BFF1303: ELECTRICAL / ELECTRONICS ENGINEERING

Alternating Current Circuits : Power Analysis

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Alternating Current Circuit (AC)-AC Power Analysis

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Contents:

- Outcome
- Instantaneous and Average Power
- Apparent Power
- Power Factor
- Complex Power
- Power Factor Correction

Outcomes

Convert time-domain sinusoidal voltages and currents to phasor notation, and vice versa; represent circuit using impedance

Solve steady-state ac circuits, using phasors and complex impedances.

Learn complex power notation; compute apparent power, real, and reactive power for complex load.

Draw the power triangle, and compute the capacitor size required to perform power factor correction on a load.

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Instantaneous Power

- The instantaneous power $p(t)$ absorbed by an element is the product of instantaneous voltage $v(t)$ across the element and the instantaneous current $i(t)$ through it.

$$p(t) = v(t)i(t)$$

- The instantaneous power (in watt) is the power at any instant of time.

- It is rate at which element absorbs energy.

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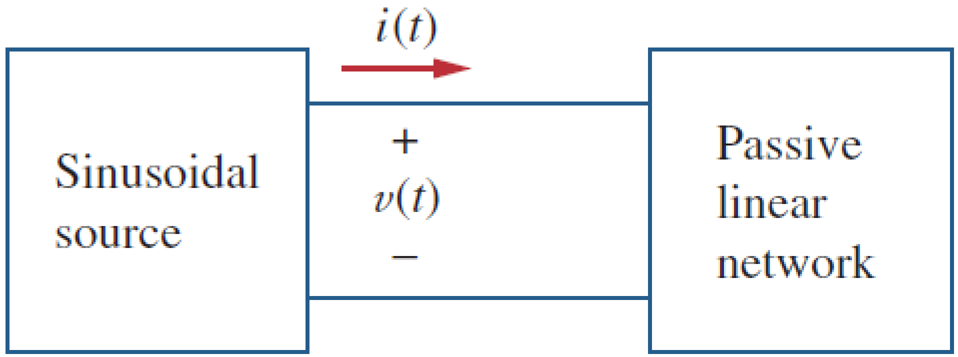
Instantaneous and Average Power



Let

$$v(t) = V_m \cos(\omega t + \theta_v)$$

V_m: Peak value; θ_v: Phase angle of voltage

$$i(t) = I_m \cos(\omega t + \theta_i)$$

I_m: Peak value; θ_i: Phase angle of current

$$p(t) = v(t)i(t) = V_m I_m \cos\left[(\omega t + \theta_v)(\omega t + \theta_i)\right]$$

