DYNAMICS

Planar Kinetics of a Rigid Body (Rotation About a Fixed Axis)

by:

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Planar Kinetics – Rotation about a fixed axis

- Aims
  - To discuss the force and acceleration method of a rigid body undergoing rotation about a fixed axis.

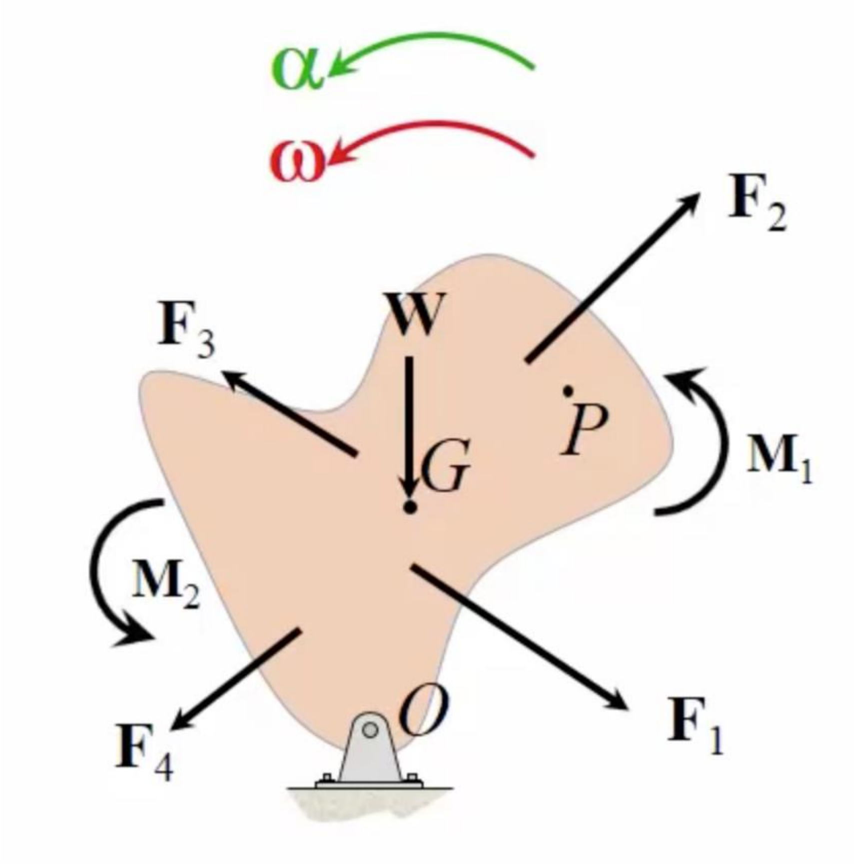
- Expected Outcomes
  - Students are able to determine the forces and moments, acceleration and angular acceleration of a rigid body undergoing rotation about a fixed axis.

- References
  - Engineering Mechanics: Dynamics 12th Edition, RC Hibbeler, Prentice Hall

