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DYNAMICS

Planar Kinetics of a Rigid Body (Work and Energy)

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Chapter Description

- Aims
  - To determine the kinetic energy of a moving body.
  - To introduce several types of work done by a moving body.
  - To discuss on the Principle of Work and Energy.
  - To explain on the Conservation of Energy.
- Expected Outcomes
  - Students are able to calculate the kinetic energy and work done by external forces on a rigid body in motion.
  - Students are able to determine utilise the Principle of Work and Energy and the Conservation of Energy to solve kinetic problems.
- References
  - Engineering Mechanics: Dynamics 12th Edition, RC Hibbeler, Prentice Hall

Contents

Planar Kinetics of Rigid Body

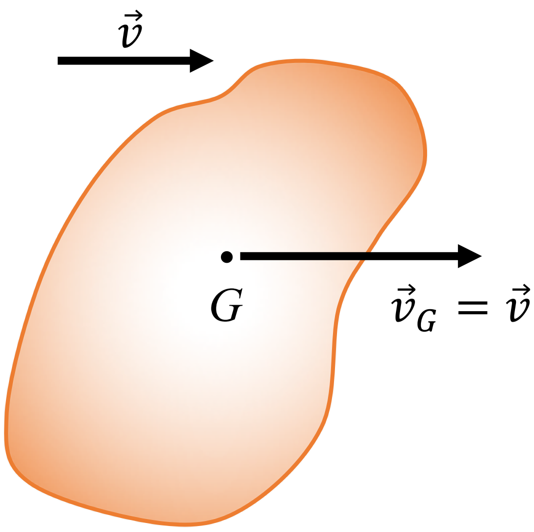
- Planar Kinetics
  - Force & Acceleration $F = ma$
  - Work & Energy
    - Kinetic Energy, *T*
    - Work, *U*
    - Principle of Work & Energy
    - Conservation of Energy
  - Impulse & Momentum

Kinetic Energy

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

Translational KE — Rotational KE

If only TRANSLATION occur



Translation

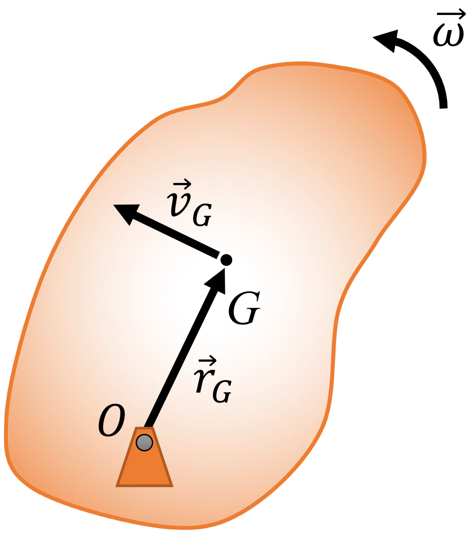
$$T = \frac{1}{2} m v_G^2$$

Kinetic Energy

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

Translational KE — Rotational KE

For ROTATION ABOUT A FIXED AXIS



Rotation about a fixed axis

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

about Point *O*

$$T = \frac{1}{2} I_O \omega^2$$

I_O can be calculated using the **parallel axis theorem**

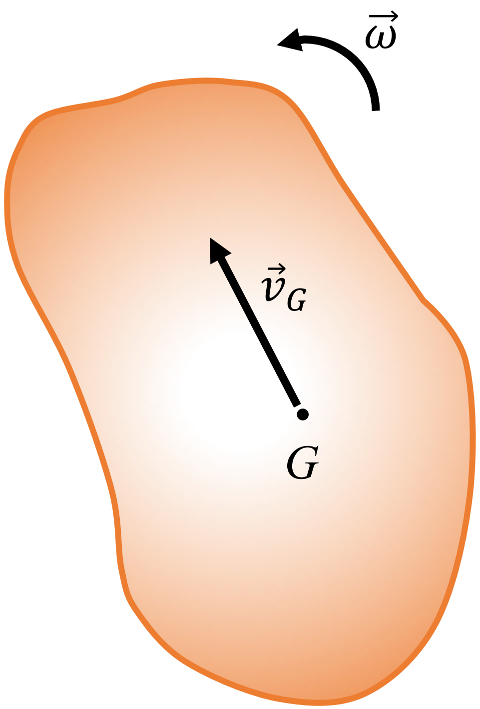
Kinetic Energy

KINETIC ENERGY IS ALWAYS POSITIVE !