Apply trigonometric identity

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

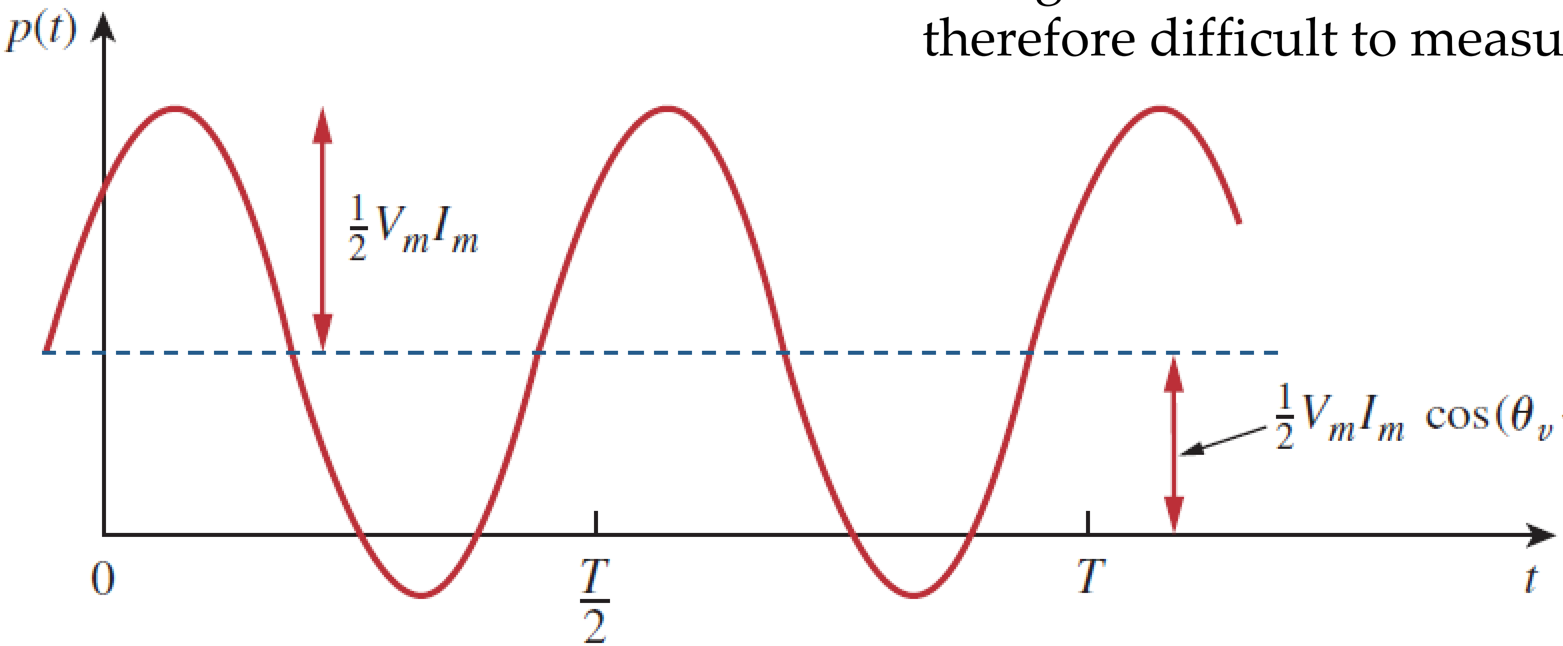
Instantaneous and Average Power

Sinusoidal function

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Time independent

- The instantaneous power changes with time and is therefore difficult to measure



Average Power

- The average power (in watts) is the average of the instantaneous power over 1 period.

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

$$P = \frac{1}{2}V_m I_m \cos\theta \quad \Rightarrow \quad \theta = \theta_v - \theta_i$$

$$P = V_{rms} I_{rms} \cos\theta$$

- A resistive load (R) absorbs power at all times, while reactive load (L or C) absorbs zero average power

Example #1

Given that

$$v(t) = 120\cos(377t + 45^\circ)\,\text{V} \qquad i(t) = 10\cos(377t - 10^\circ)\,\text{A}$$

Find the instantaneous power and the average power absorbed.

Solution

Instantaneous power

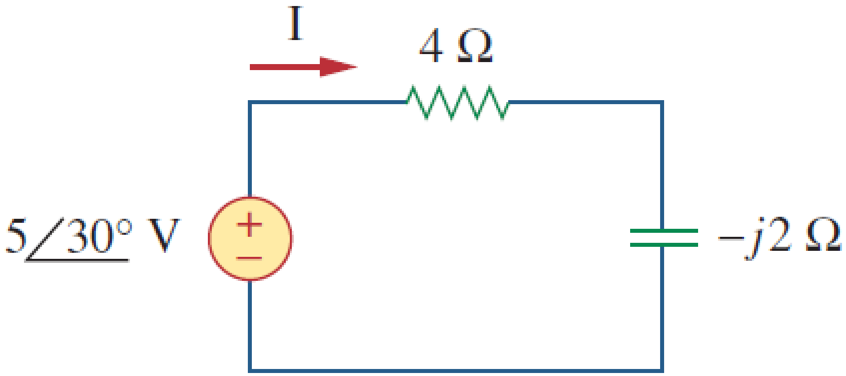
$$p(t) = v(t)i(t)$$

$$p(t) = \left[120\cos(377t + 45^\circ)\right]\left[10\cos(377t - 10^\circ)\right]$$

$$p(t) = 344.2 + 600\cos(754t + 35^\circ)\,\text{W}$$

Example #2

For the circuit shown, find the average power supplied by the source and the average power absorbed by the resistor.



Average power supplied by source

Solution

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{5\angle 30^\circ}{4 - j2}$$

$$\mathbf{I} = 1.118\angle 56.57^\circ \text{ A}$$

$$P = \frac{1}{2} V_m I_m \cos\theta$$

$$P = \frac{1}{2}(5)(1.118)\cos(30^\circ - 56.57^\circ)$$

$$P = 2.5\,\text{W}$$

Example #2

Average power supplied by resistor

$$P = \frac{1}{2} V_{m,R} I_m \cos\theta$$

$$V_{m,R} = 4\mathbf{I}$$

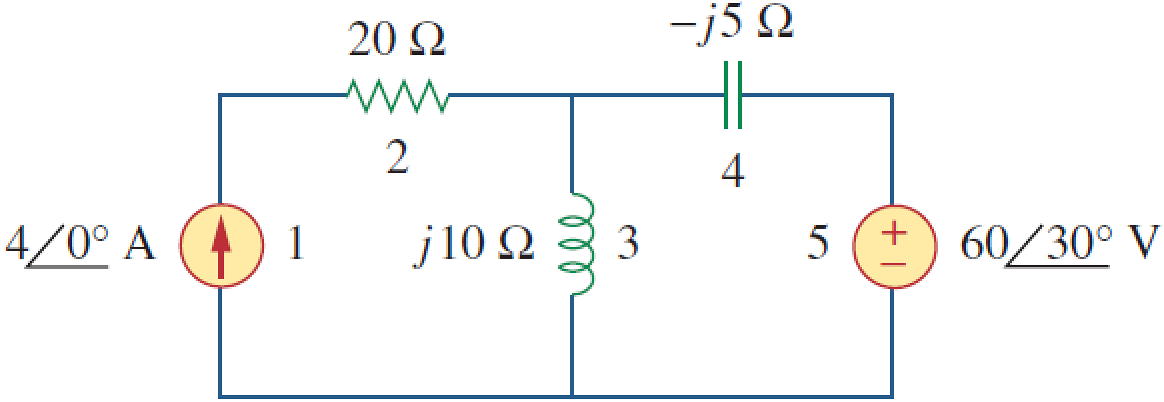
$$V_{m,R} = 4.472\angle 56.57^\circ \text{ V}$$

$$P = \frac{1}{2}(4.472)(1.118)\cos(56.57^\circ - 56.57^\circ)$$

$$P = 2.5 \text{ W}$$

Example #3

Determine the average power generated by each source and the average power absorbed by each passive element in the circuit shown