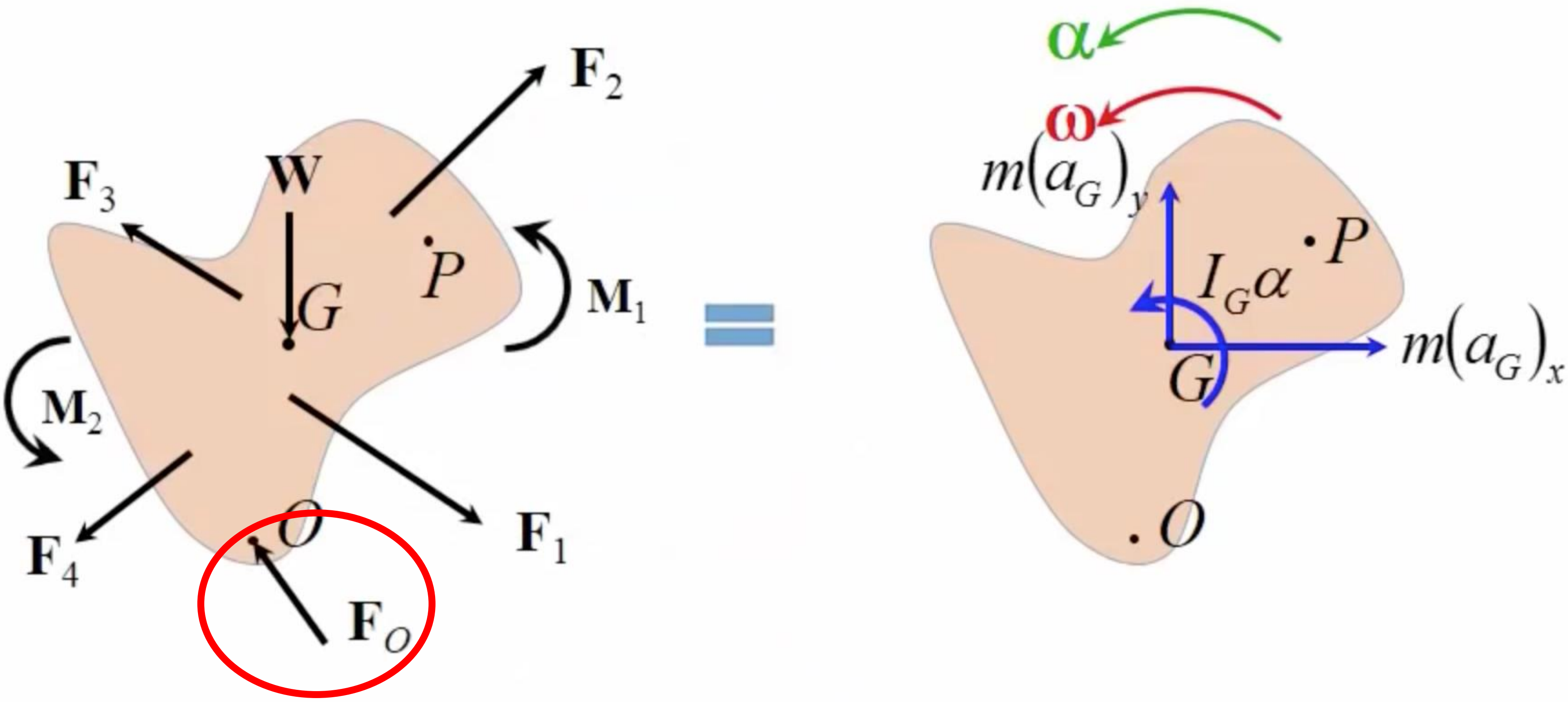
Contents

- Rotation about a fixed axis about mass centre *G*
- Rotation about a fixed axis about Point *O*
- Example calculation