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

Translational KE — Rotational KE

For GENERAL PLANE MOTION



General Plane Motion

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

Or, if the location of *IC* can be identified

$$T = \frac{1}{2} I_{IC} \omega^2$$

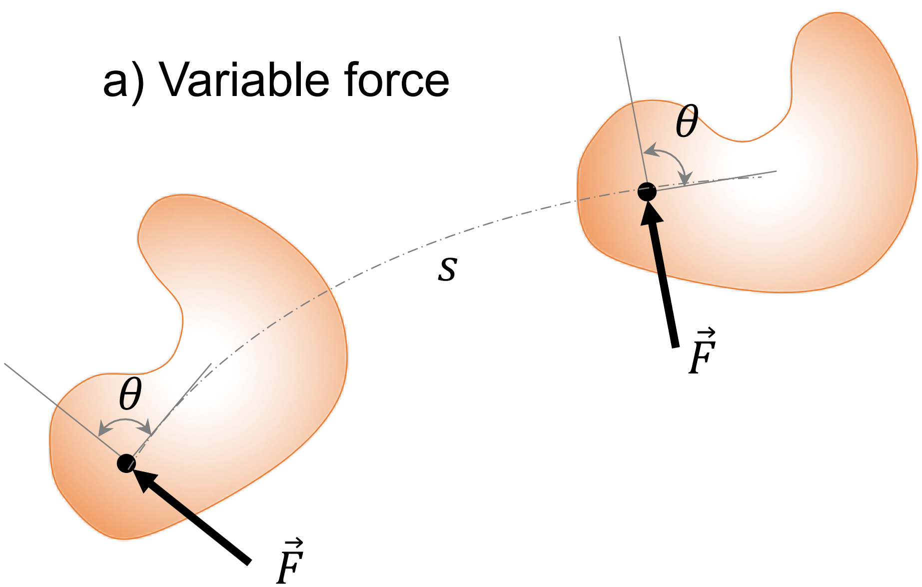
I_{IC} can be calculated using the **parallel axis theorem**

