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BFF1303: ELECTRICAL / ELECTRONICS ENGINEERING

Alternating Current Circuits : Basic Law

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Alternating Current Circuit (AC)-Basic Laws & Circuit Techniques

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Contents:

Outcomes

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Convert time-domain sinusoidal voltages and currents to phasor notation, and vice versa; represent circuit using impedance

Solve steady-state ac circuits, using phasors and complex impedances.

Learn complex power notation; compute apparent power, real, and reactive power for complex load.

Draw the power triangle, and compute the capacitor size required to perform power factor correction on a load.

Kirchhoff's Law

- This principles used in dc analysis, are also applicable in the phasor domain.

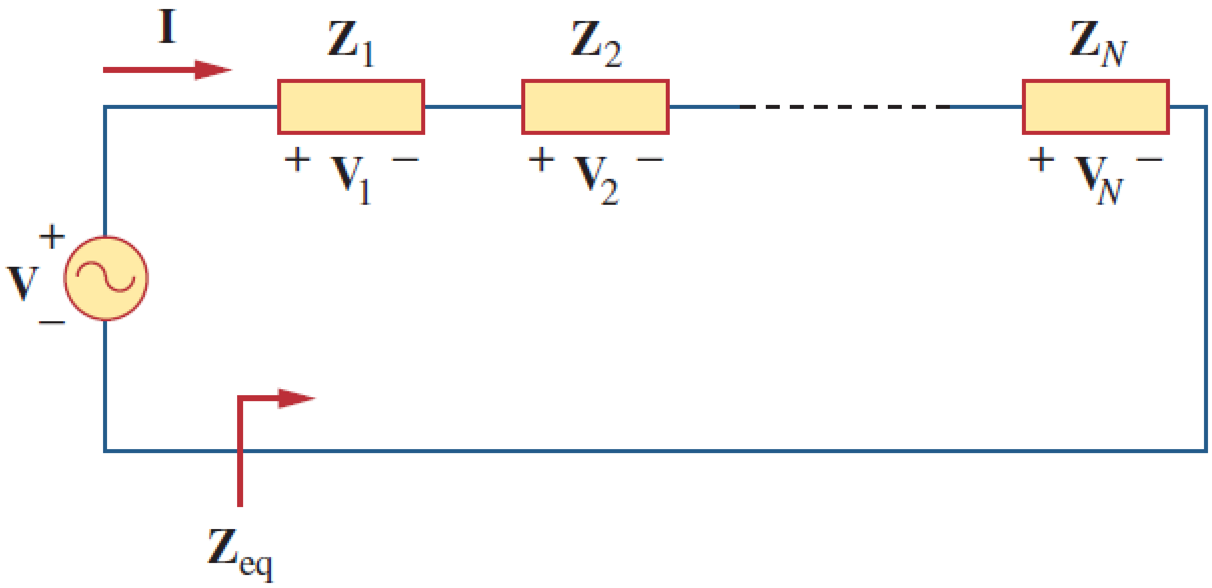
- The difference is simply the voltages, currents and resistance/inductance/capacitance are converted to phasor and impedance.

- Kirchhoff's current law (KCL) – that the algebraic sum of phasor currents entering a node (or a closed boundary) is zero.

- Kirchhoff's voltage law (KVL) – the algebraic sum of all phasor voltages around a closed path (or loop) is zero.

$$\sum_{n=1}^{N} \mathbf{I}_n = 0 \qquad \sum_{m=1}^{M} \mathbf{V}_m = 0$$

- Consider the following figure, the same current **I** flow through the impedance



- Applying KVL around the loop

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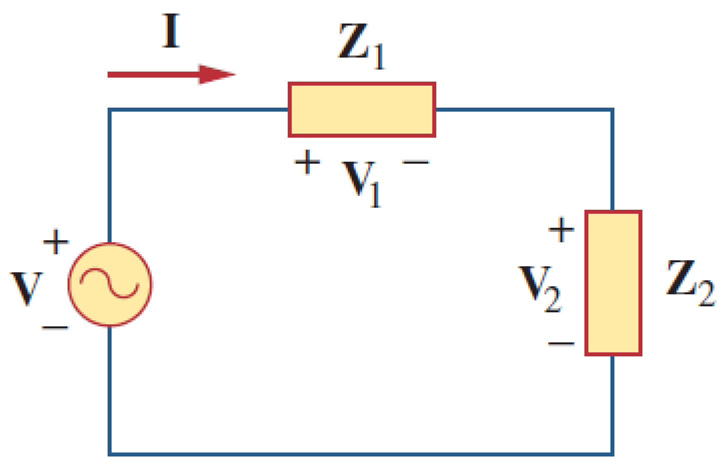
- The equivalent impedance at the input terminals

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}}{\mathbf{I}} = \left(\mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N\right)$$

- Any number of impedances connected in series is the sum of the individual impedances.

