BFF1303: ELECTRICAL / ELECTRONICS ENGINEERING

Direct Current Circuits : Circuits Theorems

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Direct Current Circuit (DC)- Circuit Theorems

BFF1303 ELECTRICAL/ELECTRONICS ENGINEERING

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Contents:

- Outcomes
- Superposition
- Source Transformation
- Thevenin’s Theorem
- Norton’s Theorem
- Maximum Power Transfer

Outcomes

Apply the principle of superposition to linear circuits.

Compute Thevenin and Norton equivalent for network containing linear resistors, independent and dependent sources.

Use equivalent-circuit ideas to compute the maximum power transfer between a source and a load.

Superposition

- The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or current through) that element due to each independent source acting alone.

- To apply the principle, two things should keep in mind:
  - Consider only one independent source at a time while all other independent sources are turned off. This implies that replacing every voltage source by 0 V (or short circuit) and every current source by 0 A (or an open circuit).
  - Dependent sources are left intact because they are controlled by circuit variables.

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Steps to apply superposition principle:

i. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques previously covered.

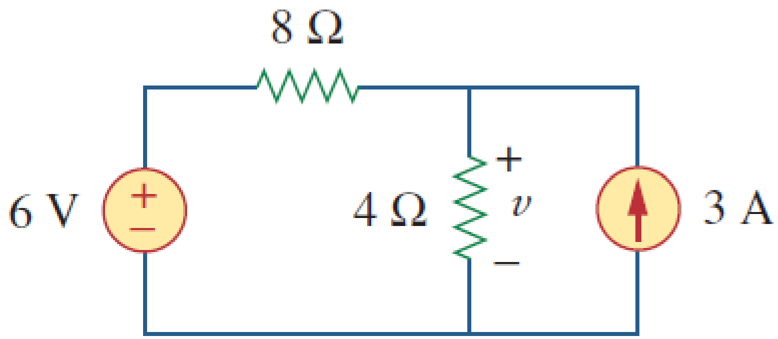
ii. Repeat step 1 for each of the other independent sources.

iii. Find the total contribution by adding algebraically all the contributions due to the independent sources.

- Disadvantage: may involve more work but help reduce a complex circuit to simpler circuits.

Example #1

Use the superposition principle to find *v*, in the circuit given



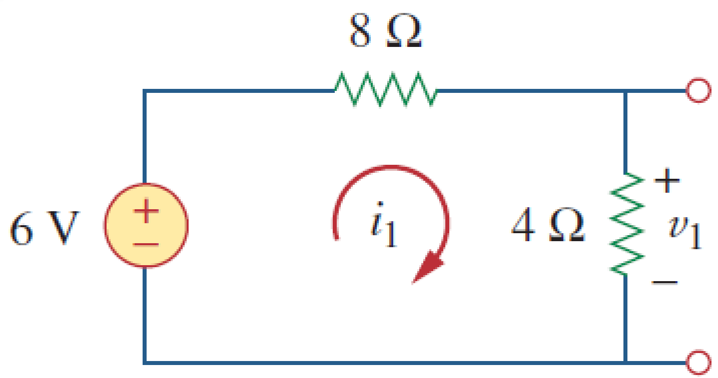
Solution

Since there are two sources, let

$$v = v_1 + v_2 \quad \text{where} \quad \begin{aligned} v_1 &= \text{due to the 6 V voltage source} \\ v_2 &= \text{due to the 3 A current source} \end{aligned}$$

