QUESTION 4

[20 Marks]

QUESTION 5

(a) Let G be a set of integers. Determine whether G is a group by checking all the properties of group if the operation is a subtraction.

(7 Marks)

(b) Determine whether $H = \left\{ \begin{bmatrix} a & b \\ 0 & a-b \end{bmatrix} \middle| a, b \in \mathbb{Z} \right\}$ is a subgroup of 2×2 real matrices under addition.

(6 Marks)

(c) Let $\mathbb{Z}_7$ be a group.

(i) Construct a Cayley Table for group $\mathbb{Z}_7$ under multiplication.

(ii) Assume that $\langle \mathbb{Z}_7, +, \times \rangle$ is a ring. Show that $\langle \mathbb{Z}_7, +, \times \rangle$ is a field.

(7 Marks)

[20 Marks]

END OF QUESTION PAPER