DYNAMICS

Planar Kinetics of a Rigid Body (Impulse and Momentum)

by:
Dr. Mohd Hasnun Arif HASSAN
Faculty of Manufacturing Engineering
mhasnun@ump.edu.my

Impulse and Momentum Method

- Aims
  - To introduce the Principle of Impulse and Momentum.
  - To explain about the conservation of momentum.
  - To learn about eccentric impact.
- Expected Outcomes
  - Students are able to use the principle of impulse and momentum to solve planar kinetics problem.
  - Students are able to apply the conservation of momentum in solving problems.
  - Students are able to determine the velocity of rigid body experiencing eccentric impact.
- References
  - Engineering Mechanics: Dynamics 12th Edition, RC Hibbeler, Prentice Hall

Contents

- Principle of Linear Impulse and Momentum
- Conservation of Momentum
- Eccentric Impact

Principle of Linear Impulse & Momentum

$$\Sigma \mathbf{F} = m\mathbf{a}_G = m(d\mathbf{v}_G/dt)$$

$$\Sigma \mathbf{F} = \frac{d}{dt}(m\mathbf{v}_G)$$

$$\Sigma \int_{t_1}^{t_2} \mathbf{F}\, dt = m(\mathbf{v}_G)_2 - m(\mathbf{v}_G)_1$$

Linear Impulse **Linear Momentum**

Principle of Angular Impulse & Momentum

$$\Sigma M_G = I_G\alpha = I_G(d\omega/dt)$$

$$\Sigma M_G = \frac{d}{dt}(I_G\omega)$$

$$\Sigma \int_{t_1}^{t_2} M_G\,dt = I_G\omega_2 - I_G\omega_1$$

Angular Impulse **Angular Momentum**

$$\Sigma M_O = I_O\alpha \rightarrow \Sigma \int_{t_1}^{t_2} M_O\,dt = I_O\omega_2 - I_O\omega_1$$

