DISCRETE MATHEMATICS AND APPLICATIONS

Logic 1

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Chapter Description

- Chapter Outline

  3.1 Propositional Logic

  3.2 Logical Connectives

  3.3 Propositional Equivalences

- Aims
  - Identify whether a statement is a proposition or not.
  - Solve problems involving logical connectives, determine truth value and construct truth table.
  - Verify that a proposition is a tautology, contradiction, contingency or logically equivalents.

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References

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- Epp S.S, Discrete Mathematics with Applications, (Fourth Edition), Thomson Learning, 2011
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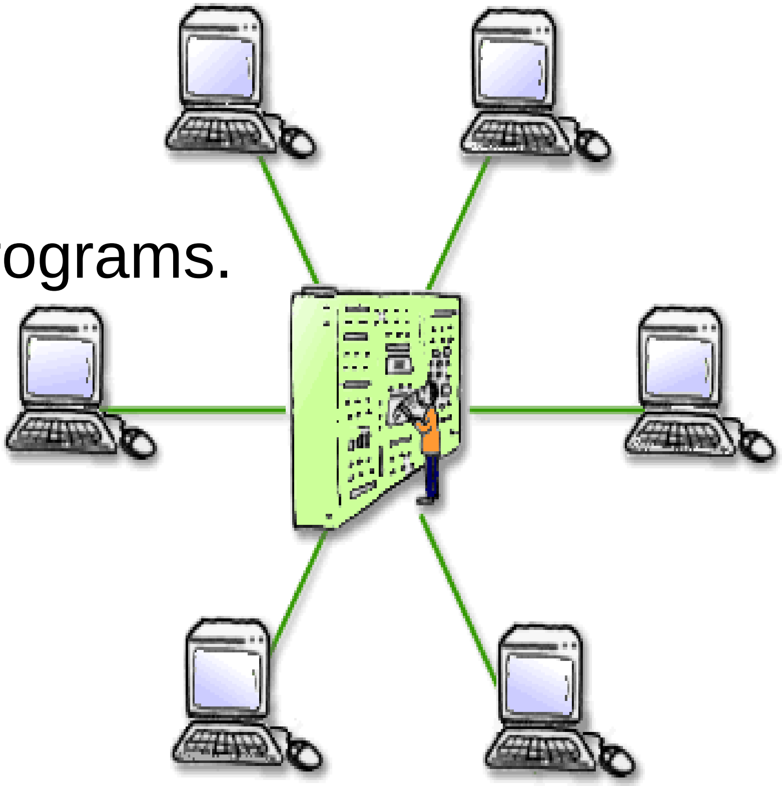
- Walls W.D., A beginner's guide to Discrete Mathematics, Springer, 2013
- Chandrasekaren, N. & Umaparvathi, M., Discrete Mathematics, PHI Learning Private Limited, Delhi, 2015

Applications of Logic in Computer Science

- Design of computing machines.
- Programming languages.
- Design of computer circuits.
- The construction of computer programs.



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Propositions

- Proposition - a declarative sentence that is either true or false, but not both.
- In other words, a statement is an idea of proposition.

True

or

False

Not Both

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Propositions: Example

Which of the following are propositions?

1. I love Mathematics Discrete and Applications subject.

   This is a proposition
2. We have to drink 8 glasses of plain water every day.

   This is a proposition
3. How are you today?

   This is not a proposition (Question)
4. Please sit down.

   This is not a proposition (Command)
5. 9+9=19

   This is a proposition

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Compound proposition

- Many mathematical statements are constructed by combining the propositions.
- Logical connectives are used to combine proposition to form new propositions by using logical operators (*not*, *and* and *or*).
- This new propositions are called **compound propositions** or also known as nonatomic propositions.
- Note that a single proposition is called as an atomic proposition.

Propositional Variables

- In logic, variables are used to represent propositions.
- The letters *p,q,r,s,…* are known as propositional variables that are used to denote the variables.

Example:

p: Making just a few changes in your lifestyle can help you live longer.

q: Healthy eating is about eating smart.

Truth Value & Truth Table

- **Truth value** - the truth or falsity of a proposition.
- **Truth table** - shows the relationships between the truth values of propositions.

Truth Value

True - T

False – F

T and F are called propositional constants.

Example:

p : A grass is a green color. (p = T)

q : Today is a National Day. (q = F)

Logical Connectives (Not, And & Or)

Example

Let

p: Today is a Thursday.

q: You will get a summon if you drive over 110 km/h.

Not *p* : Today is not a Thursday.

q and *p*: You will get a summon if you drive over 110 km/h and today is a Thursday.

p or *q* : Today is a Thursday or you will get a summon if you drive over 110 km/h.

Types of Logical Connectives

- Negation
- Conjunction
- Disjunction
- Implication/ Conditional
- Biconditional

Negation

- We read as "not p" or "opposite of p".
- Denoted by $\sim p$ or $\neg p$.

Note that the number of possible rows in a truth table is given by 2^n where n is the number of propositional variables.

Truth Table

p	$\sim p$
T	F
F	T

Example:

State the negation of the following statements.

(a) $p : 9 + 1 < 11$

(b) q : I love to eat an apple.

(a) $\sim p : 9 + 1$ *is not less than* 11. *That is* $\sim p : 9 + 1 \geq 11$.

(b) $\sim q$: *I do not love to eat an apple.*

Conjunction

- We read as "*p* and *q*".
- Denoted by $p \wedge q$

Note that the proposition is TRUE only when both *p* and *q* are true.

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example:

Determine the conjunction of the propositions *p* and *q*.

(a) *p* : Today is a sunny day.
(b) *q* : I bring an umbrella to school.

