Mathematics for Management

Chapter 4: Exponential & Logarithmic Functions

by
Nor Alisa Mohd Damanhuri and Rahimah Jusoh
Faculty of Industrial Sciences & Technology

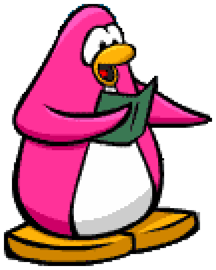
Content:

- 4.1 Exponential Functions
- 4.2 Logarithmic Functions
- 4.3 Exponential & Logarithmic Equations

Expected Outcome:

Upon successful completion of this course, students will have the ability to:

1. Identify the different between exponential and logarithmic function.
2. Solve the exponential and logarithmic equation by using the properties.



Exponential Function

Definition:

- The function f defined by

$$f(x) = b^x$$

where $b>0$, $b \neq 1$ and the exponent x is any real number. This mathematical expression is called an exponential function with base b.

Rules of Exponent

If $a > 0$, $b > 0$, where m and n are real numbers , then

1) $a^m a^n = a^{m+n}$

2) $\dfrac{a^m}{a^n} = a^{m-n}$

3) $(a^m)^n = a^{mn}$

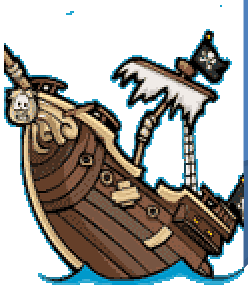
4) $(ab)^n = a^n b^n$

5) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

6) $a^1 = a$

7) $a^0 = 1$

8) $a^{-n} = \dfrac{1}{a^n}$



Exercises:

Suppose that the population of a certain country grows at an annual rate of 2%. If we measure population in millions and time in years, then

$$P(t) = P_0 e^{rt}$$

with $P_0 = 3$ and $r = 0.2$.

(a) If the current population is 3 million, what will the population be in 10 years?

(b) How long will it take the population to reach 5 million?

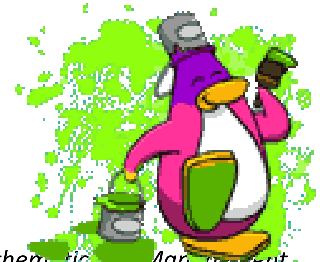
Logarithmic Functions

Definition:

If $b > 0$ and $b \neq 1$, for a positive value of *x* the expression

$$\log_b x$$

(read "**the logarithm to the base *b* of *x***") denotes that exponent to which *b* must be raised to produce *x*.



- Logarithmic functions can also be viewed as inverses of exponential functions.

$$y = \log_b x \quad \text{if and only} \quad b^y = x$$

For example:

$$\log_2 16 = 4 \quad \leftrightarrow \quad 2^4 = 16$$

$$\log_5 125 = 3 \quad \leftrightarrow \quad 5^3 = 125$$

$$\log_b \left(b^x\right) = x \quad \text{for all real values of } x$$

$$b^{\log_b x} = x \quad \text{for } x > 0$$

Properties of Logarithms

(a) $\log_b(mn) = \log_b m + \log_b n$

(b) $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

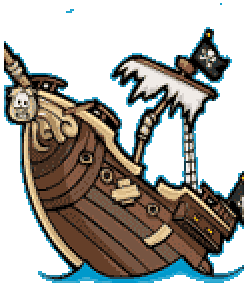
(c) $\log_b m^r = r\log_b m$

(d) $\log_b \frac{1}{m} = -\log_b m$

(e) $\log_b 1 = 0$

(f) $\log_b b = 1$

(g) $\log_b m = \frac{\log_a m}{\log_a b}$



Example:

(1) Solve the following logarithmic functions by using properties

a) $\log 56 = \log(8 \times 7)$

$= \log 8 + \log 7$ $\quad (\textit{property} : \log_b (mn) = \log_b m + \log_b n)$

b) $\log \frac{9}{2} =$

$=$ ________________________________

c) $\log 64 =$

$=$ ________________________________

(2) Converting from Exponential to Logarithmic Form

$a)\ 5^2 = 25$ $\quad \therefore\ \log_5 25 = 2$ $\quad (property: y = \log_b x \Leftrightarrow b^y = x)$

$b)\ 10^0 = 1$ $\quad \therefore$ ______________

$c)\ 3^4 = 81$ $\quad \therefore$ ______________

(3) Converting from Logarithmic to Exponential Form

a) $\log_{10} 1000 = 3$ $\quad \therefore\ 10^3 = 1000$ $\quad (property: y = \log_b x \Leftrightarrow b^y = x)$

b) $\log_{64} 8 = \frac{1}{2}$ $\quad \therefore$ ______________

c) $\log_2 \frac{1}{16} = -4$ $\quad \therefore$ ______________

Exercises:

