Mathematics for Management

Chapter 6: Compound Interest

by

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Expected Outcome:

Upon the completion of this course, students will have the ability to:

1. Obtain the compound interest and continuous compounding by using the formula.
2. Classify the difference between effective rate and nominal rate.
3. Obtain the present value by manipulating from the previous formula.

Compound Interest

What is compound interest?

- It is arises when interest is added to the principal which changes time to time
- It is paid on the original principal and on the accumulated past interest
- Compound interest is not always calculated per year, it could be per month, per day or per week.

Formula Compound Interest

$$S = P\left(1 + \frac{i}{a}\right)^{n \times a}$$

where

S: the compound interest/future value in (RM)
P: the principal (the amount of money borrowed/invested) in (RM)
i: the interest rate in (%)
n: the length of time in years
a: numbers of times the interest is compounded per year

Differences Between Compound & Simple Interest

Compound Interest	Simple Interest
Based on principal which grows from one interest interval to another	Based on original principal which does not change from time to time
Compound amount function is exponential	Simple amount function is linear
Allows a principal amount to grow at a faster rate	The principal amount grows at a slower rate

Example:

(a) $S = P\left(1 + \frac{i}{1}\right)^{n \times 1}$ → annually compounding (once in a year)

(b) $S = P\left(1 + \frac{i}{4}\right)^{n \times 4}$ → quarterly compounding (4 times in a year)

(c) $S = P\left(1 + \frac{i}{12}\right)^{n \times 12}$ → monthly compounding (12 times in a year)

Exercise:

State the suitable formula for the given compounding:

(i) Compounding semiannually

(ii) Compounded every 3 months

(iii) Daily compounding

Example:

RM1000 is invested in a local bank for three years. If the investment earns 8% **compounded annually,** calculate the interest received after three years of investment.

Solution:

State the value of

$P = RM1000$

$i = 8\%$

$n = 3$ years

a = annually (once in a year) $= 1$

By using the formula,

$$S = P\left(1+\frac{i}{1}\right)^{n \times 1}$$

$$= RM1000\left(1+\frac{0.08}{1}\right)^{3 \times 1}$$

$$= RM1259.71$$

Hence, the total interest $I = S - P$

$= RM1259.71 - RM1000$

$= RM259.71$

Exercises:

- How much would you need to invest now, to get RM10 000 in 10 years at 8% interest rate compounded every months.

- What is the interest rate compounded monthly that will make RM1000 become RM2000 in five years?

- How long does it take a sum of money to double itself at 14% compounded annually?

Effective, Nominal & Equivalent Rates

What is effective and nominal rates?

Effective rate : interest is calculated once a year

Nominal rate : interest is calculated more than once a year

- Two rates are equivalent if they yield the same future value

Example:

RM100 is invested for one year. If the interest rate is **9.04% compounded annually** and **8.75% compounded quarterly**, find the amount after one year. Is it two of these interest rate are equivalent?

Solution:

$$S_1 = P\left(1+\frac{i}{1}\right)^{n\times 1}$$

$$= \text{RM}100\left(1+\frac{0.0904}{1}\right)^{1\times 1}$$

$$= \text{RM}109.04 \text{ (effective rate)}$$

$$S_2 = P\left(1+\frac{i}{4}\right)^{n\times 4}$$

$$= \text{RM}100\left(1+\frac{0.0875}{4}\right)^{1\times 4}$$

$$= \text{RM}109.04 \text{ (nominal rate)}$$

The future values for these two rates are same, so these interest rates are equivalent.

Exercise:

- RM200 is invested for three years. If the both interest rate is 5% compounded annually and compounded every 2 months respectively, find the amount after three years. Is it two of these interest rate are equivalent?

Continuous Compounding

What is continuous compounding?

- Compound of interest is done on a continuous basis

$$S = P(e^{it})$$

where

S: the future value
P: the original principal
i: the continuous compounding rate in (%)
t: the length of time in years
e: exponent (2.718282…)

Example:

Find the accumulated value of RM1000 for six months at 10% **compounded continuously**

Solution:

State the value of

$P = RM1000$

$i = 10\%$

$t = \frac{6}{12} = 0.5$

By using the formula,

$\mathbf{S = P(e^{it})}$

$= RM1000(e^{0.1(0.5)})$

$= RM1051.27$

Exercises:

- Find the amount to be deposited now so as to accumulate RM1000 in eighteen months at 10% compounded continuously.

- If the accumulated value of RM500 for 720 days compounded continuously was RM2000. What is the continuous interest?

Present Value

What is present value?

- Present value is the sum of money that you invested or borrowed
- In other word, it's same to the principal value

$$P = S\left(1+\frac{i}{a}\right)^{-n\,x\,a}$$

where

S: the compound interest/future value
P: the original principal/present value
i: the interest rate in (%)
n: the length of time in years
a: numbers of times the interest is compounded per year

Example:

A **debt RM3000** will mature in **three years** time. Find

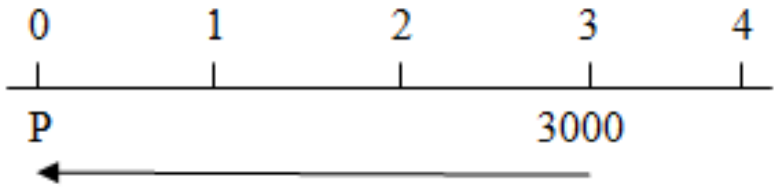
(a) the present value of this debt

(b) the value of this debt at the end of the first year,

(c) the value of this debt at the end of four years

assuming money is worth **14% compounded semi-annually**

Solution:

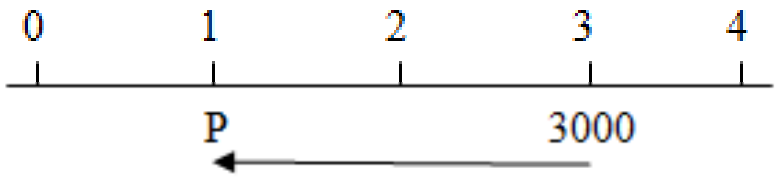


$$\text{(a)}\ P = S\left(1+\frac{i}{a}\right)^{-n\,x\,a}$$

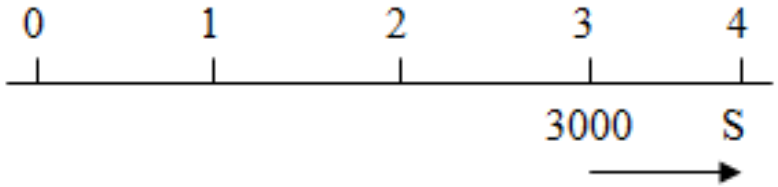
$$= RM3000\left(1+\frac{0.14}{2}\right)^{-3x\,2}$$

$$= RM1999.03$$

(b)



(c)



THE END

~THANK YOU~

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