Rotation about a fixed axis

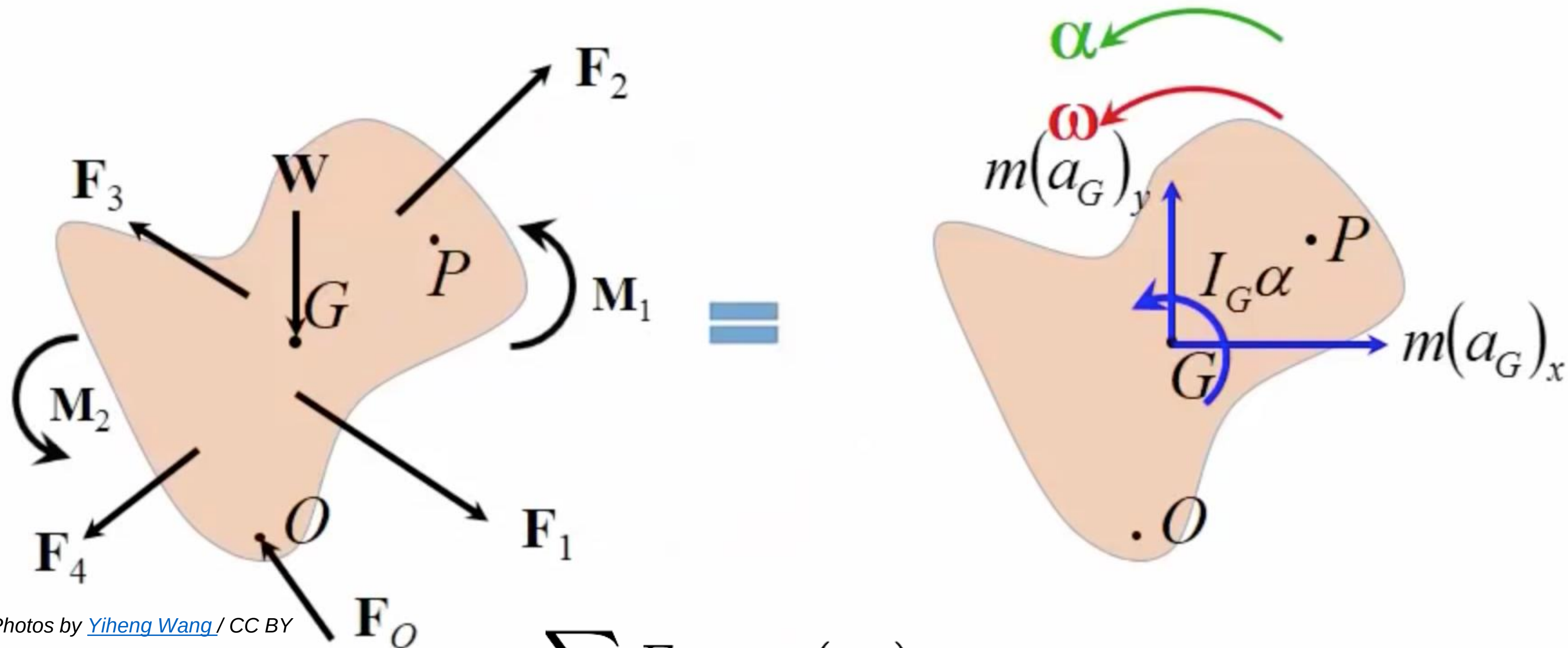


Rotation about a fixed axis



Support reaction force

Rotation about a fixed axis



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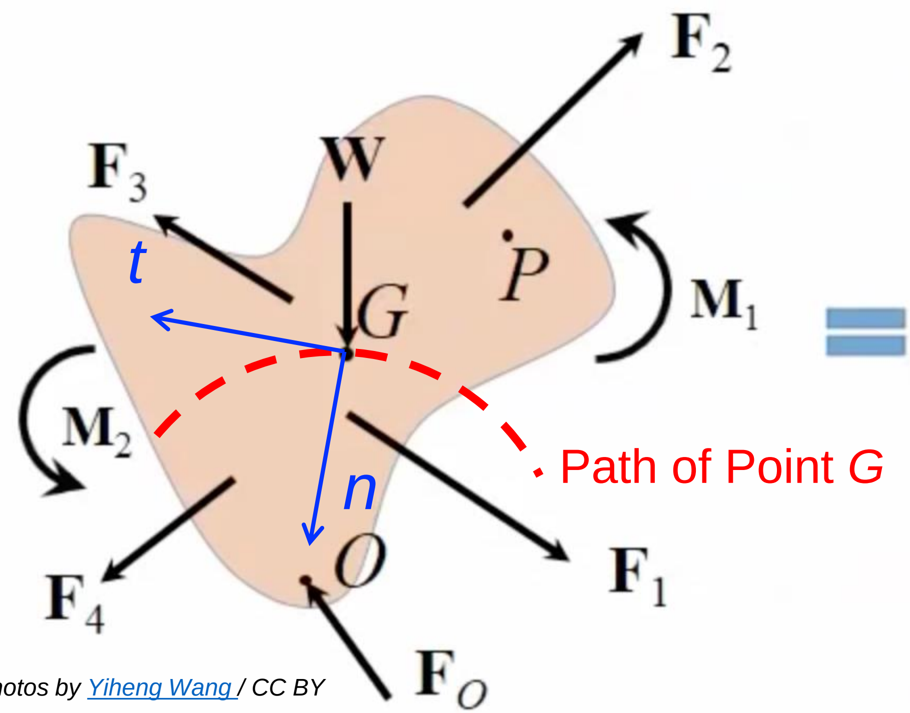
$$\sum F_x = m(a_G)_x$$

$$\sum F_y = m(a_G)_y$$

$$\sum M_P = \sum (\mathscr{M}_k)_P$$

Rotation about a fixed axis



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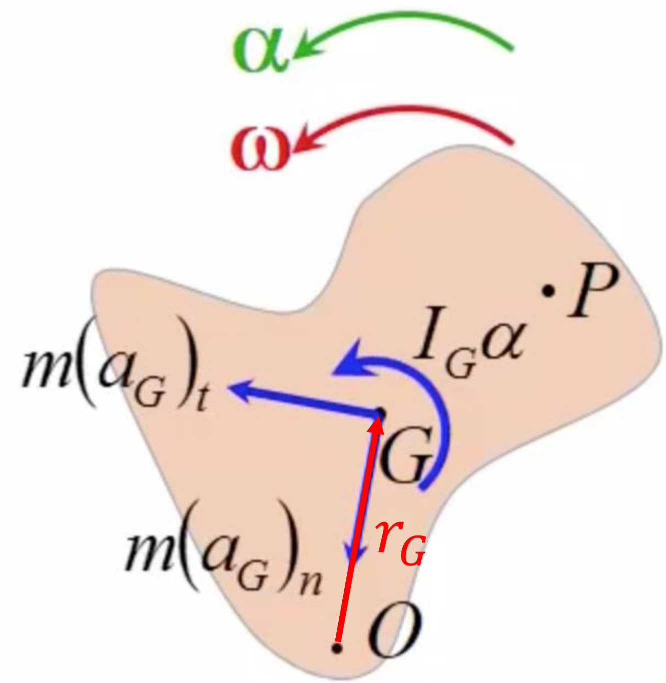