Work, *U* → *U* is the symbol for work

A body will do work when it undergoes displacement in the direction of the force

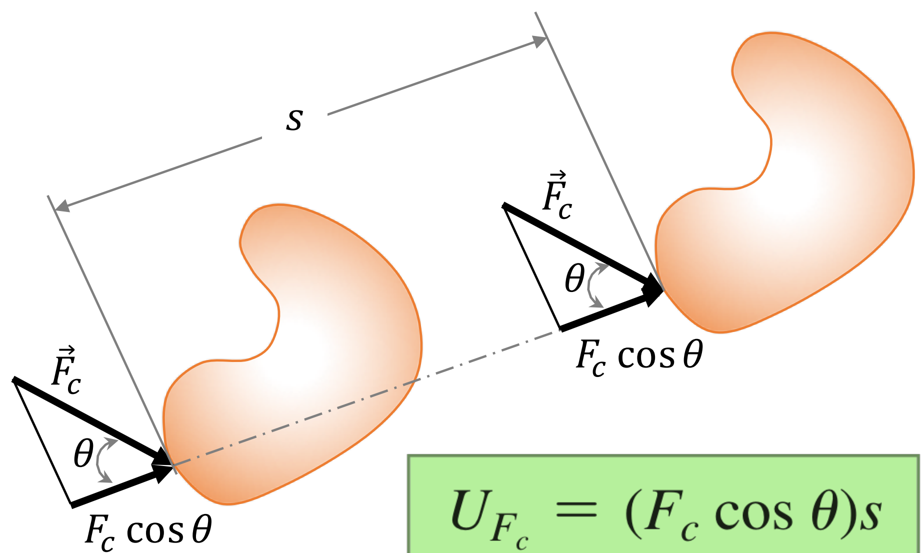
① The work of a **force**

a) Variable force



$$U_F = \int \mathbf{F} \cdot d\mathbf{r} = \int_s F \cos\theta \, ds$$

b) Constant force

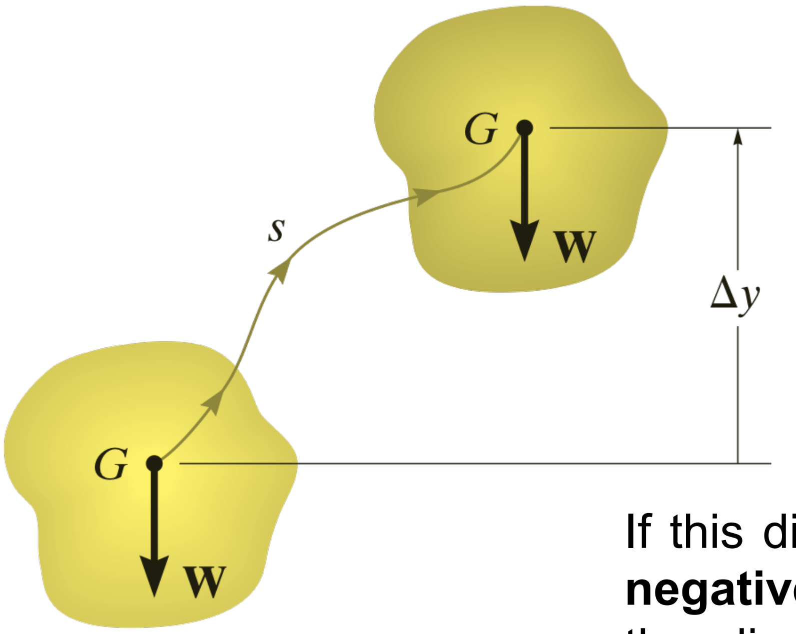


$$U_{F_c} = (F_c \cos\theta)s$$

Work, U → U is the symbol for work

A body will do work when it undergoes displacement in the direction of the force

② The work of **weight**



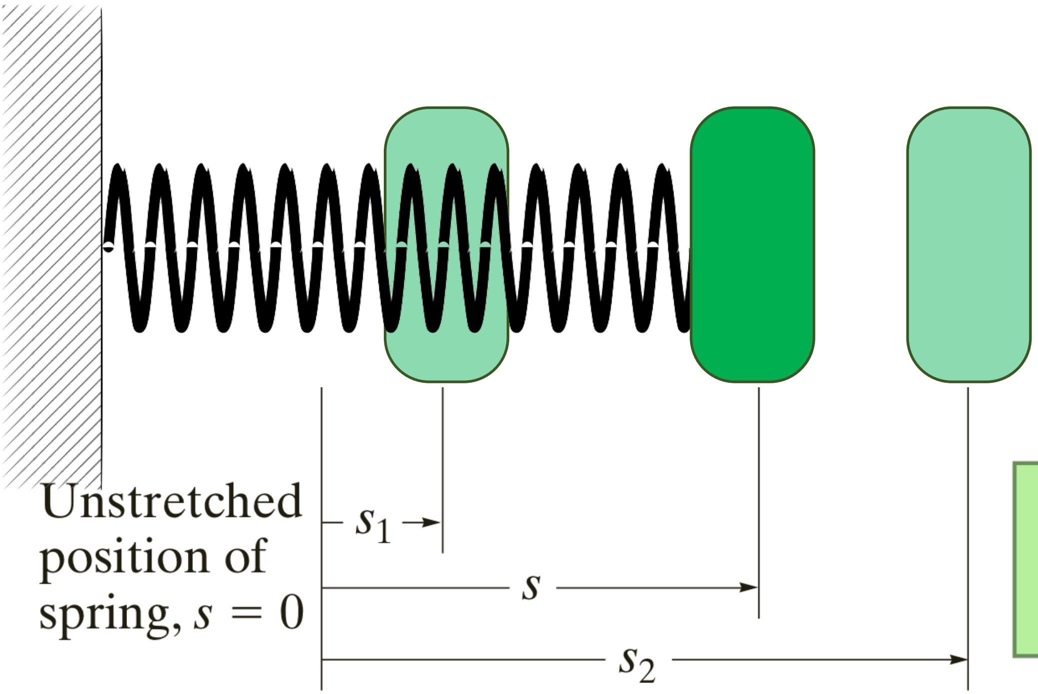
$$U_W = -W\,\Delta y$$