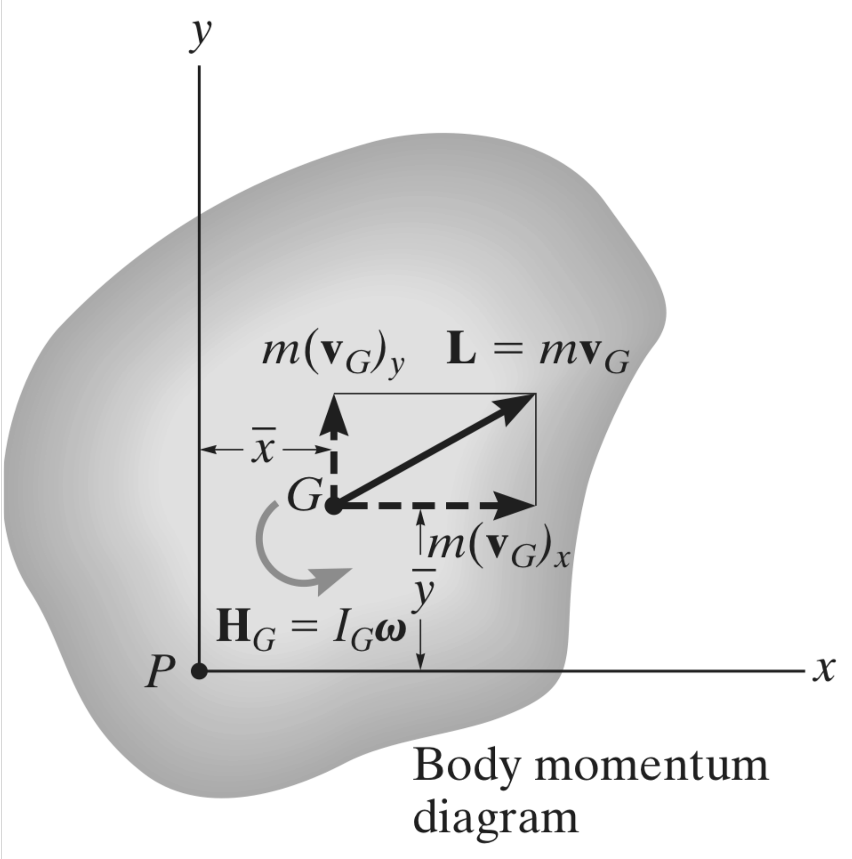
Linear and Angular Momentum

Linear momentum

$$\mathbf{L} = m\mathbf{v}_G$$

Angular momentum

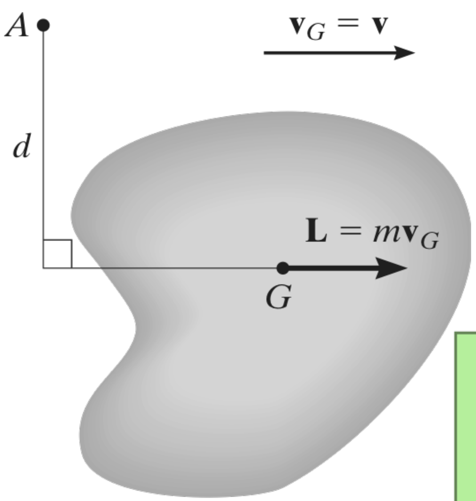
$$H_G = I_G\omega$$



$$H_P = I_G\omega + m(v_G)_x dy - m(v_G)_y dx$$

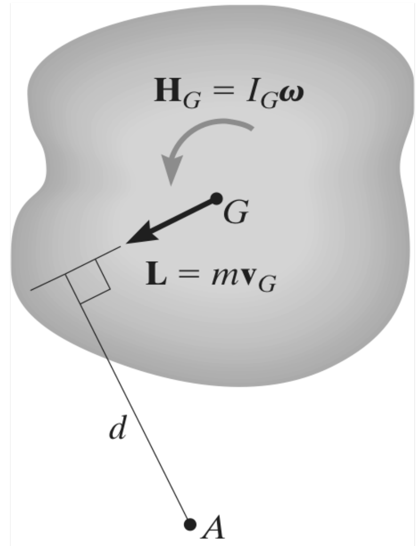
Identical to the kinetic moments about point *P*

Linear and Angular Momentum (cont'd)



Translation

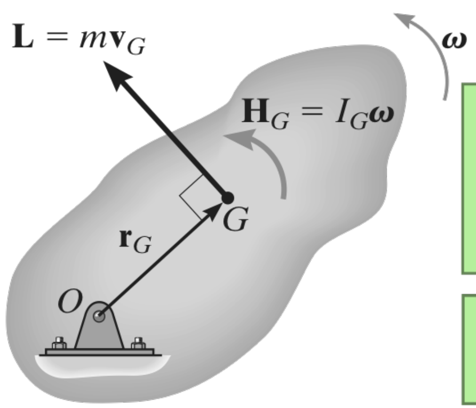
$$L = mv_G$$
$$H_G = 0$$



General plane motion

$$L = mv_G$$
$$H_G = I_G\omega$$

$$H_{IC} = I_{IC}\,\omega$$



Rotation about a fixed axis

$$L = mv_G$$
$$H_G = I_G\omega$$

$$H_O = I_O\,\omega$$

Principle of Impulse and Momentum

Universiti Malaysia PAHANG
Engineering • Technology • Creativity

$$L_1 + \sum I_{1-2} = L_2$$ Principle of Linear Impulse and Momentum

$$(H_P)_1 + \sum \int_{t_1}^{t_2} M_P\, dt = (H_P)_2$$ Principle of Angular Impulse and Momentum

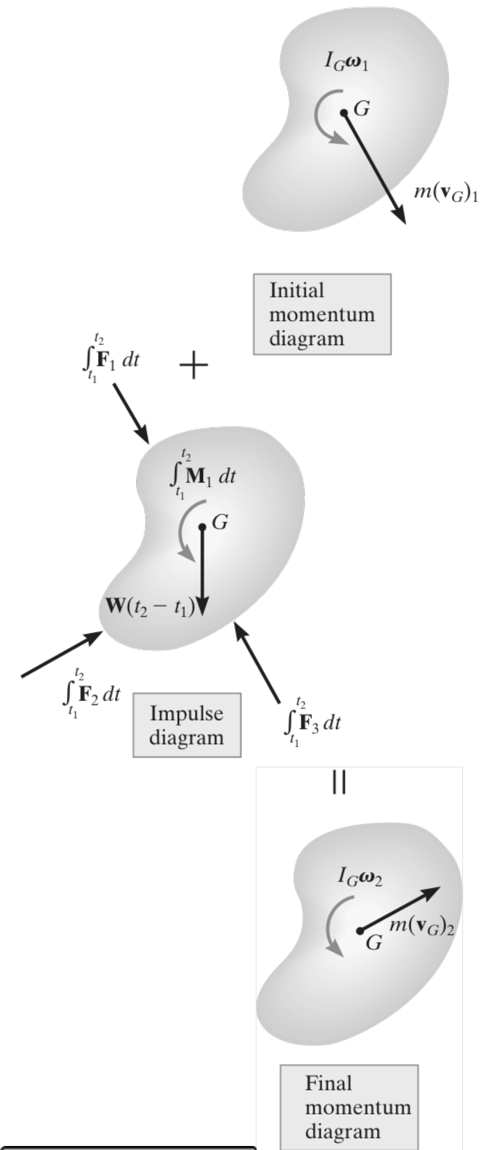
$$m(v_{Gx})_1 + \sum \int_{t_1}^{t_2} F_x\, dt = m(v_{Gx})_2$$