$$\sum F_n = m(a_G)_n \qquad a_n = \omega^2 r$$

$$\sum F_t = m(a_G)_t \qquad a_t = \alpha r$$

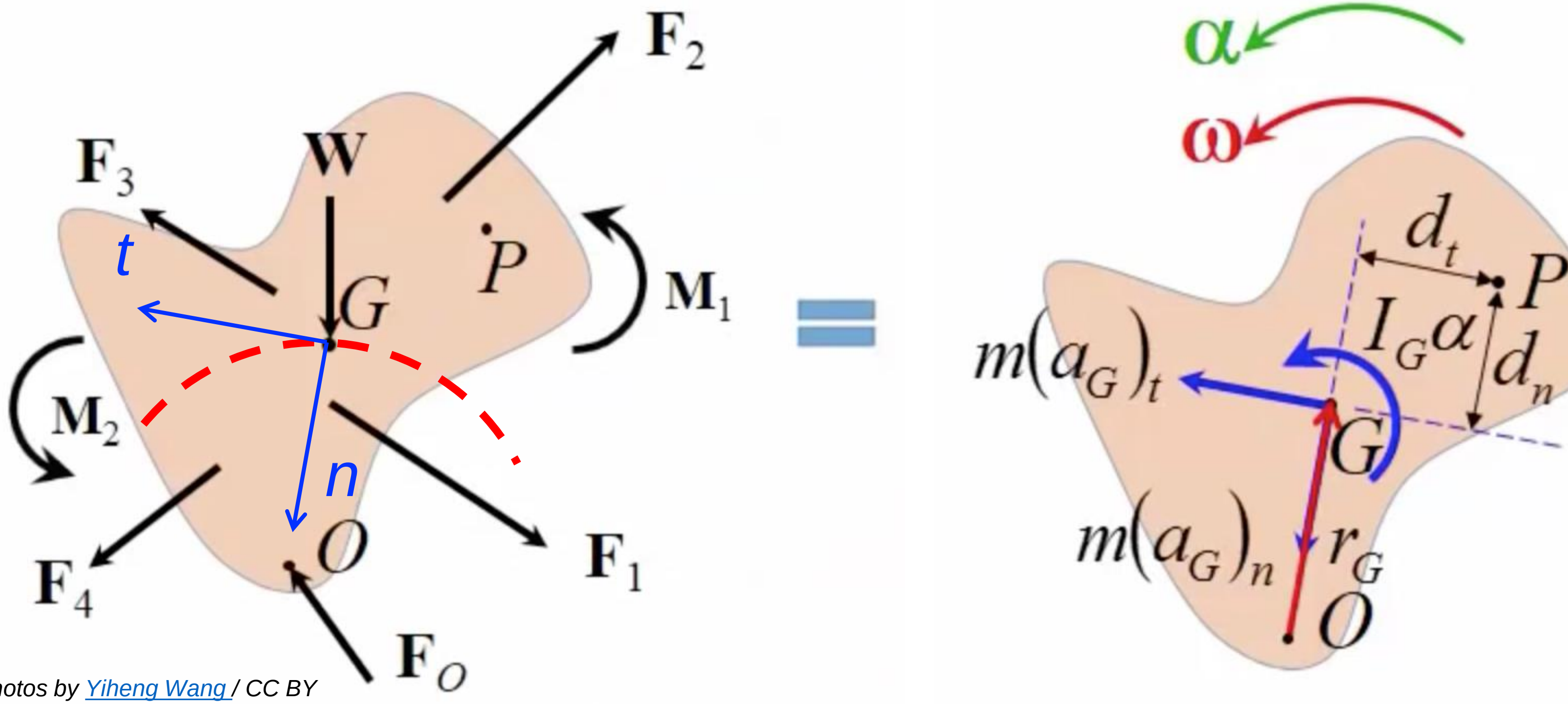
$$\sum M_P = \sum (\mathscr{M}_k)_P$$

$$\sum F_n = m\omega^2 r_G$$

$$\sum F_t = \alpha r_G$$

$$\sum M_P = \sum (\mathscr{M}_k)_P$$

Rotation about a fixed axis