- If $N = 2$



- The current through impedances

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

Series Resistors and Voltage Division

- Since

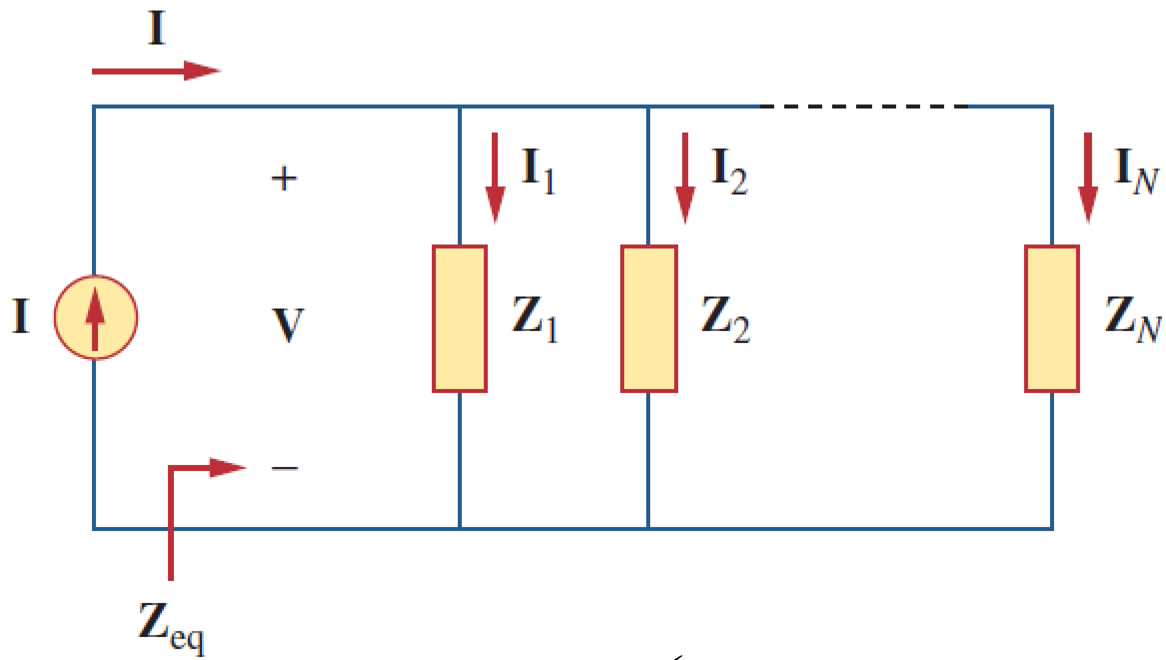
$$\mathbf{V}_1 = \mathbf{Z}_1\mathbf{I} \qquad \mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}$$

- Then

$$\mathbf{V}_1 = \left(\frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\right)\mathbf{V}, \qquad \mathbf{V}_2 = \left(\frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\right)\mathbf{V}$$

Principle of voltage division.

- The voltage across each impedance is the same. Applying KCL at top node

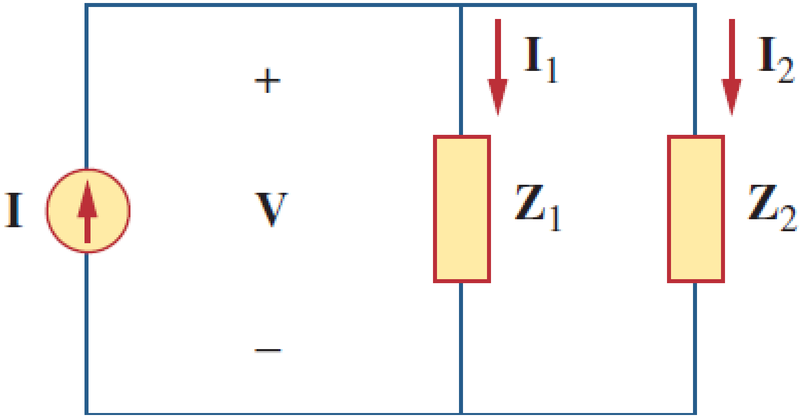


$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_N = \mathbf{V}\left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}\right)$$

- The equivalent impedance

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{\mathbf{V}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}$$

- When $N = 2$



$$\mathbf{Z}_{eq} = \frac{1}{1/\mathbf{Z}_1} + \frac{1}{1/\mathbf{Z}_2} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