Example #1

To obtain v_1 set the **current source to zero (open circuit)**



Applying KVL to the loop

$$-6+12i_1=0$$

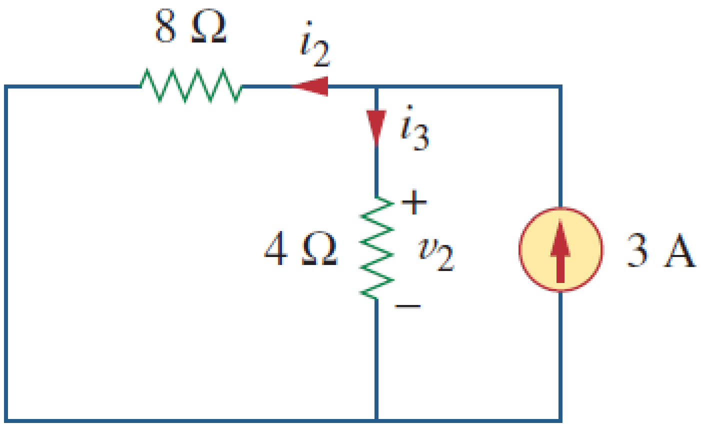
$$i_1=0.5\,\text{A}$$

Thus, give the value $v_1=4i_1=2\,\text{V}$

$$v_1=\left(\frac{4}{4+8}\right)6=2\,\text{V}$$

Example #1

To obtain v_2 set the voltage source to zero (short circuit)



By using current division

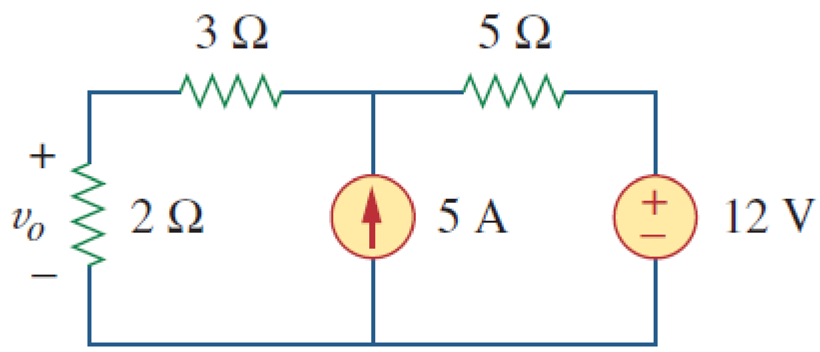
$$i_3 = \left(\frac{8}{4+8}\right)3 = 2\,\text{A}$$