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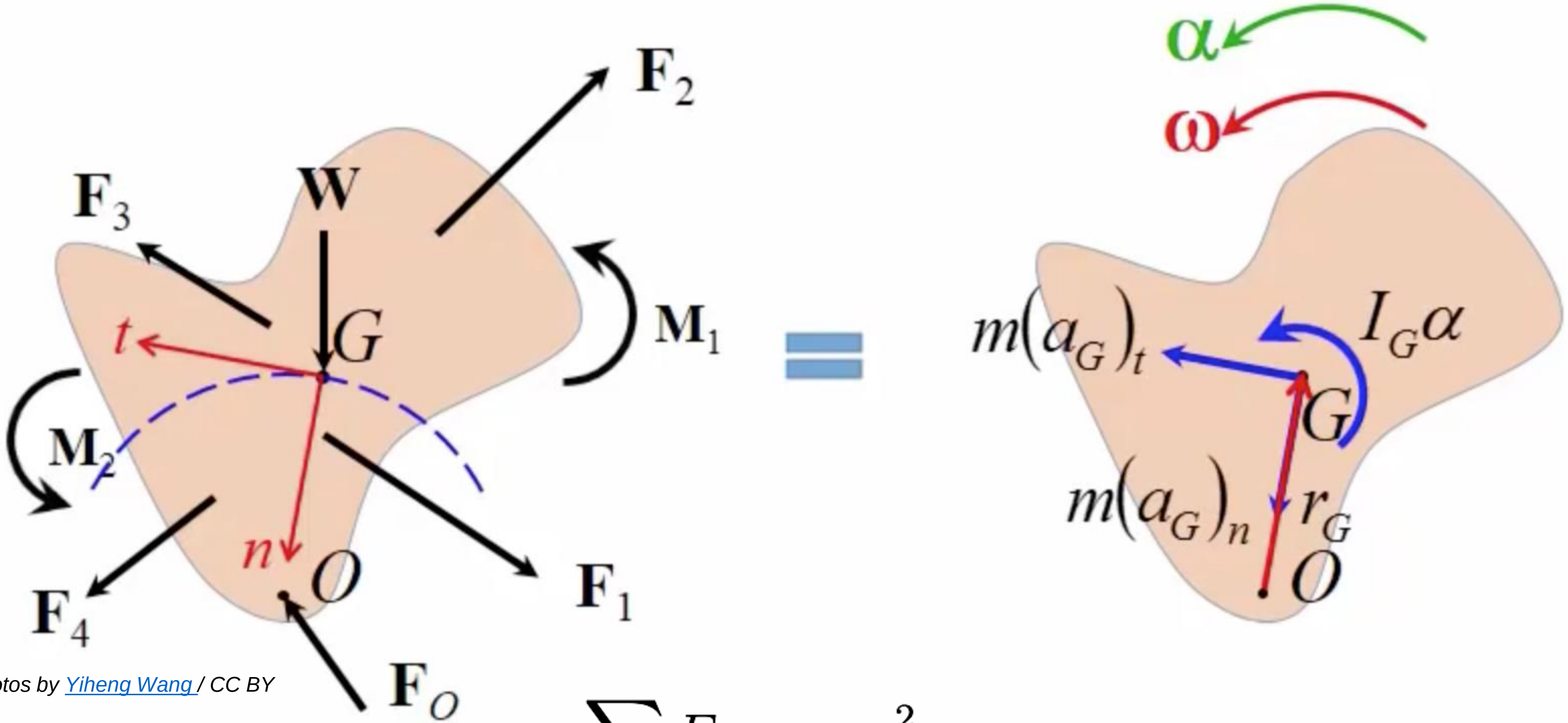
$$\sum F_n = m\omega^2 r_G$$

$$\sum F_t = \alpha r_G$$

$$\sum M_P = \sum (\mathscr{M}_k)_P$$

$$\sum (\mathscr{M}_k)_P = m(a_G)_n \cdot d_t - m(a_G)_t \cdot d_n + I_G\alpha$$

Rotation about a fixed axis



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$$\sum F_n = m\omega^2 r_G$$

