Universiti Malaysia PAHANG
Engineering • Technology • Creativity

Mathematics for Management

Chapter 3: Inequalities

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Content:

- 3.1 Introduction to Inequalities
- 3.2 Linear Inequalities and Its Application
- 3.3 Absolute Value

Expected Outcome:

Upon the completion of this course, students will have the ability to:

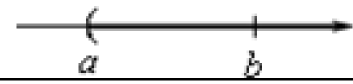
1. Solve the problems involving inequalities and absolute values
2. Construct the linear inequalities based on the applied problems.

Inequalities

What is the inequalities?

$>$	is greater than
$<$	is less than
$\geq$	is greater than or equal to
$\leq$	is less than or equal to

Interval Notation

Inequality	Graph	Interval Notation
$a \le x \le b$		$[a, b]$
$a < x < b$		(a, b)
$a \le x < b$		$[a, b)$
$a < x \le b$		$(a, b]$
$x > a$		(a, ∞)
$x \ge a$		$[a, \infty)$
$x < b$		$(-\infty, b)$
$x \le b$		$(-\infty, b]$

Rules for Manipulating Inequalities

Rule 1:
If the same number is added to or subtracted from both sides of inequality, the resulting of the inequality has the same sense as the original inequality.

If $a < b$, then $a + c < b + c$ and $a - c < b - c$

Example: **4 < 5 then 4+2 < 5+2 and 4-2 < 5-2, c = 2**

Rule 2:
If both sides of inequality are multiplied or divided by the same **positive** number, then the resulting inequality has the same sense as the original inequality.

If $a < b$ and $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

Example: **4 < 6 then 4(2) < 6(2) and 4/2 < 6/2, c = 2 (positive number)**

Rule 3:
If both sides of inequality are multiplied or divided by the same **negative** number, then the resulting inequality has **reverse** sense as the original inequality.

$$\text{If } a < b \text{ and } c < 0 \text{, then } ac > bc \text{ and } \frac{a}{c} > \frac{b}{c}$$

Example: **4 < 6 then 4(-2) > 6(-2) and 4/-2 > 6/-2, c = -2 (negative number)**

Rule 4:
Any side of inequality can be replaced by an expression equal to it.

$$\text{If } a < b \text{ and } a = c \text{, then } c < b$$

Example: **a < 5 and a = 3, then 3 < 5**

Rule 5:
If the sides of an inequality are either both side positive or both negative, then their respective reciprocals are unequal in the **reverse** sense.

$$\text{If } a < b \text{, then } \frac{1}{a} > \frac{1}{b}$$
$$\text{If} -a < -b \text{, then } -\frac{1}{a} > -\frac{1}{b}$$

Example: **4 < 6 then 1/4 > 1/6, if -3 < -2 then -1/3 > -1/2**

Rule 6:
If both sides of an inequality are positive and we raise each side to the same positive power, then the resulting inequality has the same sense as the original inequality. Thus

$$\text{If } 0 < a < b \text{ and } n > 0 \text{, then } a^n < b^n \text{ and } \sqrt[n]{a} < \sqrt[n]{b}$$

where we assume that will be n positive integer in the latter inequality.

Example: **4 < 9 then $4^2 < 9^2$ and $4^{1/2} < 9^{1/2}$, n = 3 (positive number)**

Linear Inequalities & Its Application

What is the linear inequalities?

- A linear inequalities in the variable x is an inequality that can be written in the form $ax+b>0$ or $ax+b<0$ where a, b are constants and a not equal to zero.

When solving an inequality:

- ✓ The same quantity can be added to each side
- ✓ The same quantity can be subtracted from each side
- ✓ The same **positive** quantity can be multiplied or divided to each side
- ✓ If the quantity that been multiplied or divided to each side of inequality is **negative**, the inequality symbol must be reversed (refer to Rule 3).

Example:

Solve the inequality $7x - 2 > 0$.

Solution:

Adding 2 to both sides give

$$7x > 2 \qquad \text{(Rule 1)}$$

Dividing both sides by the positive number 7 yields

$$x > \frac{2}{7} \qquad \text{(Rule 2)}$$

Hence, all values of x that are **greater than** $\frac{2}{7}$ satisfy $7x - 2 > 0$.

Example:

Find the range of values of x satisfying $x - 3 < 2x + 5$.

Solution:

There are many ways of arriving at the correct answer. For example, adding 3 to both sides

$$x < 2x + 8 \qquad \text{(Rule 1)}$$

Subtracting $2x$ from both sides then gives

$$-x < 8 \qquad \text{(Rule 1)}$$

Multiplying both sides by -1 will reverse the order as

$$x > -8 \qquad \text{(Rule 3)}$$

Thus, all values of x **greater than** -8 satisfy $x - 3 < 2x + 5$.