$$m(v_{Gy})_1 + \sum \int_{t_1}^{t_2} F_y\, dt = m(v_{Gy})_2$$

$$I_G\omega_1 + \sum \int_{t_1}^{t_2} M_G\, dt = I_G\omega_2$$

Principle of Impulse and Momentum

Universiti Malaysia PAHANG
Engineering • Technology • Creativity



$$m(v_{Gx})_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_{Gx})_2$$

$$m(v_{Gy})_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_{Gy})_2$$

$$I_G\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\omega_2$$

For a system of rigid bodies (more than one rigid bodies)

$$\left(\Sigma\,\begin{matrix}\text{syst. linear}\\ \text{momentum}\end{matrix}\right)_{x1} + \left(\Sigma\,\begin{matrix}\text{syst. linear}\\ \text{impulse}\end{matrix}\right)_{x(1-2)} = \left(\Sigma\,\begin{matrix}\text{syst. linear}\\ \text{momentum}\end{matrix}\right)_{x2}$$

$$\left(\Sigma\,\begin{matrix}\text{syst. linear}\\ \text{momentum}\end{matrix}\right)_{y1} + \left(\Sigma\,\begin{matrix}\text{syst. linear}\\ \text{impulse}\end{matrix}\right)_{y(1-2)} = \left(\Sigma\,\begin{matrix}\text{syst. linear}\\ \text{momentum}\end{matrix}\right)_{y2}$$

$$\left(\Sigma\,\begin{matrix}\text{syst. angular}\\ \text{momentum}\end{matrix}\right)_{O1} + \left(\Sigma\,\begin{matrix}\text{syst. angular}\\ \text{impulse}\end{matrix}\right)_{O(1-2)} = \left(\Sigma\,\begin{matrix}\text{syst. angular}\\ \text{momentum}\end{matrix}\right)_{O2}$$

Principle of Impulse and Momentum

Universiti Malaysia PAHANG
Engineering • Technology • Creativity

- When to use the principle of impulse and momentum?
  - To solve problems involving **force**, **velocity** and **time**.

- When to use the work and energy method?
  - To solve problems involving **force**, **velocity** and **displacement**.

- When to use the force and acceleration method?
  - To solve problems involving **force** and **acceleration**.

Conservation of Momentum

Conservation of Linear Momentum

$$\left(\sum \begin{matrix}\text{syst. linear} \\ \text{momentum}\end{matrix}\right)_1 = \left(\sum \begin{matrix}\text{syst. linear} \\ \text{momentum}\end{matrix}\right)_2$$

If there is no/negligible net external linear impulse

$$\sum m(\vec{v}_G)_1 = \sum m(\vec{v}_G)_2$$

Conservation of Angular Momentum

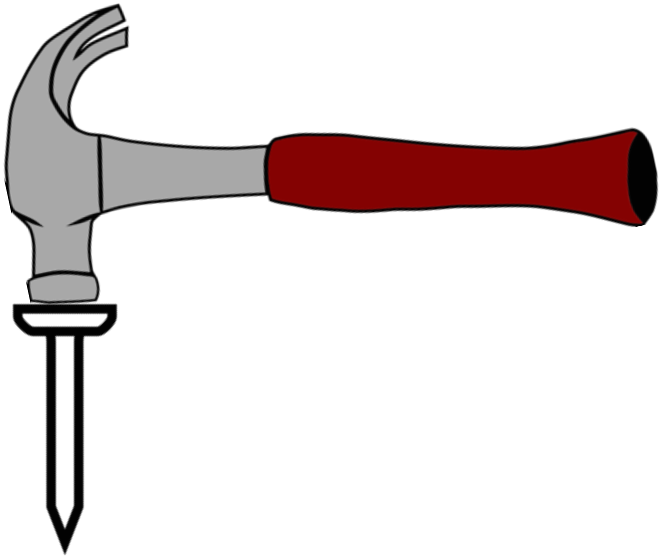
$$\left(\sum \begin{matrix}\text{syst. angular} \\ \text{momentum}\end{matrix}\right)_{O1} = \left(\sum \begin{matrix}\text{syst. angular} \\ \text{momentum}\end{matrix}\right)_{O2}$$