Answer

$$P_1 = -367.8\,\text{W} \qquad P_2 = 160\,\text{W}$$

$$P_3 = 0\,\text{W}$$

$$P_4 = 0\,\text{W}$$

$$P_5 = 207.8\,\text{W}$$

Apparent Power

- The apparent power is the product of the rms values of voltage and current.

$$S = V_{rms} I_{rms} \qquad (\text{VA})$$

- The unit is volt-amperes or VA to distinguish it from the average power, which is measured in watts.

- We know $P = V_{rms} I_{rms} \cos\theta$

- Then $P = S\cos\theta$

Power Factor

$$pf = \cos\theta = \cos(\theta_v - \theta_i)$$

$$pf = \frac{P}{S}$$

- The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance.

- The value of pf ranges between 0 – 1.

- For a purely resistive load, the voltage and current are in phase, so that $\theta_v - \theta_i = 0$ and $pf = 1$

- For a purely reactive load $\theta_v - \theta_i = \pm 90°$ and $pf = 0$

Power Factor

- For $0 < pf < 1$, the pf is said to be **leading** or **lagging**.
- **Leading** pf means that current leads voltage, which implies a **capacitive load**.
- **Lagging** pf means that current lags voltage, implying an **inductive load**.
- Leading and lagging pf can be determined by using a **phasor diagram**.
- Power factor affects the electric bills consumers pay the electric utility companies.

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- Steps to determined leads or lags using phasor diagram:
  - Make sure the amplitude of the expression is +ve value. Use the following to change from –ve to +ve.

$$-\sin A \quad \Rightarrow \quad \sin(A \pm 180°)$$

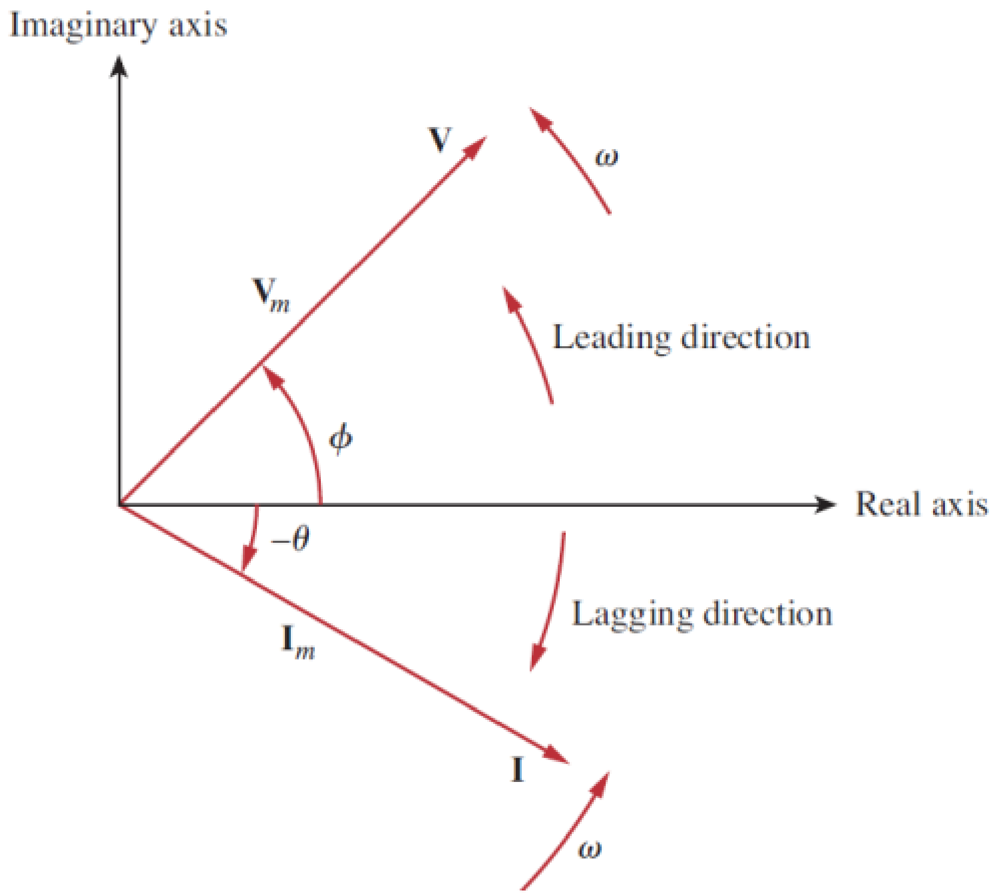
$$-\cos A \quad \Rightarrow \quad \cos(A \pm 180°)$$