Hence, $v_2 = 4i_3 = 8\,\text{V}$

Thus, $v = v_1 + v_2 = 2 + 8 = 10\,\text{V}$

Example #2

By using the superposition principle find v_o, in the circuit given

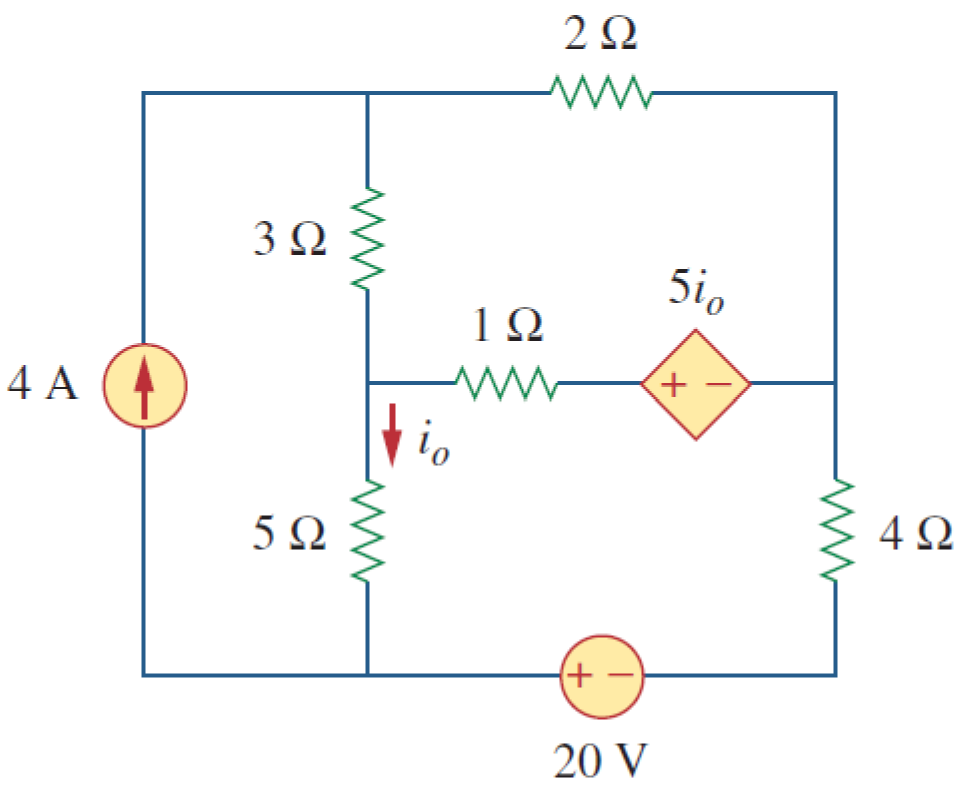


Answer

$$v_o = 7.4\,\text{V}$$

Example #3

Find i_o, in the circuit given using the superposition principle



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Solution

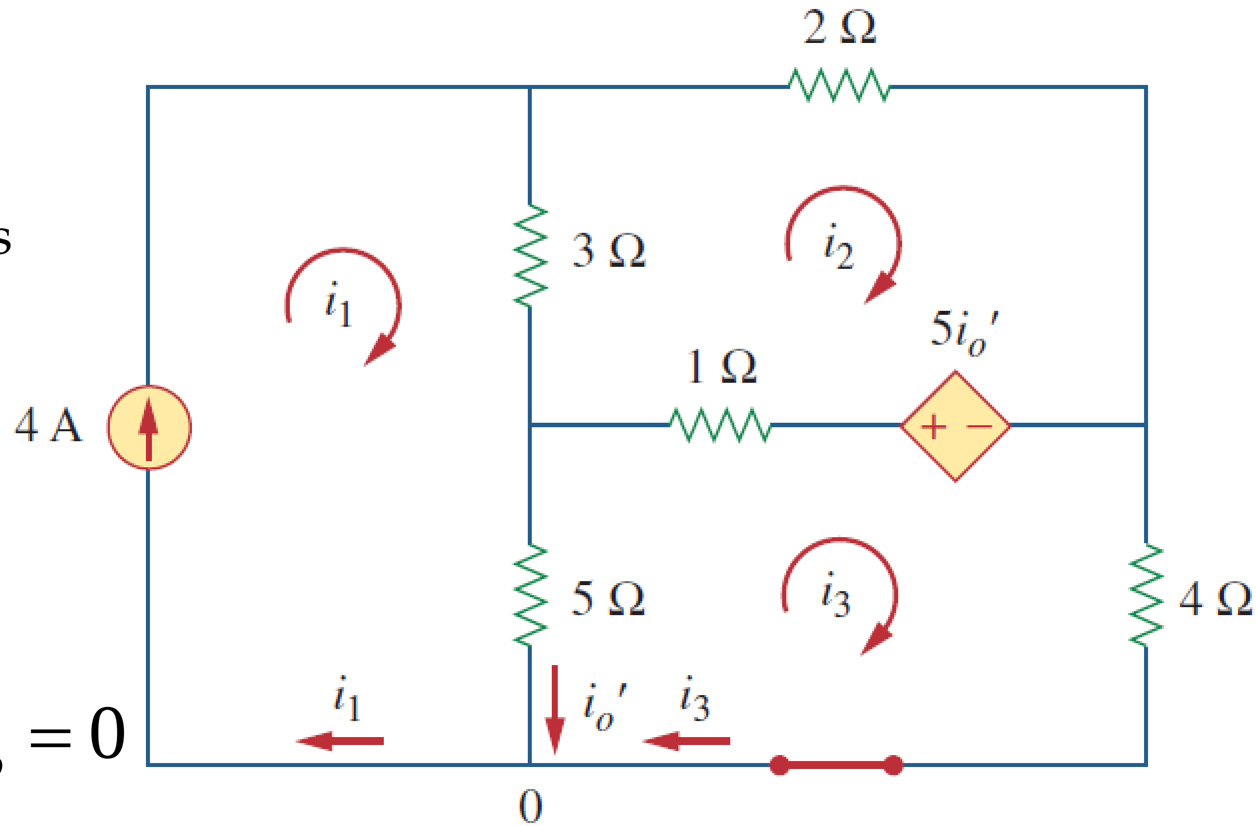
Since the circuit involves a dependent source, which must be left intact