If there is no/negligible net external angular impulse

$$\sum (H_P)_1 = \sum (H_P)_2$$

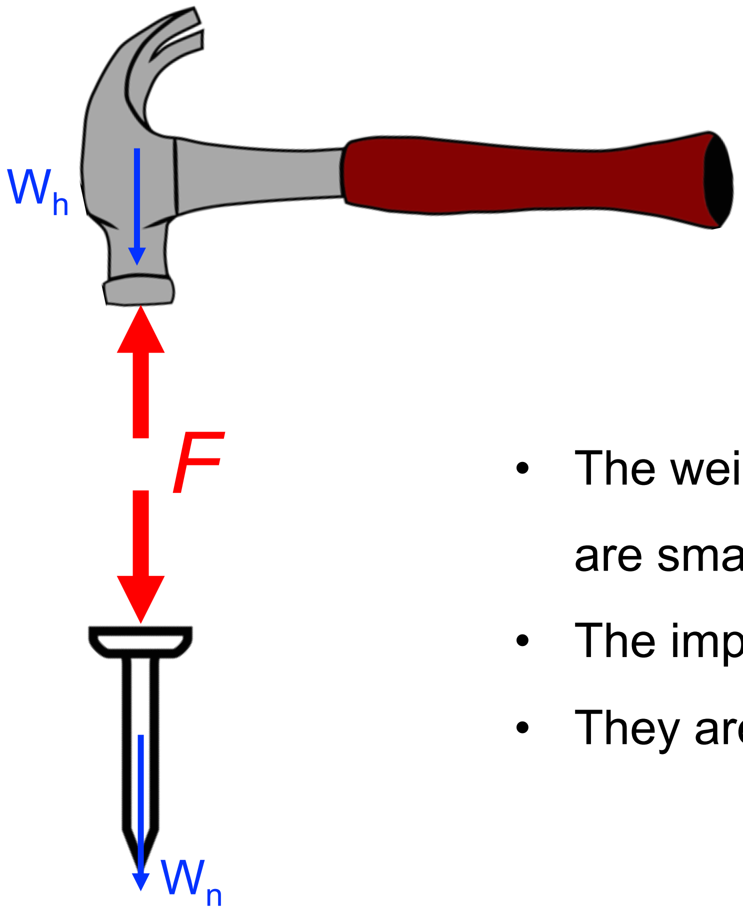
Conservation of Momentum

It may be possible to apply conservation of linear momentum when the linear impulses are small or ***nonimpulsive*** (**small forces** acting over very **short periods of time** e.g., weight of a body).



Conservation of Momentum

It may be possible to apply conservation of linear momentum when the linear impulses are small or ***nonimpulsive*** (**small forces** acting over very **short periods of time** e.g., weight of a body).

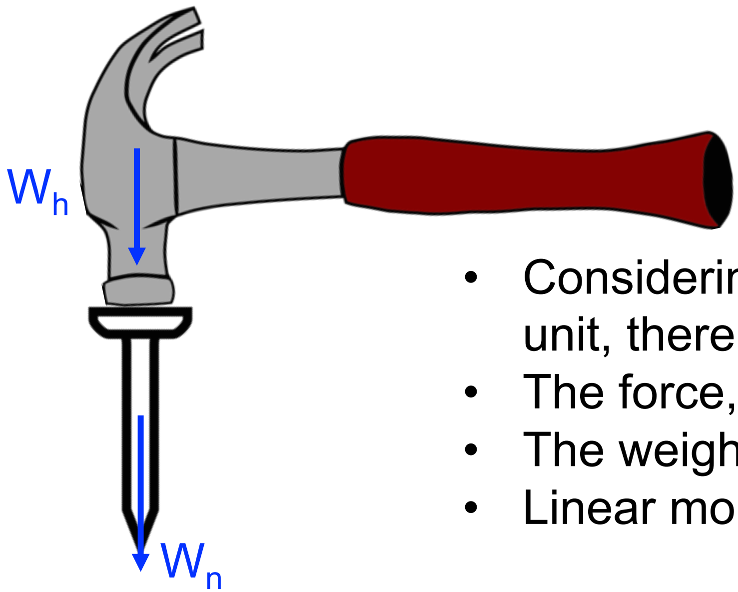


- The weight of the hammer and nail are small relative to the force, F.
- The impact duration is very short.
- They are considered non-impulsive.