Example:

Solve the inequality $\frac{4}{x} < 3 + x$

Solution:

$$\frac{4}{x} - 3 - x < 0$$

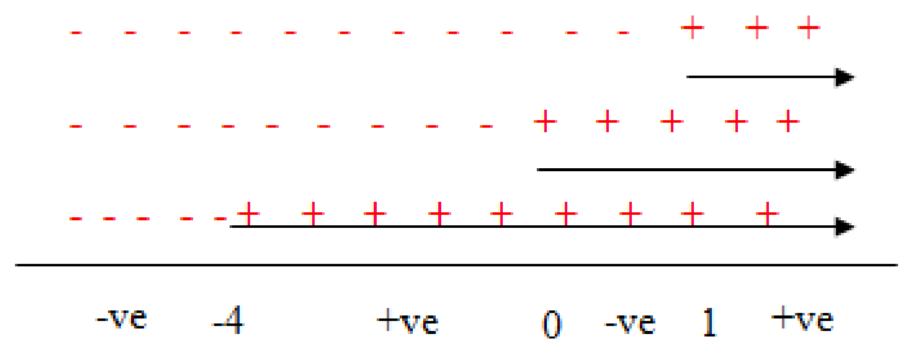
$$\frac{4 - 3x - x^2}{x} < 0$$

$$\frac{x^2 + 3x - 4}{x} > 0 \quad \textbf{change symbol}$$

$$\frac{(x+4)(x-1)}{x} > 0 \quad \textbf{factorize}$$

$x + 4 > 0$ $\qquad$ $x - 1 > 0$ $\qquad$ $x > 0$

$x > -4$ $\qquad$ $x > 1$



$\therefore -4 < x < 0 \quad or \quad x > 1$

Or it can be written as $(-4, 0) \cup (1, \infty)$

Exercises:

1. Solve the inequality $5-3x \le 3$
2. Solve the inequality $4x+6 > 3x+7$
3. Solve the inequality $3x-5 \le 3-x$
4. Find the range of x if $\dfrac{-3-2x}{x} \ge -x$

Example (application):

For a company that manufactures aquarium heaters, the combined cost for labor and material is RM21 per heater. Fixed cost (costs incurred in a given period, regardless output) is RM70 000. If the selling price of heater is RM35, how many must be sold for the company to earn a profit?

$$\text{Profit} = \text{Total revenue} - \text{Total cost}.$$

Let q be the number of heaters sold. Then, their cost is $21q$.

Then,

$$\text{Total cost} = 21q + 70000$$

$$\text{Total revenue from the sale of } q \text{ heaters} = 35q$$

We want Profit > 0,

$$\therefore \text{Total revenue} - \text{Total cost} > 0$$

$$35q - (21q + 70000) > 0$$

$$14q > 70000$$

$$q > 5000$$

Therefore, at least 5001 heaters must be sold to earn a profit.

Exercise:

- A builder must decide whether to rent or buy an excavating machine. If he were to rent the machine, the rental fee would be RM3000 per month (on a yearly basis), and the daily cost (gas, oil and driver) would be RM180 for each day the machine is used. If he were to buy it, his fixed annual cost would be RM20 000 and daily operating and maintenance cost would be RM230 for each day the machine is used. What is the least number of days each year that the builder would have to use the machine to justify renting it rather than buying it?

Absolute Value

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

To solve for an absolute values problem, in general we have these formulas:

Absolute Values Inequality $(b > 0)$	Solution
$\lvert x \rvert \leq b$	$-b \leq x \leq b$
$\lvert x \rvert < b$	$-b < x < b$
$\lvert x \rvert \geq b$	$x \leq -b$ or $x \geq b$
$\lvert x \rvert > b$	$x < -b$ or $x > b$

Properties of Absolute Value

- $|ab| = |a||b|$
- $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
- $|a + b| \le |a| + |b|$
- $|a - b| \ge |a| - |b|$
- $|a - b| = |b - a|$
- $-|a| \le a \le |a|$
- $|a| \ge 0$
- $|a^2| = |a|^2 = a^2$
- $|-a| = |a| = a$
- $|a| = \sqrt{a^2} = a$

Example:

a) $|(-8).2| = |-8|.|2| = 8.2 = 16$

b) $|9-6| = |6-9| = 3$

c) $|x-2| = |2-x|$

d) $\left|\dfrac{-5}{3}\right| = \dfrac{|-5|}{|3|} = \dfrac{5}{3}$

e) $\left|\dfrac{x-4}{-7}\right| = \dfrac{|x-4|}{|-7|} = \dfrac{|x-4|}{7}$

f) $-|8| \leq 8 \leq |8|$

g) $|3x+2| < 0$ has no solution since absolute value is **strictly positive**, i.e. (≥ 0).

Example:

Solve the absolute value inequality $|2x+1|<3$.

Solution:

From the formula, this absolute value problem would have the solution of

$$-3<2x+1<3$$

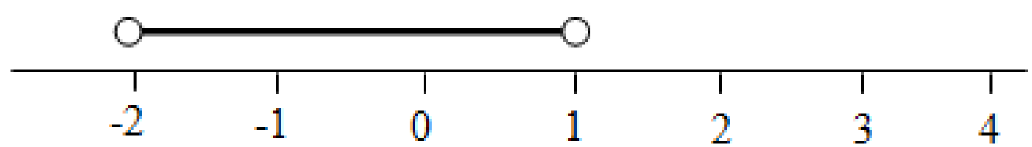
Subtract -1 yields

$$-4<2x<2$$

Divide by 2 gives

$$-2<x<1$$

Hence, the required solution is given as the number line below, which is, all the real numbers in between -2 and 1 (not including them).



Exercises:

1. Solve the absolute value of $\left|3+\frac{1}{2}x\right|>4$
2. Solve the inequality $\frac{3}{2}<|2-2x|$

THE END

~THANK YOU~

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