$$i_o = i_o^{'} + i_o^{''} \quad \text{where} \quad \begin{aligned} i_o^{'} &= \text{due to the 4 A current source} \\ i_o^{''} &= \text{due to the 20 V voltage source} \end{aligned}$$

To obtain i_o' set the voltage source to zero (short circuit)

Example #3

Apply mesh analysis to the loop



For mesh/loop 1

$$i_1 = 4\,\text{A}$$

For mesh/loop 2

$$-3i_1 + 6i_2 - i_3 - 5i_o' = 0$$

$$6i_2 - i_3 - 5i_o' = 12\,\text{A}$$

KCL at node 0

$$i_3 = i_1 - i_o' = 4 - i_o'$$

For mesh/loop 3

$$-5i_1 - i_2 + 10i_3 + 5i_o' = 0$$

$$-i_2 + 10i_3 + 5i_o' = 20\,\text{A}$$

Example #3

Then, in matrix form

$$\begin{bmatrix} 6 & -1 & -5 \\ -1 & 10 & 5 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \\ i_0^{'} \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 4 \end{bmatrix}$$

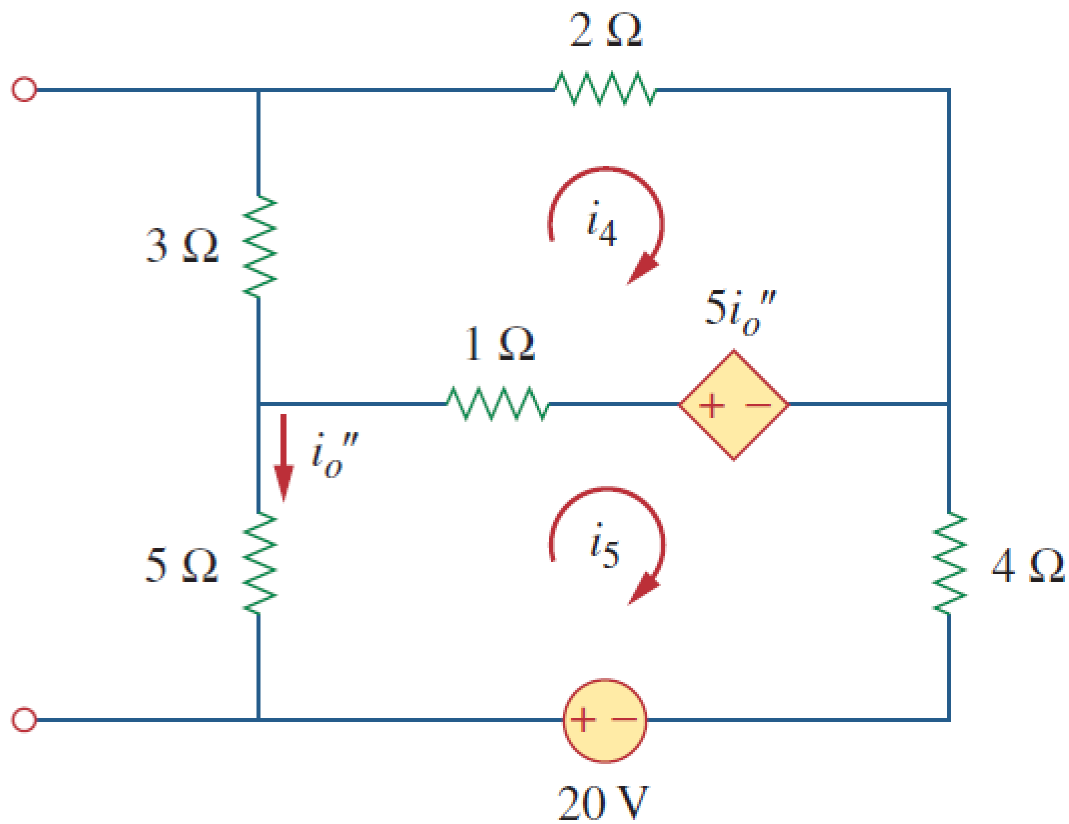
By using Cramer's rule

$$i_o^{'} = \frac{\Delta_1}{\Delta} = \frac{104}{34} = 3.0588\,\text{A}$$

To obtain $i_o^{''}$ set the current source to zero (open circuit)

Example #3

Apply mesh analysis to the loop



Mesh/loop 4

$$6i_4 - i_5 - 5i_o'' = 0$$

Mesh/loop 5

$$-i_4 + 10i_5 - 20 + 5i_o'' = 0$$

and $i_5 = -i_o''$

$$6i_4 - 4i_o'' = 0$$

$$i_4 + 5i_o'' = -20$$

Example #3

Then, in matrix form

$$\begin{bmatrix} 6 & -4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} i_2 \\ i_0^{''} \end{bmatrix} = \begin{bmatrix} 0 \\ -20 \end{bmatrix}$$

By using Cramer's rule

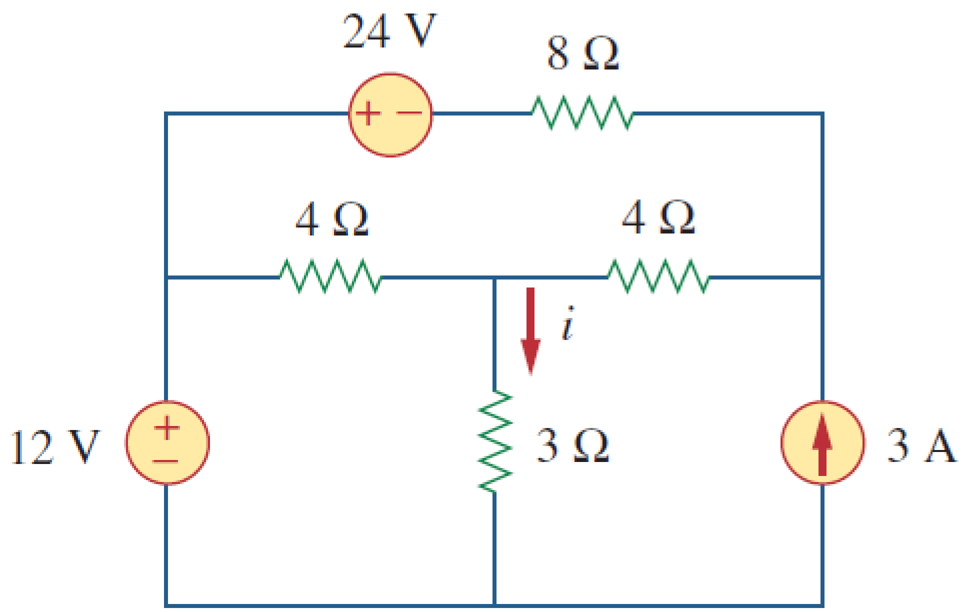
$$i_o^{''} = \frac{\Delta_1}{\Delta} = \frac{-120}{34} = -3.5294\,\text{A}$$