- Since

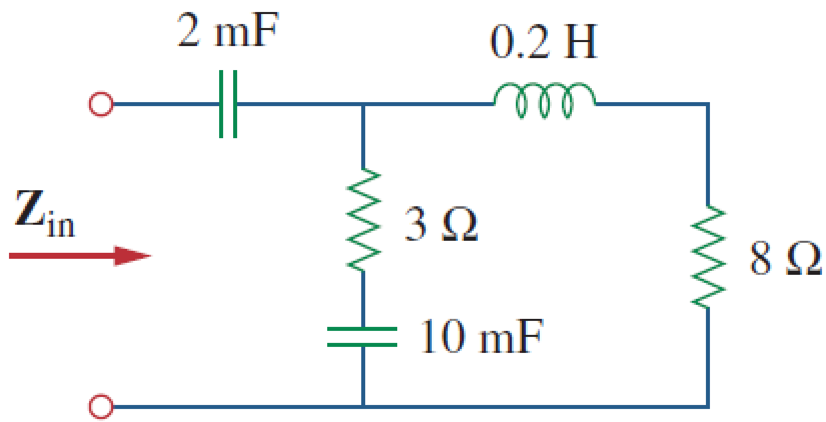
$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{eq} = \mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2$$

- The current in the impedances are

$$\mathbf{I}_1 = \left(\frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\right)\mathbf{I}, \qquad \mathbf{I}_2 = \left(\frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\right)\mathbf{I}$$

Example #1

Find the input impedance of the circuit shown. Assume that the circuit operates at $\omega = 50$ rad/s



Solution

To get $\mathbf{Z}_{in}$, we combine resistors, resistor-capacitor and resistor-inductor in series and in parallel.

Example #1

Let

$\mathbf{Z}_1$ Impedance of the 2 mF capacitor

$\mathbf{Z}_2$ Impedance of the 3 Ω resistor in series with the 10 mF capacitor

$\mathbf{Z}_3$ Impedance of the 0.2 H inductor in series with the 8 Ω resistor

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = -j10\ \Omega$$

Then

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 - j2\ \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j10\ \Omega$$

Example #1

The input impedance

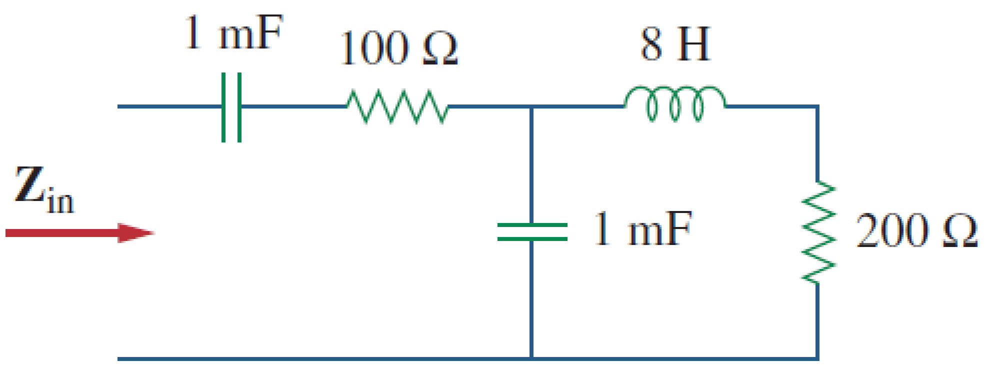
$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \left(\mathbf{Z}_2 \parallel \mathbf{Z}_3\right)$$

$$\mathbf{Z}_{in} = -j10 + \frac{\left(3 - j2\right)\left(8 + j10\right)}{11 + j8}$$

$$\mathbf{Z}_{in} = 3.22 - j11.07\ \Omega = 11.52\angle -73.78^\circ\ \Omega$$

Example #2

Determine the input impedance of the circuit in figure shown at $\omega = 10$ rad/s.

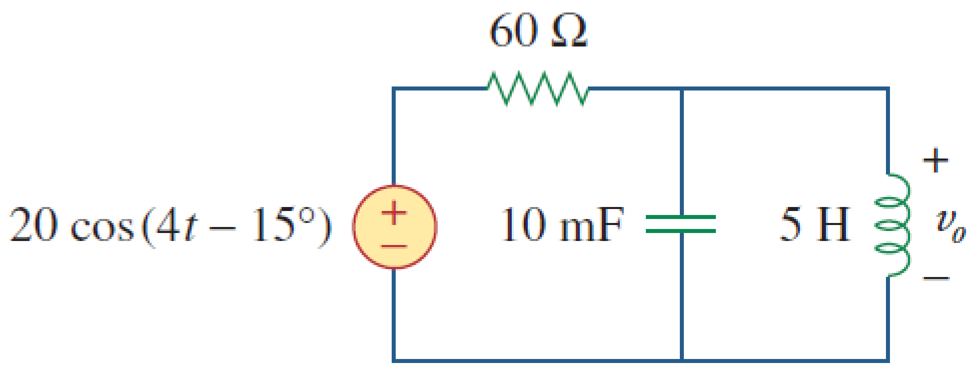


Answer

$$\mathbf{Z}_{in} = 149.52 - j195\ \Omega = 245.73\angle 52.52°\ \Omega$$

Example #3

Determine $v_o(t)$ for the given circuit.