  - Make sure the function is in sine function. Change the cos function into sine function by using

$$\cos A \quad \Rightarrow \quad \sin(A + 90°)$$

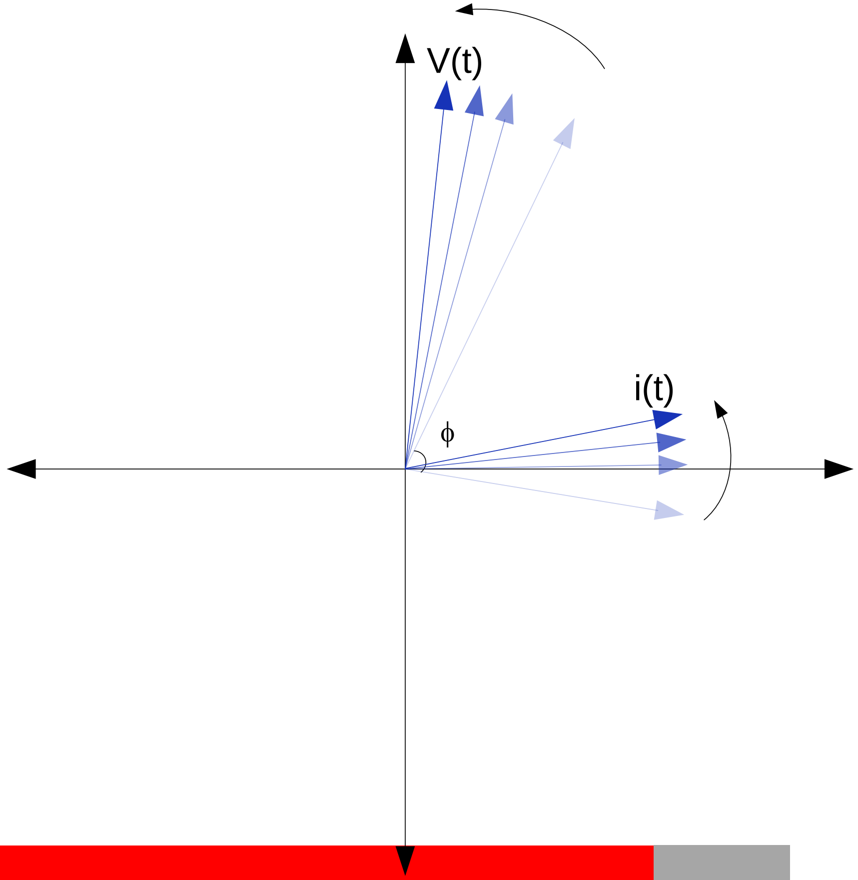
Power Factor

- Draw the phasor diagram.



Power Factor

- From the phasor diagram, rotate (anti-clockwise) both of the drawn expression to determined the leading or lagging pf. From the rotation if the current follows the voltage then it is said that the current lags voltage, thus the pf is lagging.



Example #4

Given that current and voltage for series-connected load. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

$$i(t) = 4\cos(100\pi t + 10°)\,\text{A} \qquad v(t) = 120\cos(100\pi t - 20°)\,\text{V}$$

Solution

Apparent power $S = V_{rms} I_{rms}$

$$S = \left(\frac{120}{\sqrt{2}}\right)\left(\frac{4}{\sqrt{2}}\right) = 240\,\text{VA}$$

Power factor $pf = \cos\theta$

$$pf = \cos(\theta_v - \theta_i) = 0.866$$

The pf is leading because the current leads the voltage.

Example #4

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle -20^\circ}{4\angle 10^\circ}$$

$$\mathbf{Z} = 25.98 - j15\ \Omega$$

The load impedance **Z** can be modeled by a 25.98 Ω resistor in series with a capacitor with