If this displacement is *upward*, the work is **negative**, since the weight is opposite to the displacement. If the displacement is *downward* (-*y*) the work becomes **positive**.

Work, *U* → *U* is the symbol for work

A body will do work when it undergoes displacement in the direction of the force

③ The work of a **spring**

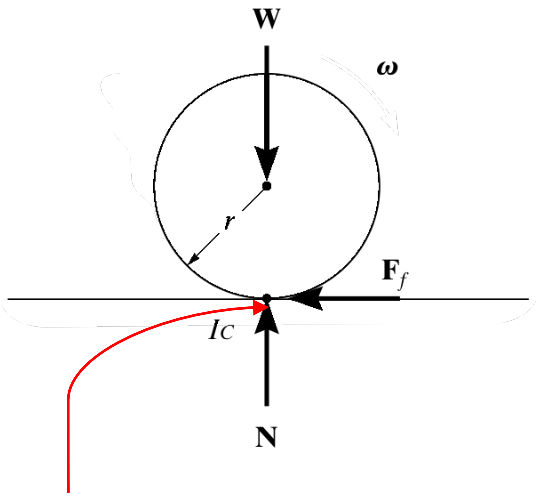


In both cases (*compressed* or *stretched*), the work of a spring is **NEGATIVE** since the displacement of the body is in the opposite direction of the force.

$$U_s = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)$$

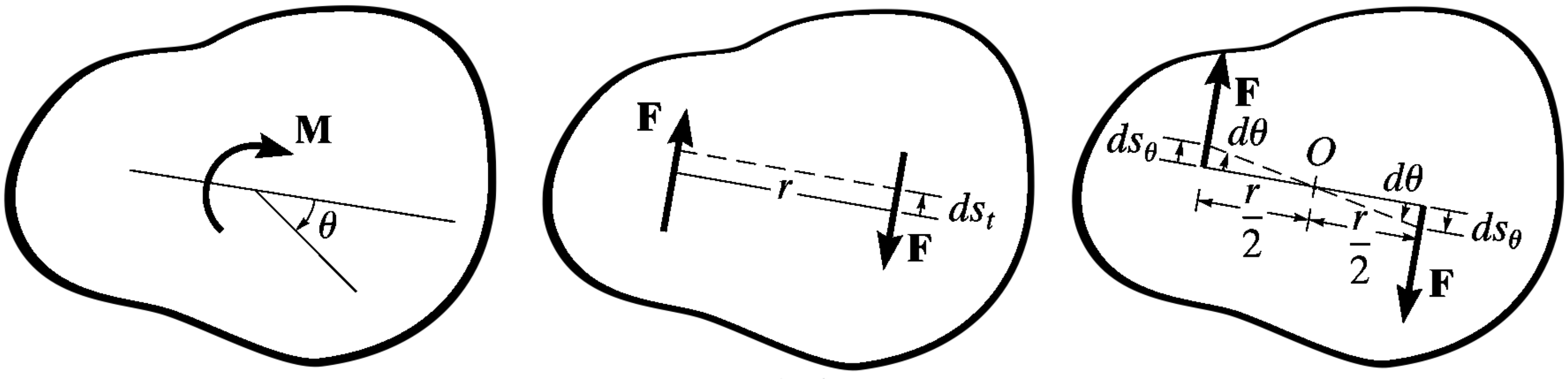
Forces that do NO WORK!