$p \wedge q$: *Today is a sunny day and I bring an umbrella to school.*

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Disjunction

- We read as "*p* or *q*".
- Denoted by $p \vee q$.

Note that the proposition is FALSE only when both *p* and *q* are false.

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example:

Find the disjunction of the propositions *p* and *q*.

(a) *p* : Today is a sunny day.
(b) *q* : I bring an umbrella to school.

$p \vee q$: *Today is a sunny day or I bring an umbrella to school.*

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Implication/Conditional (i)

- We read as "*if p then q* ".
- Denoted by $p \to q$.

Bear in mind that the proposition is TRUE only when both p and q are true and also p is false.

Truth Table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Example:

p : Today is a raining day.
q : I will bring an umbrella to school.

$p \to q$: *If today is a raining day then I will bring an umbrella to school.*

Note: p is called hypothesis and q is called as a conclusion.

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Implication/Conditional (ii)

On the other hand, $p \rightarrow q$ can also be read as:

1. " if *p*, *q* "
2. " *p* is sufficient for *q* "
3. " *q* if *p* "
4. " *q* when *p* "
5. " a necessary condition for *p* is *q* "
6. " *q* unless ~ *p*"
7. " *p* implies *q* "
8. "*p* only if *q* "
9. " a sufficient condition for *q* is *p* "
10. " *q* whenever *p* "
11. " *q* is necessary for *p* "
12. " *q* follows from *p* "

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Implication: Converse, Contrapositive & Inverse

Related implications that can be formed from $p \to q$:

- Converse : $q \to p$
- Contrapositive : $\sim q \to \sim p$
- Inverse : $\sim p \to \sim q$

Same truth value as $p \to q$

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Implication: Example

Let p: It is Saturday.
q: Today is a holiday.

Converse: $q \rightarrow p$

- If today is a holiday, then it is Saturday.

Contrapositive: $\sim q \rightarrow \sim p$

- If today is not a holiday, then it is not Saturday.

Inverse: $\sim p \rightarrow \sim q$

- If it is not Saturday, then today is not a holiday.

Biconditional

- We read as "*p if and only if q*".
- Denoted by $p \leftrightarrow q$.

Truth Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Note that the proposition is TRUE only when both *p* and *q* have the same truth value.

Another terms of $p \leftrightarrow q$:

- "*p* iff *q*"
- "if p then q, and conversely"
- " *p* is necessary and sufficient condition for *q* ".

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Biconditional: Example

Let p : You can watch a movie
q : You buy a ticket

$p \leftrightarrow q$: "You can watch a movie if and only if you buy a ticket."

This statement is TRUE if p and q are either both true or false.

- If you buy a ticket and can watch a movie or
- If you do not buy a ticket and you cannot watch a movie.

The statement is FALSE when p and q have an opposite truth values.

- If you do not buy a ticket, but you can watch a movie (such as when you get a free ticket).
- If you buy a ticket and cannot watch a movie (such as when you missed the movie).

Precedence of Logical Operators

- In general, we will use parentheses to identify the order in logical operators.

Logical Operator	*Precedence*
$\sim$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Example:

Construct the truth table of the compound proposition $(p \wedge \sim q) \rightarrow (\sim p \vee q)$.

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Precedence of Logical Operators: Example

Find the truth value of the proposition $(\sim q \wedge p) \leftrightarrow (\sim p \vee q)$.

p	q	$\sim q$	$(\sim q \wedge p)$	$\sim p$	$(\sim p \vee q)$	$(\sim q \wedge p) \leftrightarrow (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	T	F

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Tautology, Contradiction and Contingency

- The classification of compound propositions are according to their possible values.

Tautology	Truth value of the compound proposition that is always True.
Contradiction	Truth value of the compound proposition that is always False.
Contingency	Truth value of the compound proposition that is neither tautology nor contradiction.

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Tautology, Contradiction and Contingency: Example 1

Find the truth value of the proposition.

q	$\sim q$	$\sim q \wedge q$	$q \vee \sim q$	$q \rightarrow \sim q$
T	F	F	T	F
F	T	F	T	T

Contradiction

Tautology

Contingency

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Tautology, Contradiction and Contingency: Example 2

Show that the proposition $(p \wedge \sim q) \leftrightarrow (\sim p \vee \sim q)$ is a contingency.

p	q	$\sim q$	$p \wedge \sim q$	$\sim p$	$\sim p \vee \sim q$	$(p \wedge \sim q) \leftrightarrow (\sim p \vee \sim q)$
T	T	F	F	F	F	T
T	F	T	T	F	T	T
F	T	F	F	T	T	F
F	F	T	F	T	T	F

$\therefore (p \wedge \sim q) \leftrightarrow (\sim p \vee \sim q)$ is a contingency.

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Logical Equivalences

- Different compound propositions that have the same truth values.
- The compound propositions *p* and *q* are *known as* *logically equivalent if* $p \leftrightarrow q$ *is a tautology.*
- Denoted by $p \equiv q$.

Logical Equivalences: Example 1

Show that the proposition $\sim(p \wedge \sim q)$ and $\sim p \vee q$ are logically equivalent.

p	q	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$	$\sim p$	$\sim p \vee q$
T	T	F	F	T	F	T
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	F	T	T	T

$$\therefore \quad \sim(p \wedge \sim q) \equiv \sim p \vee q.$$

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Logical Equivalences: Example 2

Show that the proposition $\sim(p \rightarrow q) \equiv p \wedge \sim q.$

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$	$\sim q$	$p \wedge \sim q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

$$\therefore \sim(p \rightarrow q) \equiv p \wedge \sim q.$$

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