$$X_C = -\frac{1}{\omega C}$$

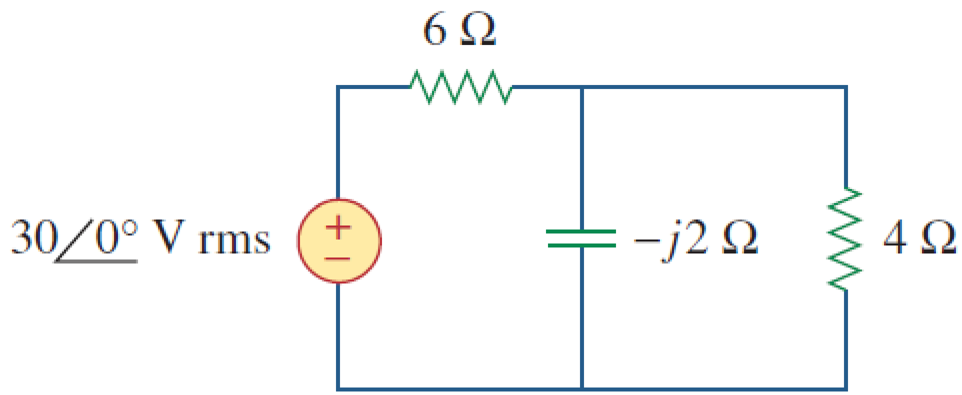
$$-15 = -\frac{1}{\omega C}$$

$$C = \frac{1}{(100\pi)15}$$

$$C = 212.2\ \mu\text{F}$$

Example #5

Determine the pf and average power delivered by the source



Solution

The total impedance is

$$\mathbf{Z} = 6 + 4 \parallel (-j2) = 6 + \frac{-j2 \times 4}{4 - j2} = 6.8 - j1.6 = 7\angle -13.24^{\circ}$$

Example #5

Thus

$$\mathbf{I}_{rms} = \frac{\mathbf{V}_{rms}}{\mathbf{Z}} = \frac{30\angle 0^\circ}{7\angle -13.24^\circ} = 4.286\angle 13.24^\circ \text{A}$$

The power factor,

$$pf = \cos(\theta_v - \theta_i) = \cos(0^\circ - 13.24^\circ) = 0.9734$$

Which is leading power factor

The average power supply by the source is

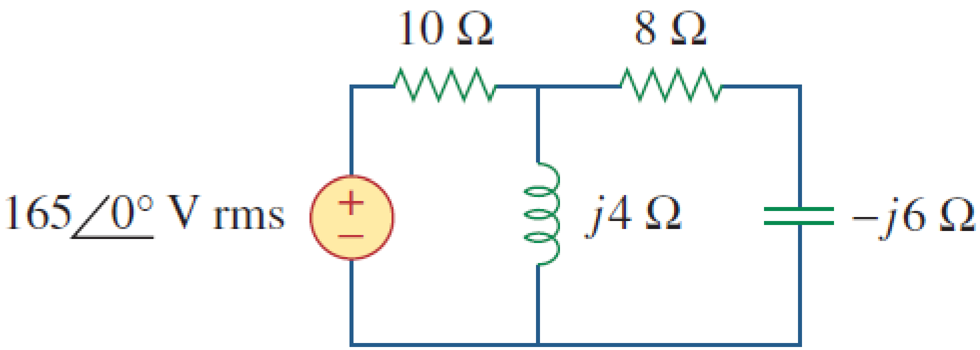
$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$= 30(4.286)(0.9734)$$

$$= 125\,\text{W}$$

Example #6

Determine the pf and average power delivered by the source



Answer

$$pf = 0.936 \quad \text{lagging}$$

$$P = 2.008 \text{ kW}$$

Complex Power

- Complex power is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P and its imaginary is reactive power Q

- Consider the following

$$v(t) = V_m \cos(\omega t + \theta_v) = V_m \angle \theta_v$$
$$i(t) = I_m \cos(\omega t + \theta_i) = I_m \angle \theta_i$$

Complex Power

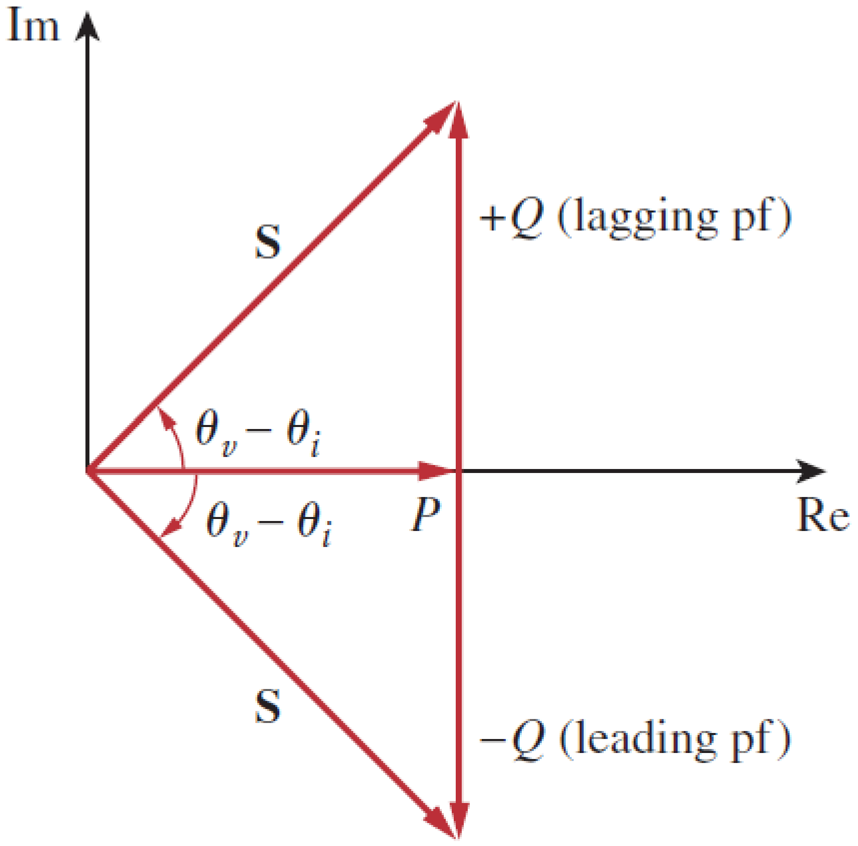
$$Complex\ power = V_{rms} I_{rms}^{*}$$
$$= V_{rms} I_{rms} \angle(\theta_v - \theta_i) \quad (\mathrm{VA})$$
$$= |S| \angle(\theta_v - \theta_i)$$
$$= P + jQ$$
$$= \left(\sqrt{P^2 + Q^2}\right) \angle(\theta_v - \theta_i)$$