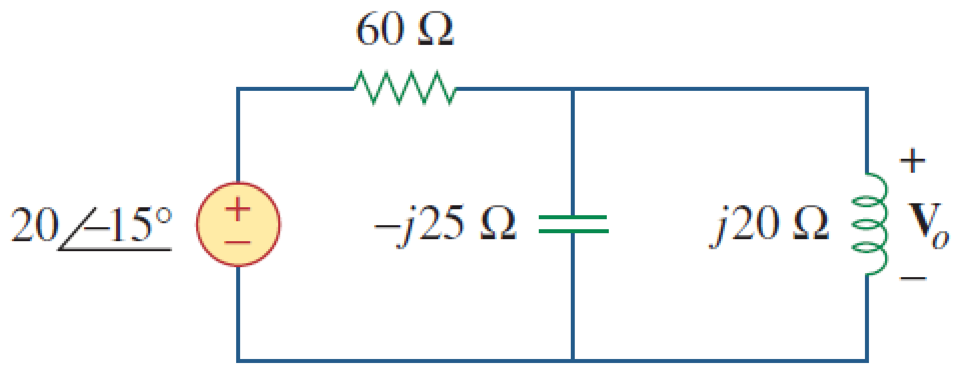


Solution

Transform the time domain equivalent to phasor form

$$\mathbf{V}_s = 20\angle -15^\circ \text{ V} \quad \mathbf{Z}_C = \frac{1}{j\omega C} = -j25\ \Omega \quad \mathbf{Z}_L = j\omega L = j20\ \Omega$$

Example #3



Let

$\mathbf{Z}_1$ Impedance of the 60 Ω resistor

$\mathbf{Z}_2$ Impedance of the parallel combination of the 10 mF capacitor and the 5 H inductor

$$\mathbf{Z}_1 = 60\ \Omega \qquad \mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L = j100\ \Omega$$

Example #3

By using voltage divider

$$\mathbf{V}_o = \left(\frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \right) \mathbf{V}$$

$$\mathbf{V}_o = \left(\frac{j100}{60 + j100} \right) 20\angle -15^\circ$$

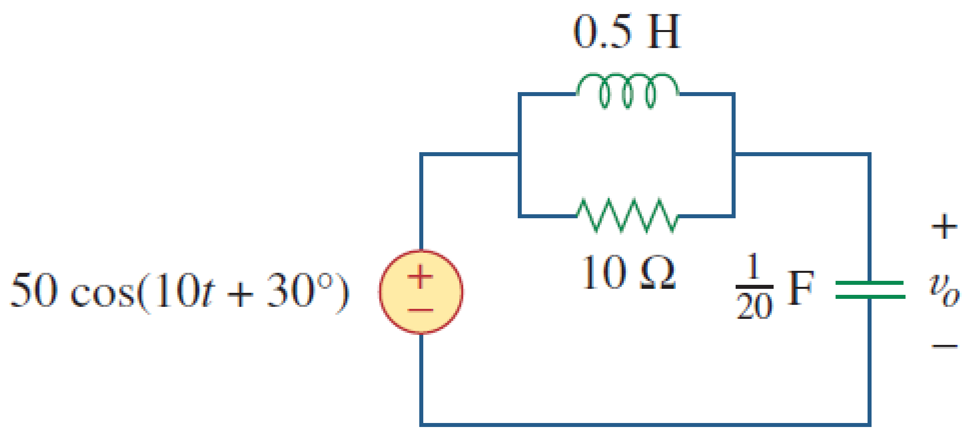
$$\mathbf{V}_o = 17.15\angle 15.96^\circ \text{ V}$$