$$\sum F_t = m\alpha r_G$$

$$\sum M_G = I_G\alpha$$

About Centre of Gravity *G*

Rotation about a fixed axis

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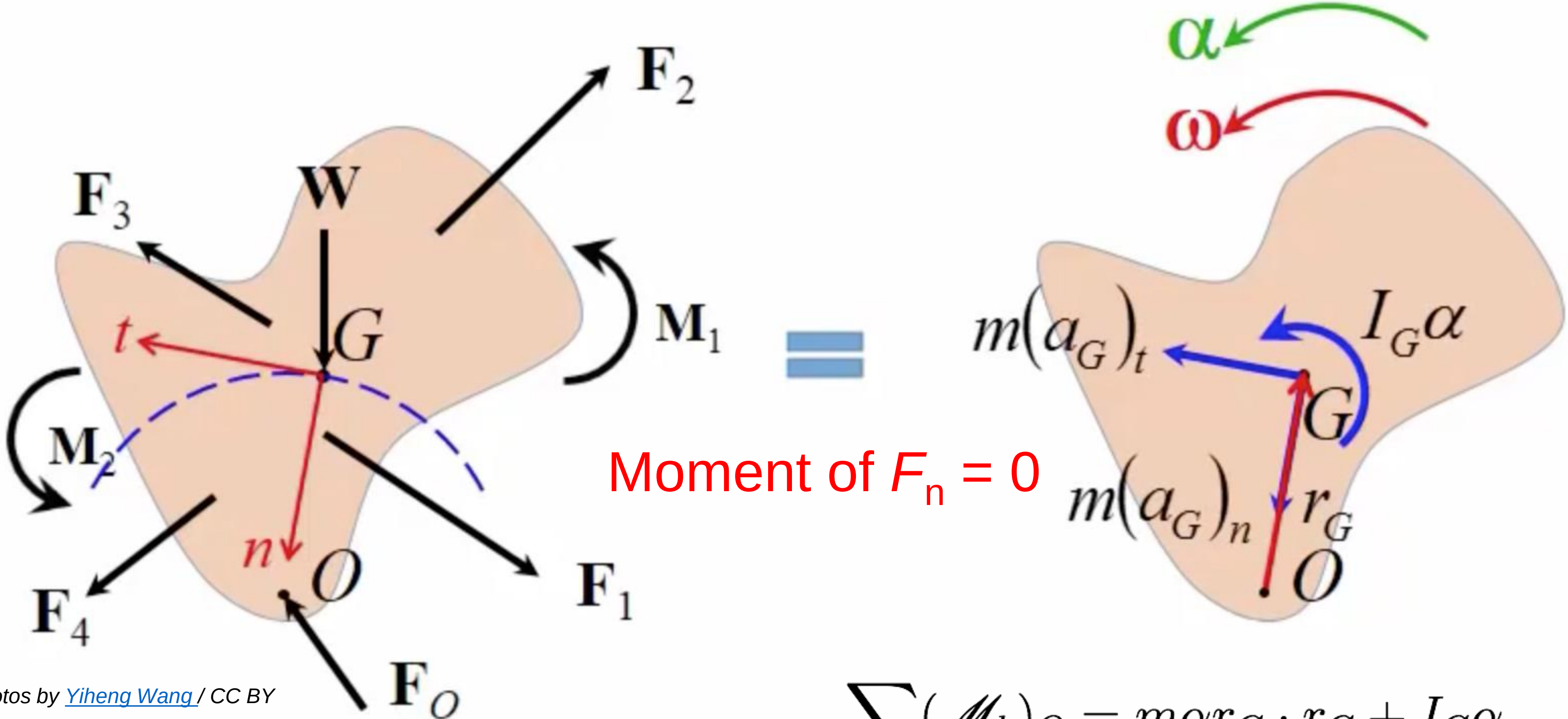