- Where Q is the reactive power (VAR)

- If
  - $Q = 0$ for resistive loads (unity pf).
  - $Q < 0$ for capacitive loads (leading pf).
  - $Q > 0$ for inductive loads (lagging pf).

Complex Power

- Power triangle:



Example #7

Given that

$$v(t) = 60\cos(\omega t - 10^\circ)$$

$$i(t) = 1.5\cos(\omega t + 50^\circ)$$

Find the complex and apparent powers, the real and reactive powers and the power factor.

Solution $Complex\ power = V_{rms} I_{rms} \angle(\theta_v - \theta_i)$

$$= \frac{60}{\sqrt{2}} \frac{1.5}{\sqrt{2}} \angle(-10^\circ - 50^\circ)$$

$$= 45\angle -60^\circ \text{ VA}$$

$$= 22.5 - j38.97 \text{ VA}$$

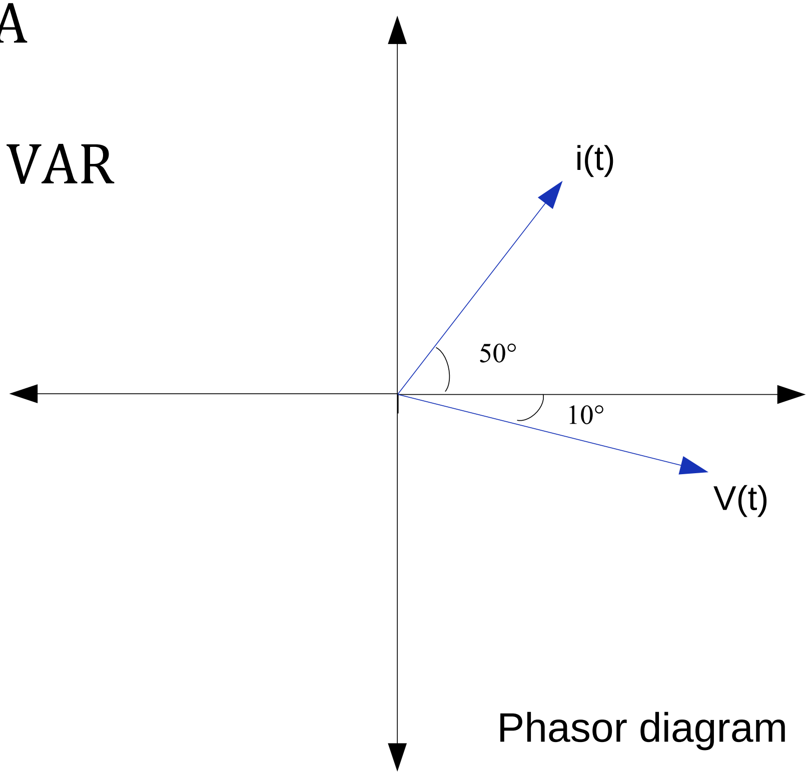
Example #7

Power factor,

$$pf = \cos(-10° - 50°) = 0.5$$

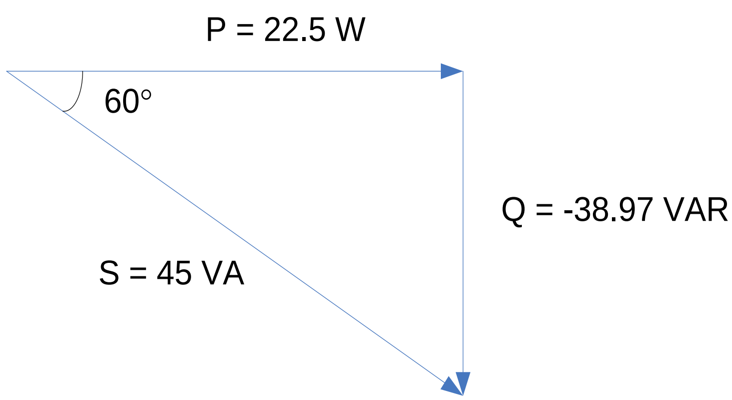
Thus the apparent power, $S = 45$ VA
real power, $P = 22.5$ W
reactive power, $Q = -38.97$ VAR

From the phasor diagram $i(t)$ leads $v(t)$, thus the pf is leading.