In time domain

$$v_o(t) = 17.15\cos(4t + 15.96^\circ) \text{ V}$$

Example #4

Find $v_o(t)$ in the given circuit



Answer

$$v_o(t) = 35.36\cos(10t - 105^\circ)\,\text{V}$$

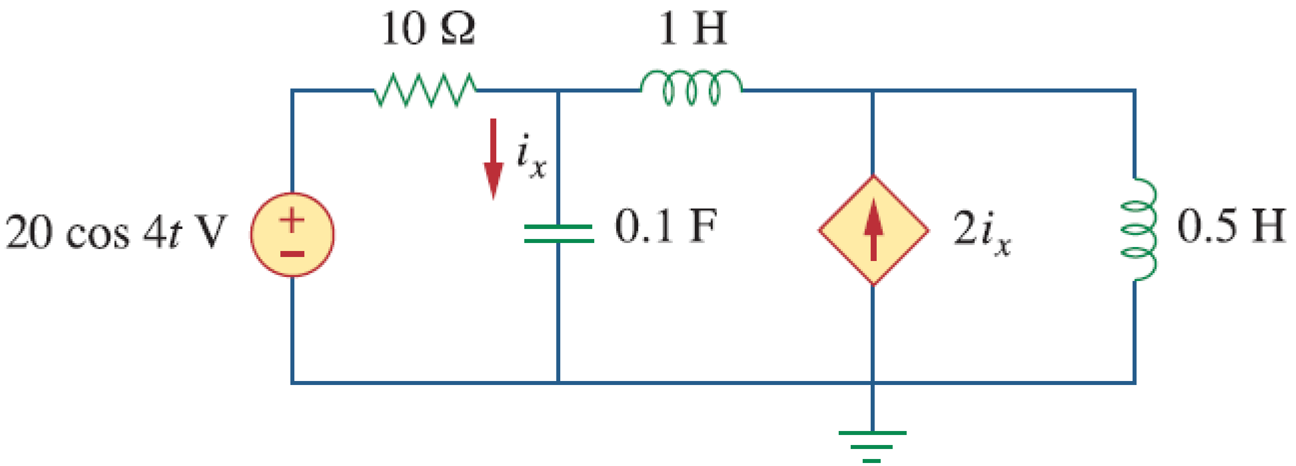
Nodal Analysis

- The basis of nodal analysis is Kirchhoff's Current Law.

- Since KCL is valid for phasors, we can analyze ac circuit by nodal analysis.

Example #5

Find i_x in the given circuit using nodal analysis



Solution

Convert the circuit to phasor form

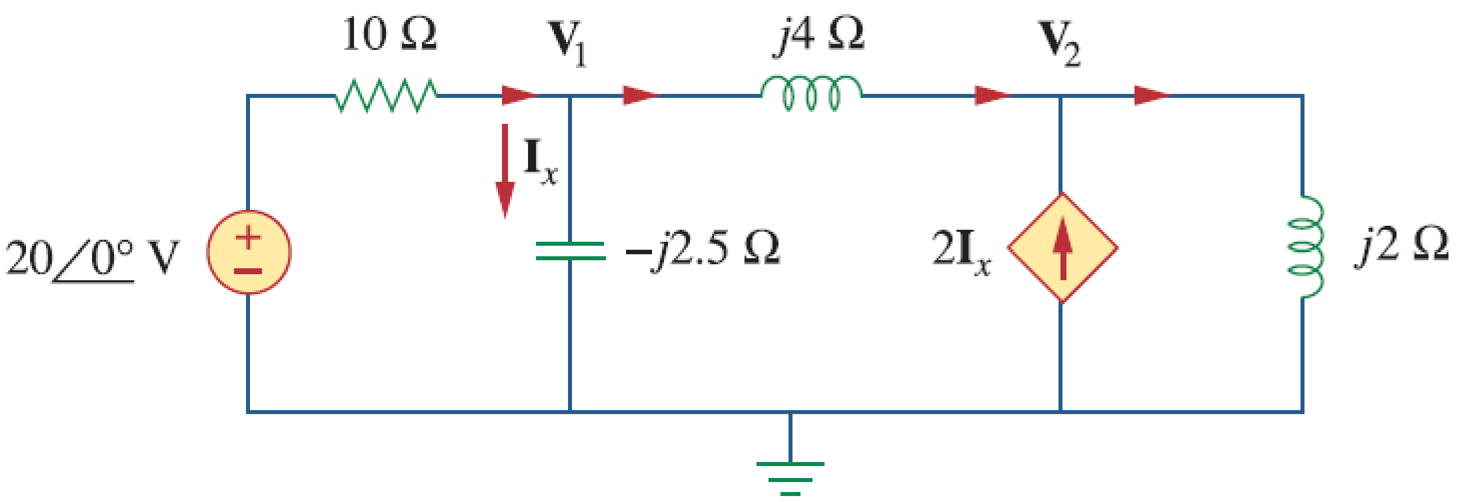
Example #5

$$20\cos 4t \Rightarrow 20\angle 0^\circ$$

$$1\text{H} \Rightarrow j\omega L = j4$$

$$0.5\text{H} \Rightarrow j\omega L = j2$$

$$0.1\text{F} \Rightarrow \frac{1}{j\omega C} = -j2.5$$



Example #5

Applying KCL at node $\mathbf{V}_1$

$$\frac{20-\mathbf{V}_1}{10}=\frac{\mathbf{V}_1}{-j2.5}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j4}$$

$$(1+j1.5)\mathbf{V}_1+j2.5\mathbf{V}_2=20$$

Applying KCL at node $\mathbf{V}_2$

$$2\mathbf{I}_x+\frac{\mathbf{V}_1-\mathbf{V}_2}{j4}=\frac{\mathbf{V}_2}{j}$$

And $\mathbf{I}_x = {}^{\mathbf{V}_1}/_{-j2.5}$

$$\frac{2\mathbf{V}_1}{-j2.5}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j4}=\frac{\mathbf{V}_2}{j}$$

$$11\mathbf{V}_1+15\mathbf{V}_2=0$$

Then in matrix form

$$\begin{bmatrix}1+j1.5 & j2.5\\ 11 & 15\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}=\begin{bmatrix}20\\ 0\end{bmatrix}$$

