DYNAMICS

Planar Kinetics of a Rigid Body (Mass Moment of Inertia)

by:

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Mass Moment of Inertia

- Aims
  - To introduce the meaning of inertia.
  - To determine the mass moment of inertia of a body.
  - To introduce the parallel axis theorem and radius of gyration.

- Expected Outcomes
  - Students understand about the inertia of a body.
  - Students are able to determine the mass moment of inertia of a body.

- References
  - Engineering Mechanics: Dynamics 12th Edition, RC Hibbeler, Prentice Hall

Contents

- What does inertia mean?
- Mass moment of inertia.
- Parallel axis theorem.
- Radius of gyration.
- Example calculations.

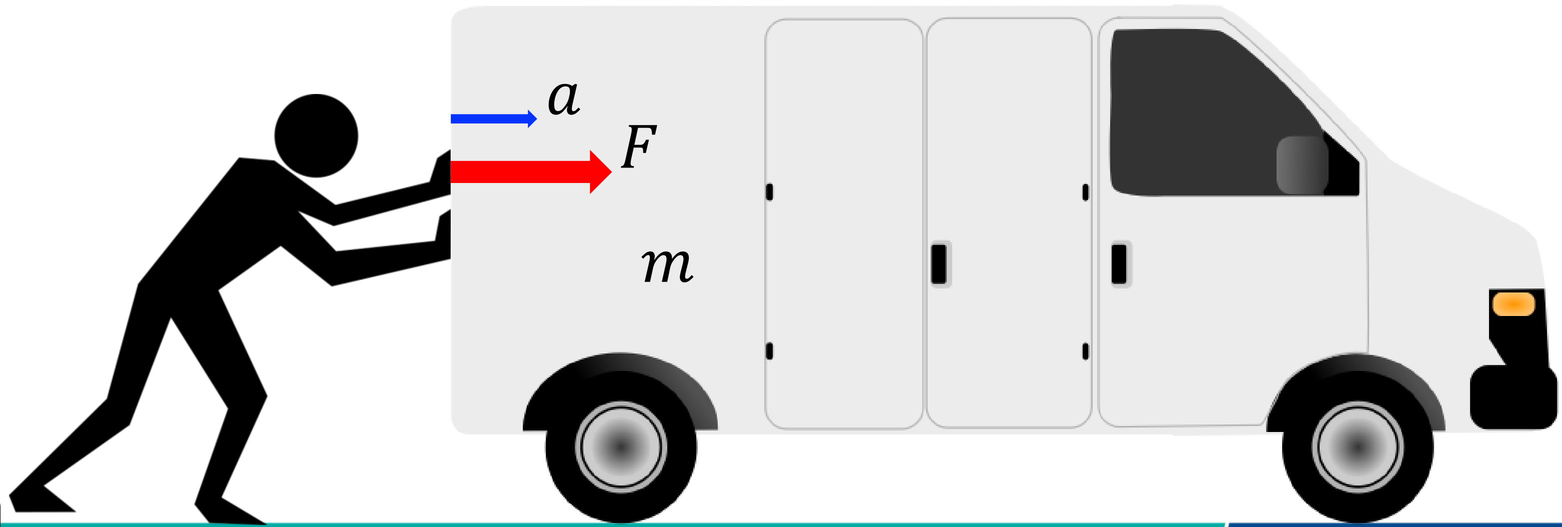
What does 'inertia' mean?

- **Inertia** – the **resistance** of an object to **change** its state of **motion**.
- The most familiar physical quantity of inertia is **mass**.
- From the Newton's 2nd Law of Motion, $F = ma$, rearrange:

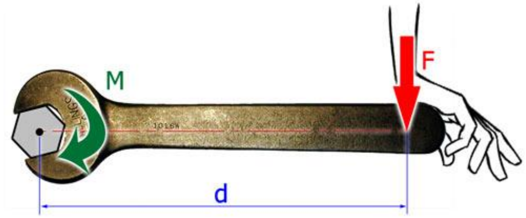
$$\vec{a} = \frac{\vec{F}}{m}$$

This shows that the **force**, F causes **translational motion**.

The **mass**, m, is the **resistance** to the **translational motion**.



What does ‘inertia’ mean?



- How about rotational motion? → **Moment of force**
- What causes rotational motion?
- What is the equation of motion for rotational motion?

$$\vec{M}_G = I_G \vec{\alpha} \quad \text{rearrange} \rightarrow \quad \vec{\alpha} = \frac{\vec{M}_G}{I_G}$$

Moment about the centre of gravity, M_G causes **rotational motion**.

- Compared to the translational motion:

$$\vec{F} = m\vec{a} \qquad \vec{a} = \frac{\vec{F}}{m}$$

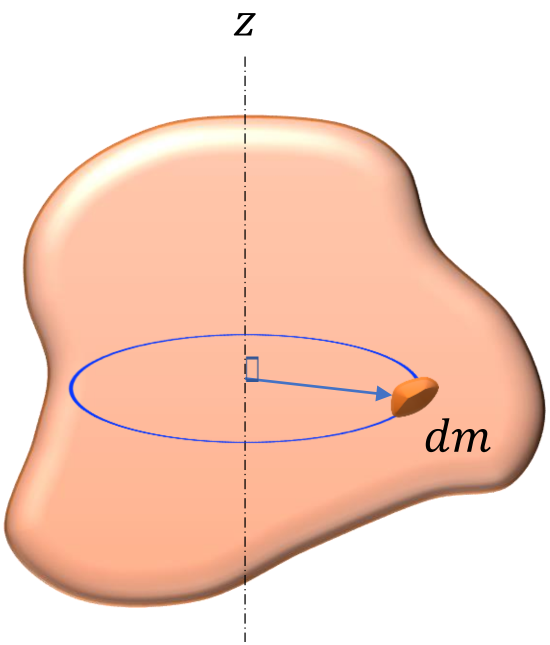
The **mass moment of inertia**, I_G, is the **resistance** to the **rotational motion**.