- Forces that act on a fixed point on a body
  - eg. Reaction at pin support about which the body rotates.
- Forces that have direction perpendicular to the displacement
  - Normal reactions acting on a body that moves along a fixed surface
  - Weight when the centre of gravity of the body moves in horizontal plane
  - Friction force acting on a body when it rolls without slipping



Since this point is IC, the work done is ZERO

The work of a Couple Moment



$$U_M = \int_{\theta_1}^{\theta_2} M\, d\theta$$

If the couple moment **M** has constant magnitude,

$$U_M = M(\theta_2 - \theta_1)$$

The angle θ is in radian

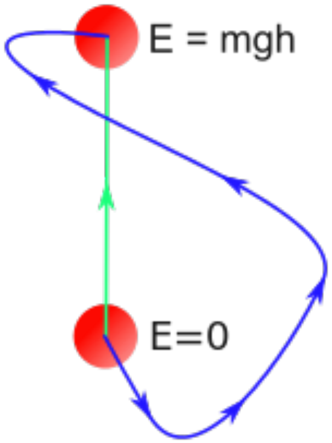
Principle of Work and Energy

$$T_1 + \sum U_{1-2} = T_2$$

This equation states that the body's **initial translational and rotational kinetic energy**, ***plus*** the **work done** by all the external forces and couple moments acting on the body as the body moves from its initial to its final position, is ***equal*** to the body's **final translational and rotational kinetic energy**.

Conservative Force

- A **conservative force** is a **force** with the property that the work done in moving a particle between two points is independent of the taken path.



- When a force system acting on a rigid body consists only of ***conservative forces***, the conservation of energy theorem can be used to solve a problem.

- Easier to apply – the work of a conservative force is ***independent of the path*** and depends only on the initial and final positions of the body.

Gravitational Potential Energy

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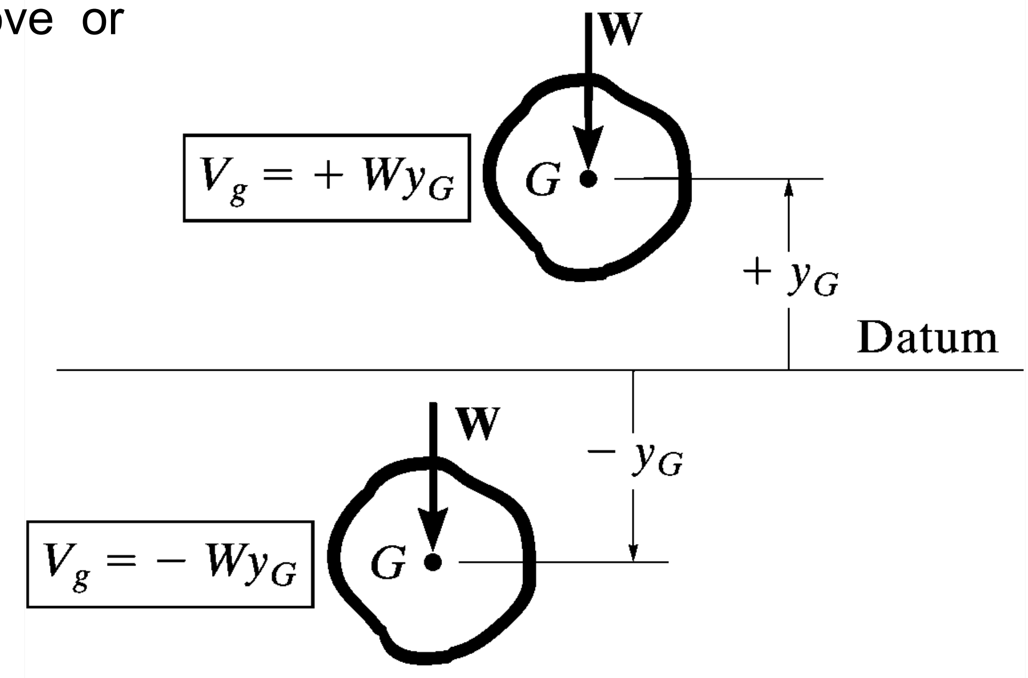
The gravitational potential energy of the body is determined by knowing the height of the body's centre of gravity above or below a horizontal datum.