Example #5

Then by using Cramer's Rule

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix}}{\begin{vmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{vmatrix}} = \frac{300}{15-j5} = 18.97\angle 18.43^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\begin{vmatrix} 1+j1.5 & 20 \\ 11 & 0 \end{vmatrix}}{\begin{vmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{vmatrix}} = \frac{-220}{15-j5} = 13.91\angle 198.3^\circ \text{ V}$$

Example #5

Then $\mathbf{I}_x$

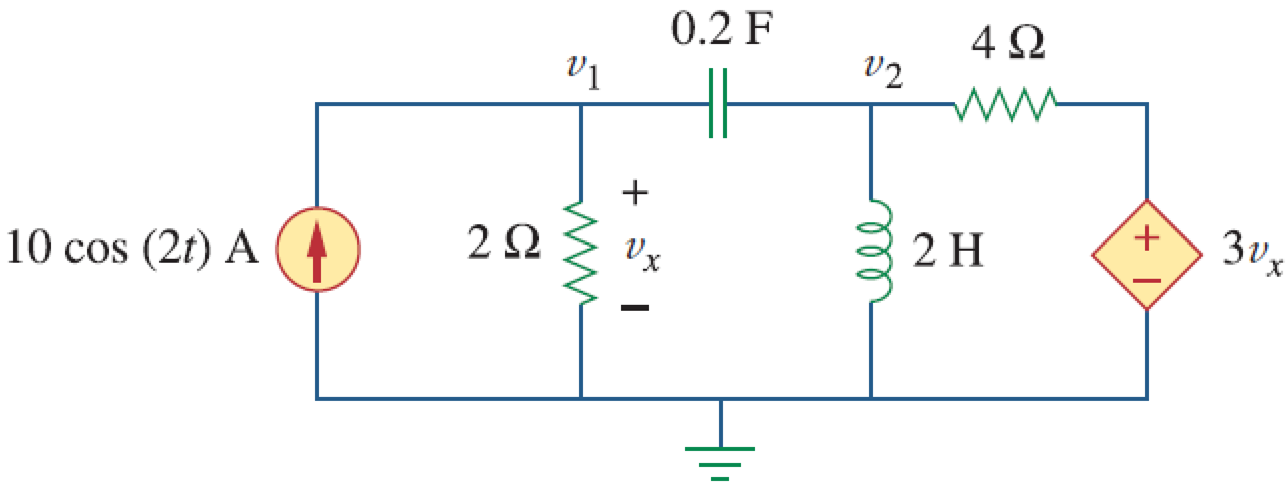
$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\angle 18.43^\circ}{2.5\angle 90^\circ} = 7.59\angle 108.4^\circ \text{ A}$$