Mass Moment of Inertia

- Mass, m → absolute.
- Mass moment of inertia, I → relative to axis.

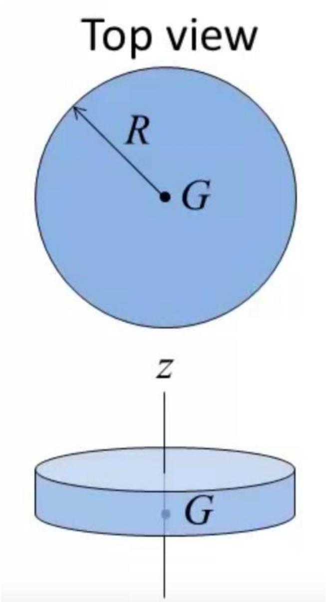
$$I_{zz} = \int_m r^2 \, dm$$

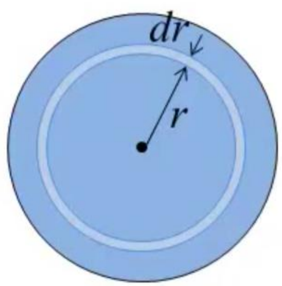
$$dm = \rho \, dV$$



Example Calculation #1

Determine the moment of inertia of the uniform thin disk shown about the *z* axis, which passes through its centre of gravity, *G*, and is perpendicular to the disk. The density of the material, ρ, is constant.





$$dA = 2\pi r\ dr$$

$$dV = \rho(2\pi r)(t)\ dr$$

$$I_{zz} = \int_m r^2\ dm$$

$$dm = \rho\ dV \rightarrow dV = \rho(2\pi r)(t)\ dr$$

$$I_{zz} = \rho 2\pi t \int_0^R r^3\ dr$$

$$= \rho 2\pi t \frac{R^4}{4} = \rho\pi t \frac{R^4}{2}$$

Since the volume of a cylinder

$$V = \pi R^2 t\ , \quad m = \rho V = \rho\pi R^2 t$$

$$I_{zz} = \frac{1}{2} m R^2$$

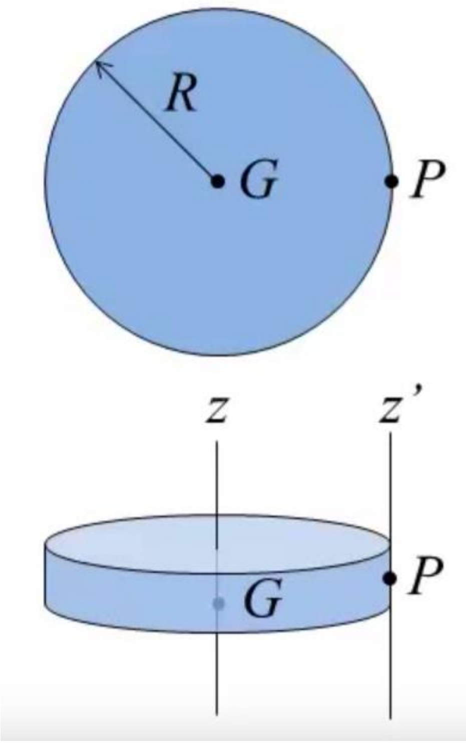
Photos by Yiheng Wang / CC BY

Parallel Axis Theorem

Moment of inertia about *z'*?

$$I = I_G + md^2$$

where *d* is the perpendicular distance between *z* and *z'* axis



$$I_{z'z'} = \frac{1}{2}mR^2 + mR^2$$

$$= \frac{3}{2}mR^2$$

Radius of Gyration

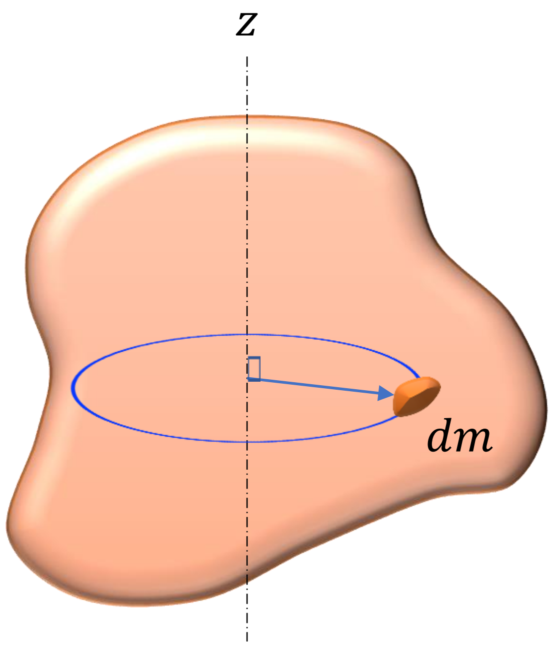
$$I_{zz} = \int_m r^2 \, dm$$

Once the mass moment of inertia is known, the **radius of gyration** can be determined using the following equation:

$$k_{zz} = \sqrt{\frac{I_{zz}}{m}} \qquad \text{[m]}$$

Normally the radius of gyration is given, the mass moment of inertia is then determined by:

$$I_{zz} = m k_{zz}^2$$



Conclusions

- Inertia is the resistance of an object to change its current state of motion.
- Mass is the inertia in a translational motion.
- Mass moment of inertia is the inertia in a rotational motion.
- To determine the mass moment of inertia about an axis not passing through the centre of gravity, the parallel axis theorem is used.
- For a non-uniform shaped object, usually the radius of gyration is given.

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"Genius is patience."

– *Sir Isaac Newton*