Phasor diagram

Power triangle



Example #8

Given that

$$\mathbf{V}_{rms} = 110\angle 85°\ \text{V}$$

$$\mathbf{I}_{rms} = 0.4\angle 85°\ \text{A}$$

Find:

i. The complex and apparent powers.
ii. The real and reactive powers.
iii. The power factor and load impedance.

Answer

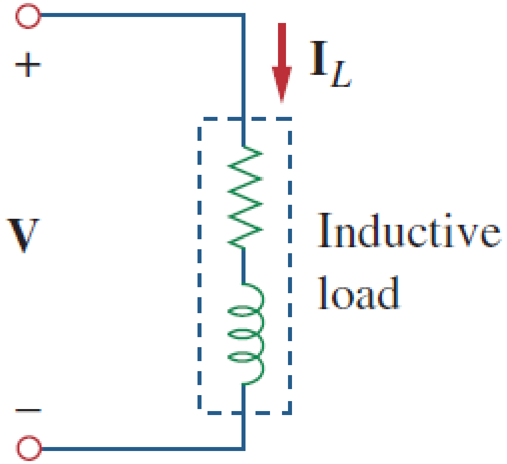
i. $44\angle 70°$ VA, 44 VA

ii. 15.05 W, 41.35 VAR

iii. 0.342 lagging, $94.06 + j258.4\,\Omega$

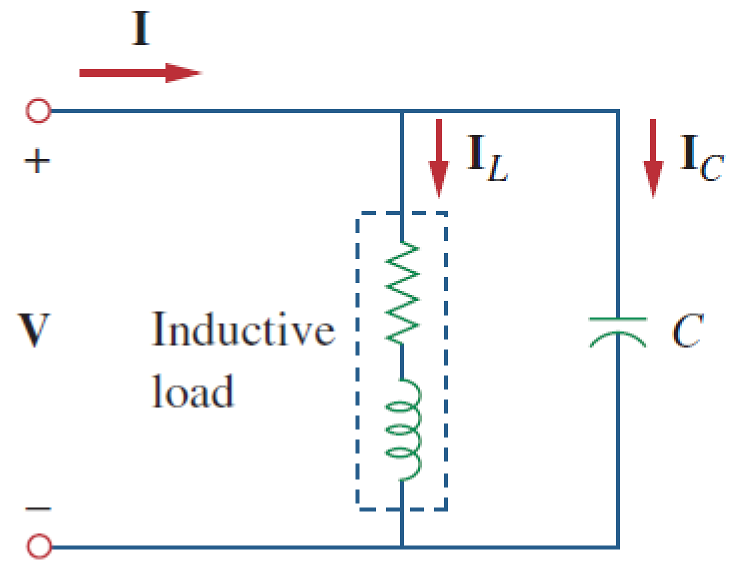
Power Factor Correction

- Most domestic loads (such as washing machines, air conditioners, and refrigerators) and industrial loads (such as induction motors) are inductive and operate at a low lagging power factor.
- Although the inductive nature of the load cannot be changed, we can increase its power factor.
- The process of increasing the power factor without altering the voltage or current to the original load is known as power factor correction.

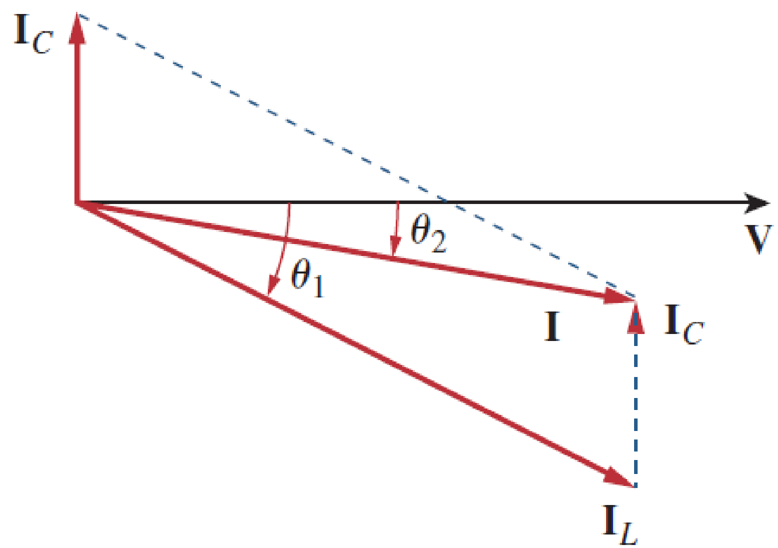


Power Factor Correction

- A load's power factor is improved or corrected by deliberately installing a capacitor in parallel with the load