In time domain

$$i_x(t) = 7.59\cos\left(4t + 108.4^\circ\right)\text{A}$$

Example #6

Find v_1 and v_2 in the given circuit using nodal analysis



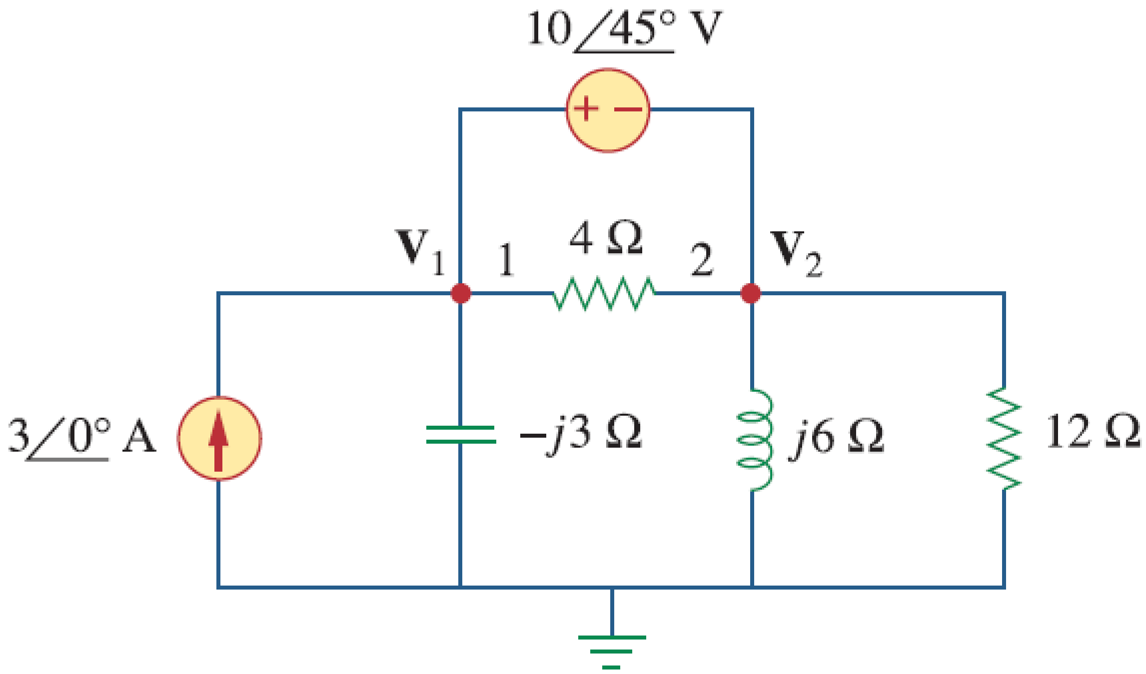
Answer

$$v_1(t) = 11.325\cos(2t + 60.01°)\ \text{V}$$

$$v_2(t) = 33.02\cos(2t + 57.12°)\ \text{V}$$

Example #7

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the following circuit



Answer

$$\mathbf{V}_1 = 25.78\angle -70.48°\ \text{V}$$

$$\mathbf{V}_2 = 31.41\angle -87.18°\ \text{V}$$

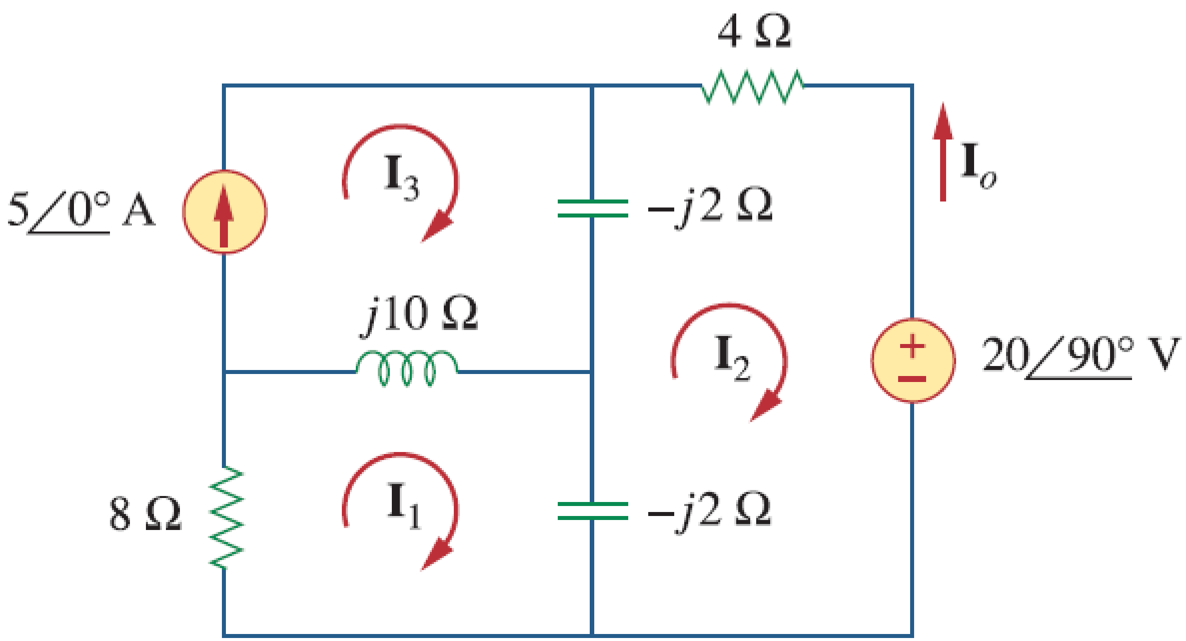
Mesh Analysis

- **Kirchhoff's Voltage Law** form the basis of mesh analysis.

- Since KVL is valid for phasors, we can analyze ac circuit by mesh analysis.

Example #8

Determine $\mathbf{I}_o$ for the given circuit by using mesh analysis.