(4) Solving logarithmic and exponential equations

a) $\log_2 x = 4$

$Solution: \; x = 2^4 = 16$

b) $\ln(x+1) = 7$

c) $\log_3 x = -3$

d) $\log_8 x = \frac{5}{3}$

e) $\log_3 x = 2^{-1}$

(5) Evaluate the following logarithms without using calculator

$(a)\ \log_3 81$

$Solution: \log_3\left(3^4\right) = 4$

$(b)\ \log_3 \sqrt{3}$

$(c)\ \log_2\left(\frac{\sqrt[3]{16}}{4}\right)$

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(6) Express $\log \dfrac{1}{x^2}$ in terms of $\log x$

Solution: $\log \dfrac{1}{x^2} = \log 1 - \log x^2$

$= 0 - 2\log x$

$= -2\log x$

(7) Write $\ln \dfrac{x}{zw}$ in terms of $\ln z, \ln x$ and $\ln w$.

Solution :

(8) Simplify

$(a)\ \ln x - \ln(x+3)$

$Solution:\ \ln x - \ln(x+3) = \ln \dfrac{x}{x+3}$

$(b)\ 2\log_5 15 - 3\log_5 4 + \dfrac{3}{2}\log_5 16$

$(c)\ \dfrac{1}{2}\log_3 16 + \dfrac{1}{3}\log_3 8 + 2$

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(9) Given that $\log 2 = 0.3010$ and $\log 7 = 0.8451$. Evaluate

(a) $\log 14$

Solution:

$$\log(2\times 7) = \log 2 + \log 7$$
$$= 0.3010 + 0.8451$$
$$= 1.1461$$

(b) $\log \frac{7}{2}$

(c) $\log 7^{\frac{4}{3}}$

Exponential & Logarithmic Functions

Definition:

- A **logarithmic equation** is an equation that contain logarithmic expression and a constant unknown.

  For example : $2\ln(x+1) = 9$

- An **exponential equation** is an equation where the unknown is an exponent

  For example : $2^{3x} = 64$

Properties

- If $\log_b m = \log_b n$ then $m = n$
- If $b^m = b^n$ then $m = n$

Example:

(1) Find x if $(25)^{x+2} = 5^{3x-4}$

Solution:

$$(25)^{x+2} = 5^{3x-4}$$
$$(5^2)^{x+2} = 5^{3x-4}$$
$$5^{2x+4} = 5^{3x-4}$$
$$2x+4 = 3x-4$$
$$x = 8$$

(2) Solve $\log_2 x = 5 - \log_2(x+4)$

Solution:

$$\log_2 x + \log_2(x+4) = 5$$
$$\log_2 x(x+4) = 5$$
$$x^2 + 4x = 2^5$$
$$x^2 + 4x - 32 = 0$$
$$(x+8)(x-4) = 0$$
$$\therefore x = -8 \quad \text{and} \quad x = 4$$

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(3) Solve $9^{x}-4(3)+3=0$

Solution:

$$\left(3^{2}\right)^{x}-4(3)+3=0$$

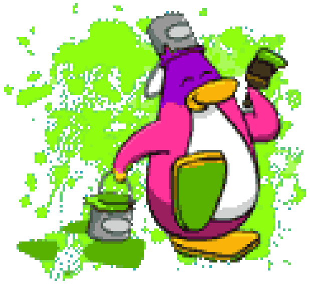
$$3^{2x}-4(3)+3=0$$

$$3^{2x}-9=0$$

$$3^{2x}=9$$

$$3^{2x}=3^{2}$$

$$\therefore x=1$$



(4) Solve $\log_4 x + 12\log_x 4 = 7$

Solution:

By changing the basis of logarithm, the given equation can be rewritten as

$$\log_4 x + 12\left(\frac{\log_4 4}{\log_4 x}\right) = 7 \qquad \textit{property}: \log_b m = \frac{\log_a m}{\log_a b}$$

Let $\log_4 x = p$. So we get

$$p + \frac{12}{p} = 7$$

$$p^2 + 12 = 7p$$

$$p^2 - 7p + 12 = 0$$

$$(p-4)(p-3) = 0$$

$$\therefore p = 3 \quad \text{or} \quad p = 4$$

Thus

$\log_4 x = 4$ or $\log_4 x = 4$

$x = 4^4$ $\qquad$ $x = 4^4$

$x = 256$ $\qquad$ $x = 64$

Exercises:

1. Solve $5+(3)4^{x-1}=12$

   Solution:

2. Solve $3^{x}\left(3^{x+2}\right)=27$

   Solution:

3. Solve $\log_3 2x^2 = 1+\log_3 x$

   Solution:

4. Find the values of x of the equation $\left(\frac{1}{4}\right)^{x^2} = 16^{x-4}$.

   Solution:

5. Solve the equation $625 \cdot 5^{x^2-1} = 5^{5x-1}$.

   Solution:

6. Solve $2\left(\log_9 x + \log_x 9\right) = 5$

   Solution:

THE END

~THANK YOU~

Author Information

noralisa@ump.edu.my
rahimahj@ump.edu.my