Moment of $F_n = 0$

$$\sum F_n = m\omega^2 r_G$$

$$\sum F_t = m\alpha r_G$$

$$\sum M_G = \sum(\mathscr{M}_k)_O$$

$$\sum(\mathscr{M}_k)_O = m\alpha r_G \cdot r_G + I_G\alpha$$

$$= (mr_G^2 + I_G)\alpha$$

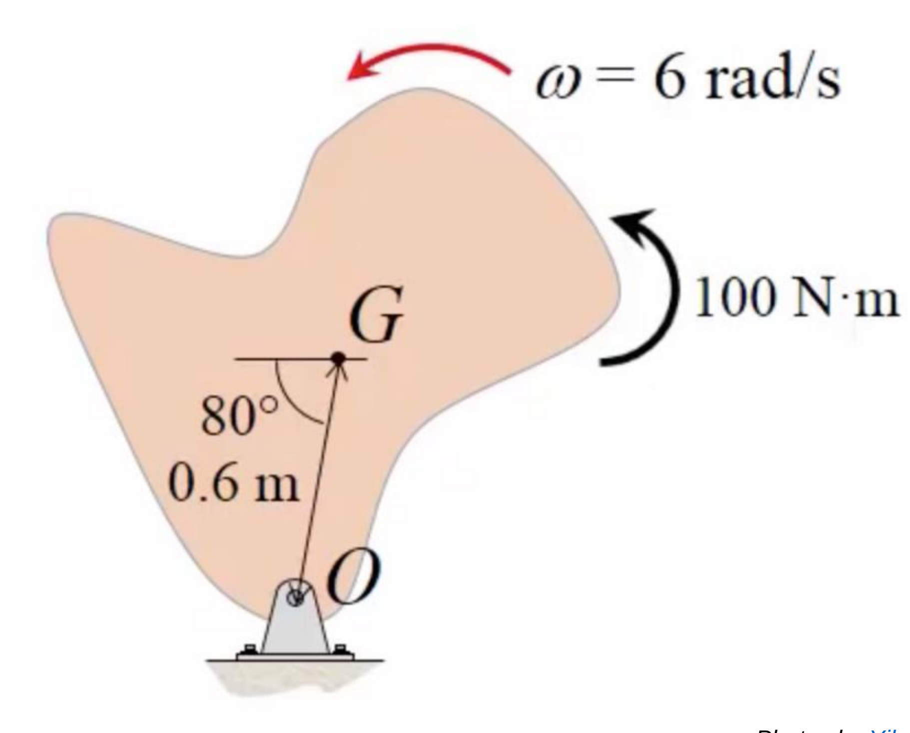
$$= I_O\alpha$$

$$I = I_G + md^2$$

About Point *O*

Example Calculation

A 50-kg object with radius of gyration, k_G = 0.4 m, about its centre of gravity *G* is **pinned at point *O*** and is subjected to the couple moment as shown. If at this instant it has angular velocity of 6 rad/s, **determine the support reaction at point *O* and its angular acceleration**.



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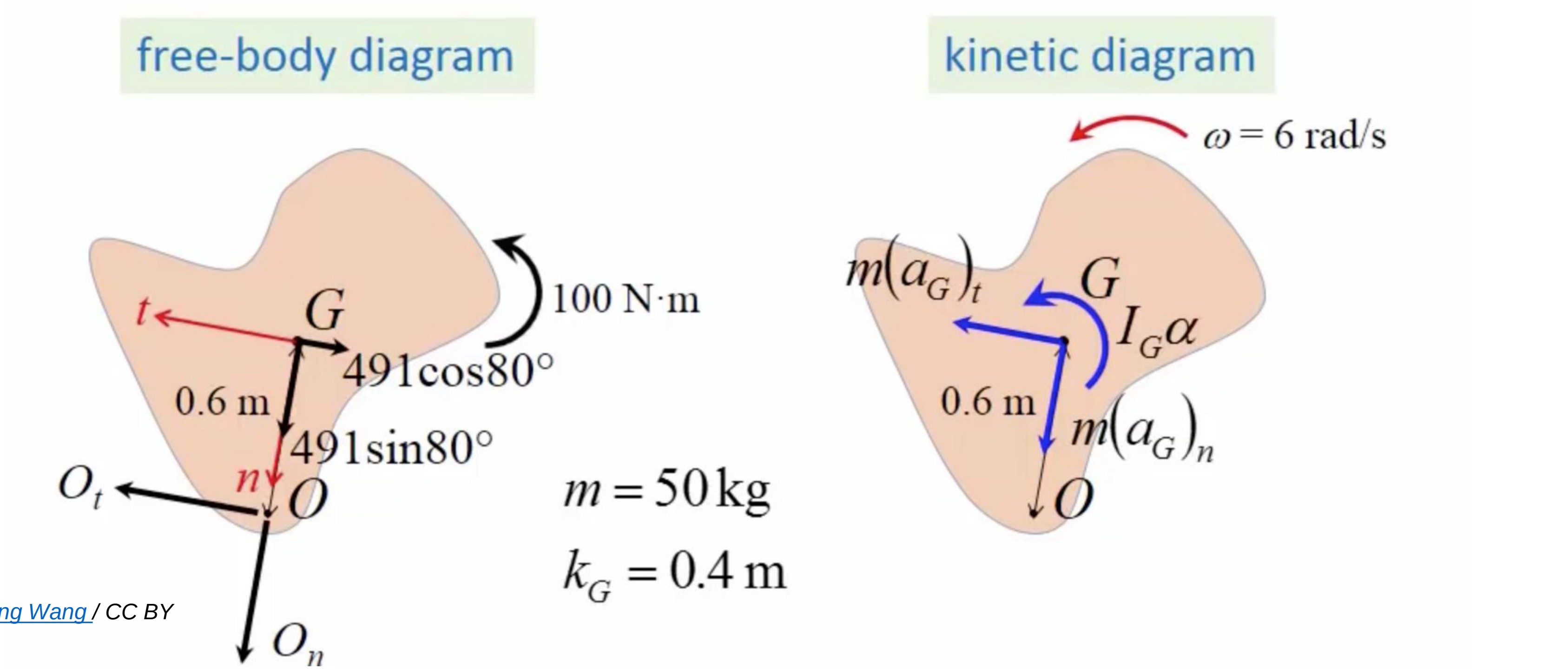
Example #1 (Moment about Point O)