Solution

Apply KVL to mesh 1

Example #8

$$\left(8+j10-j2\right)\mathbf{I}_1-\left(-j2\right)\mathbf{I}_2-\left(j10\right)\mathbf{I}_3=0$$

Apply KVL to mesh 2

$$\left(4-j2-j2\right)\mathbf{I}_2-\left(-j2\right)\mathbf{I}_1-\left(-j2\right)\mathbf{I}_3+2\angle 90^\circ=0$$

For mesh 3 $\mathbf{I}_3=5$

Then

$$\left(8+j8\right)\mathbf{I}_1+j2\mathbf{I}_2=j50$$

$$j2\mathbf{I}_1+\left(4-j4\right)\mathbf{I}_2=-j30$$

Example #8

In matrix form

$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

By using Cramer's Rule

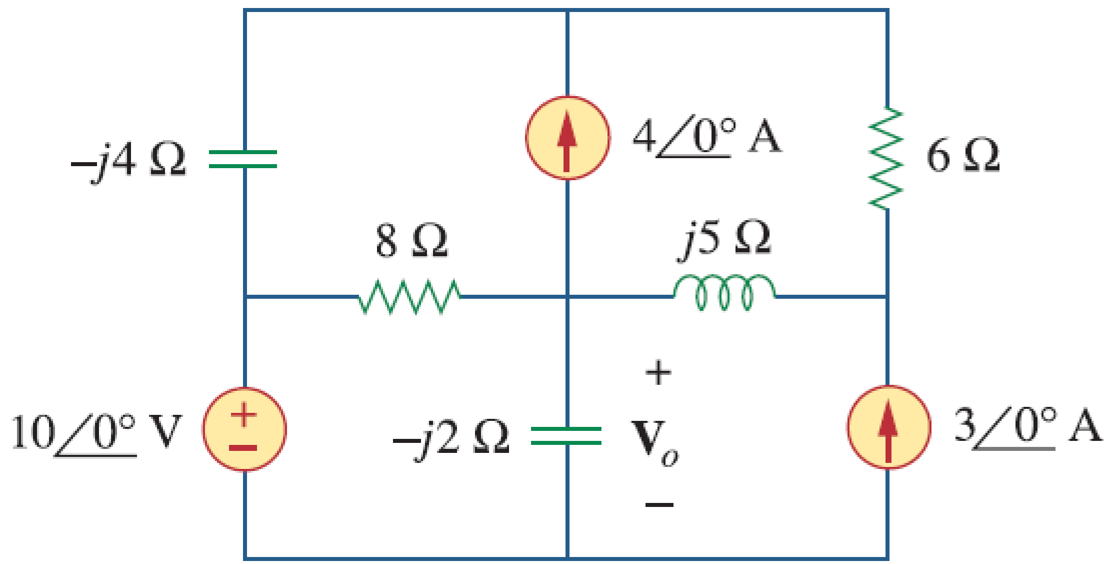
$$\mathbf{I}_2 = \frac{\begin{bmatrix} 8+j8 & j50 \\ j2 & -j30 \end{bmatrix}}{\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix}} = \frac{416.17\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{ A}$$

The desired current

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\angle 144.78^\circ \text{ A}$$

Example #9

Solve for $\mathbf{V}_o$ in the following circuit using mesh analysis



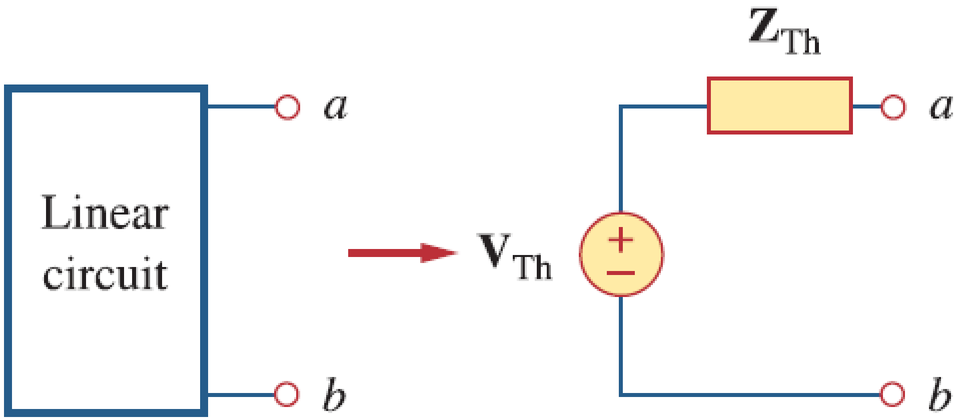
Answer

$$\mathbf{V}_o = 9.756\angle 222.32°\,\text{V}$$

- Thevenin and Norton theorem are applied to AC circuit in the same way as they are to DC circuits.

- The only additional effort arises from the need to manipulate complex number.

- If the circuit has sources operating at different frequencies the Thevenin or Norton equivalent circuit must be determined at each frequency.



$$\mathbf{V}_{Th} = \mathbf{Z}_N \mathbf{I}_N \qquad \mathbf{Z}_{Th} = \mathbf{Z}_N$$