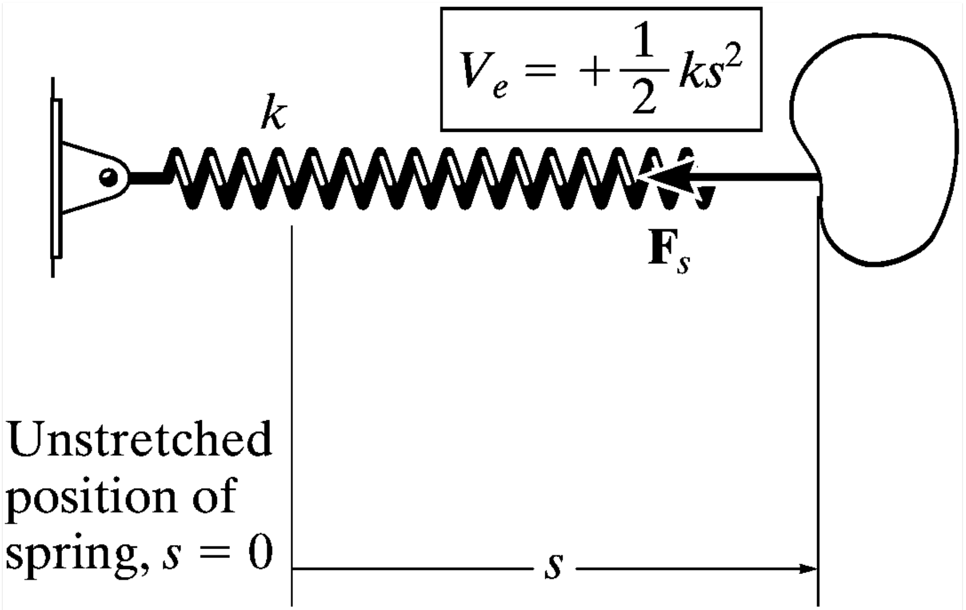
$$V_g = W y_G$$



Elastic Potential Energy

The force developed by an elastic spring is also a conservative force. The elastic potential energy which a spring imparts to an attached body when the spring is stretched or compressed from an initial undeformed position ($s = 0$) to a final position s, is

$$V_e = +\frac{1}{2}ks^2$$



Conservation of Energy

If a body is subjected to both gravitational and elastic forces, the **total potential energy** can be expressed as

$$V = V_g + V_e$$

$$T_1 + \Sigma U_{1-2} = T_2$$

→ Principle of Work and Energy

$$(\Sigma U_{1-2})_{\text{cons}} = V_1 - V_2$$

→ Total work done by the conservative forces

Rearranging the principle of work and energy,

$$T_1 + V_1 + (\Sigma U_{1-2})_{\text{noncons}} = T_2 + V_2$$

If the total work done by non-conservative forces is ZERO, then

$$T_1 + V_1 = T_2 + V_2$$

Conservation of Mechanical Energy

Planar Kinetics of a Rigid Body (Work and Energy)

"If I have ever made any valuable discoveries, it has been owing more to patient attention, than to any other talent."

– *Sir Isaac Newton*