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$$\sum F_n = O_n + 491\sin 80° = m\omega^2 r_G \qquad \therefore \quad O_n = 597\,(\text{N})$$

$$\sum F_t = O_t - 491\cos 80° = m\alpha r_G = 30\alpha \qquad \therefore \quad O_t = 142\,\text{N}$$

$$\sum M_O = 100 - 491\cos 80° \cdot 0.6 = I_O\alpha = \left(I_G + m r_G^2\right)\alpha$$

$$= \left(m k_G^2 + m r_G^2\right)\alpha = 50\left(0.4^2 + 0.6^2\right)\alpha = 26\alpha \qquad \therefore \quad \alpha = 1.88\,\text{rad/s}^2$$

Example #1 (Moment about Point *G*)

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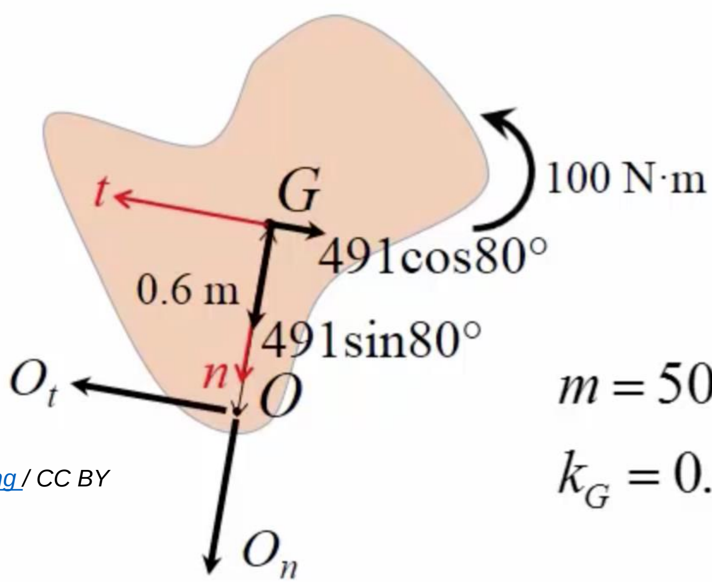
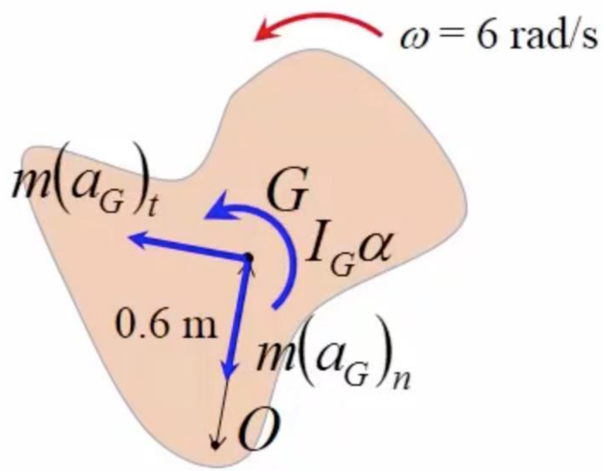
free-body diagram



kinetic diagram



$$m = 50\,\text{kg}$$

$$k_G = 0.4\,\text{m}$$

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Alternatively:

$$\sum M_G = -O_t \cdot 0.6 + 100 = I_G\alpha = 50 \cdot 0.4^2\alpha$$

$$\sum F_t = O_t - 491\cos 80^\circ = m\alpha r_G = 30\alpha$$

$$\therefore \begin{cases} \alpha = 1.88\,\text{rad/s}^2 \\ O_t = 142\,\text{N} \end{cases}$$

Planar Kinetics of a Rigid Body (Rotation About a Fixed Axis)

"Truth is ever to be found in simplicity, and not in the multiplicity and confusion of things."

– Sir Isaac Newton

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