Conservation of Momentum

Universiti Malaysia PAHANG
Engineering • Technology • Creativity

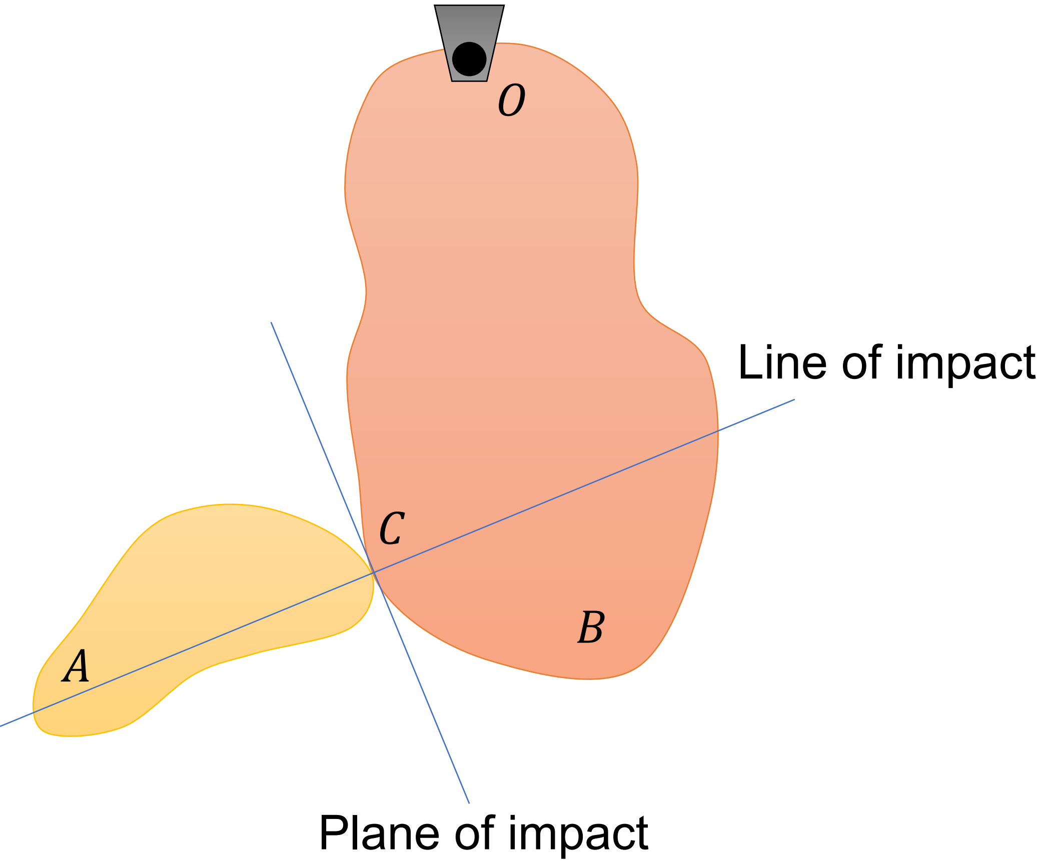
It may be possible to apply conservation of linear momentum when the linear impulses are small or ***nonimpulsive*** (**small forces** acting over very **short periods of time** e.g., weight of a body).



- Considering both rigid bodies as one unit, there is no external impulse.
- The force, F cancel out each other.
- The weights are non-impulsive.
- Linear momentum is conserved.

The angular momentum can be conserved while the linear momentum is not. Such cases occur whenever the external forces creating the linear impulse pass through either the center of mass of the body or a fixed axis of rotation.

Eccentric Impact



- Eccentric impact occurs when the line connecting the mass centers of the two bodies does not coincide with the line of impact.
- Often occurs when one or both of the bodies are constrained to rotate about a fixed axis.

Eccentric impact

- In general, a problem involving the impact of two bodies requires determining the *two unknowns* $(v_A)_2$ and $(v_B)_2$, assuming $(v_A)_1$ and $(v_B)_1$ are known.
- The *first equation* generally involves application of *the conservation of angular momentum to the two bodies*.
- The *second equation* can be obtained using the definition of the *coefficient of restitution, e*, which is a ratio of the restitution impulse to the deformation impulse.

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

Coefficient of restitution

Planar Kinetics of a Rigid Body (Impulse and Momentum)

"No great discovery was ever made without a bold guess."

– *Sir Isaac Newton*