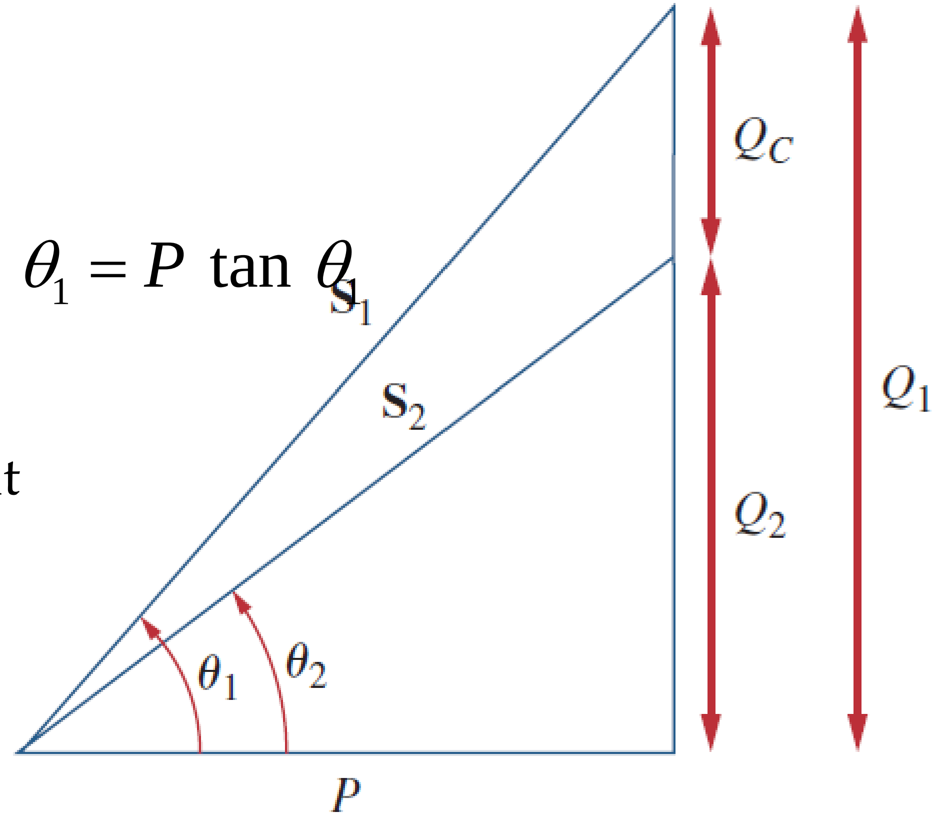


- The effect of adding the capacitor can be illustrated using either the power triangle or the phasor diagram of the currents involved



Power Factor Correction

$$P = S_1 \cos\theta_1 \qquad Q_1 = S_1 \sin\theta_1 = P\tan\theta_1$$



If we desire to increase the power factor from $\cos\theta_1$ to $\cos\theta_2$ without altering the real power, then the new reactive power

$$Q_2 = P\tan\theta_2$$

The reduction in the reactive power is caused by the shunt capacitor;

$$Q_C = Q_1 - Q_2 = P\left(\tan\theta_1 - \tan\theta_2\right)$$

The value of the required shunt capacitance C

$$C = \frac{Q_C}{\omega V_{rms}^2} = \frac{P\left(\tan\theta_1 - \tan\theta_2\right)}{\omega V_{rms}^2}$$

- Note that the real power P dissipated by the load is not affected by the power factor correction because the average power due to the capacitance is zero.

- Although the most common situation in practice is that of an inductive load, it is also possible that the load is capacitive; that is, the load is operating at a leading power factor.

- In this case, an inductor should be connected across the load for power factor correction.

$$Q_L = Q_1 - Q_2 \qquad Q_L = \frac{V_{rms}^2}{X_L} = \frac{V_{rms}^2}{\omega L} \quad \Rightarrow \quad L = \frac{V_{rms}^2}{\omega Q_L}$$

Example #9

When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

Solution

$$pf = 0.8$$

$$\cos\theta_1 = 0.8 \quad \Rightarrow \quad \theta_1 = 36.87°$$

The apparent power

$$S = \frac{P}{\cos\theta_1} = \frac{4000}{0.8}$$

$$S = 5000\,\text{VA}$$

The reactive power

$$Q_1 = S_1 \sin\theta_1 = 5000 \sin 36.87$$

$$Q_1 = 3000\,\text{VAR}$$

Example #9

$$pf_{\text{new}} = 0.95$$

$$\cos\theta_2 = 0.95 \quad \Rightarrow \quad \theta_2 = 18.19^\circ$$

The new apparent power

$$S_2 = \frac{P}{\cos\theta_2} = \frac{4000}{0.95}$$

$$S_2 = 4210.5\,\text{VA}$$

The new reactive power

$$Q_2 = S_2 \sin\theta_2 = 4210.5 \sin 18.19$$

$$Q_2 = 1314.4\,\text{VAR}$$

$$Q_C = Q_1 - Q_2 = 1685.6\ \text{VAR}$$

Then the value of capacitor

$$C = \frac{Q_C}{\omega V_{rms}^2}$$

$$C = 310.5\,\mu\text{F}$$

Example #9

Find the value of parallel capacitance needed to correct a load of 140 kVAR at 0.85 lagging pf to unity pf. Assume that the load is supplied by a 110-V (rms), 60-Hz line.

Answer

$$C = 30.69 \text{ mF}$$