Thus,

$$i_o = i_o^{'} + i_o^{''} = 3.0588 - 3.5294 = -0.4706\,\text{A}$$

Example #4

For the circuit given use the superposition principle to find *i*



Answer

$$i = 2\,\text{A}$$

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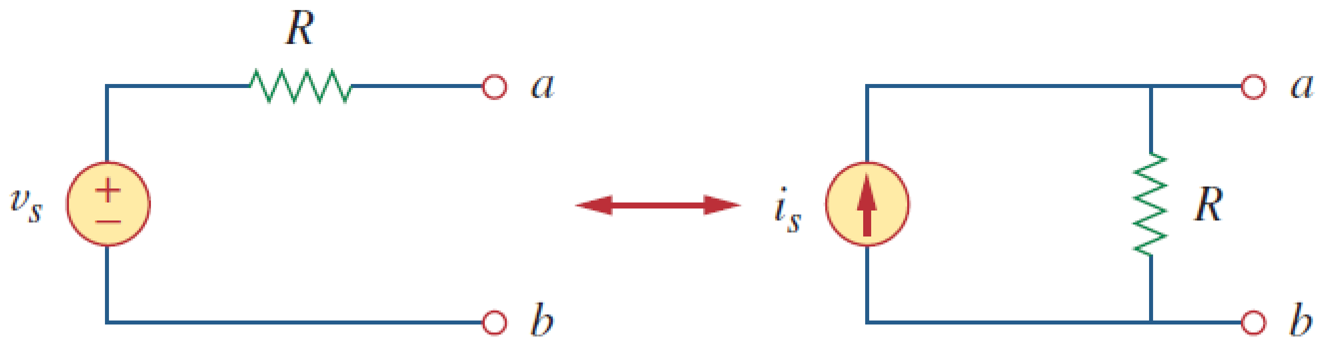
- Another tool for simplifying circuit.

- Basic to these tools is the concept of equivalence.

- A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

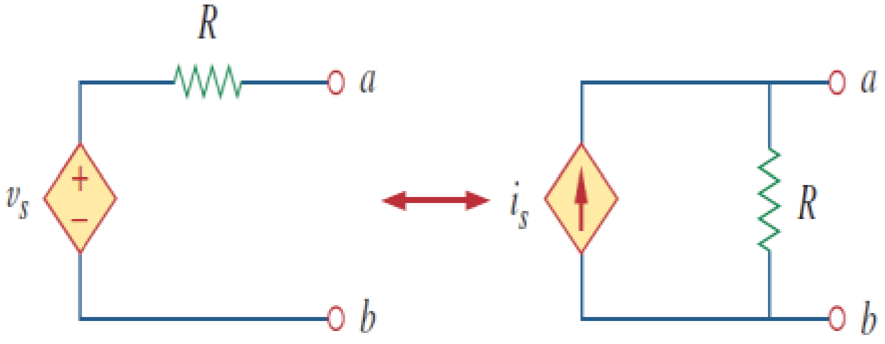


- Since the circuit are equivalent, there have the same voltage-current relation at terminals $a - b$.

- If the sources are turned off, the equivalent resistance at terminals $a - b$ in both circuits is R.

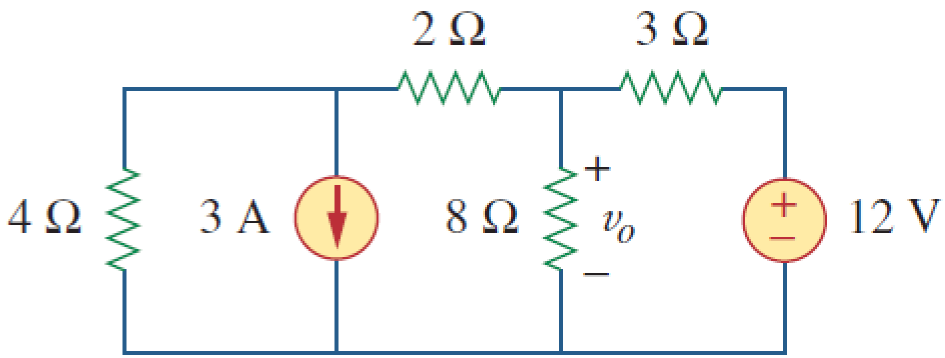
$$v_s = i_s R \qquad i_s = \frac{v_s}{R}$$

- Source transformation also applies to dependent sources, provided we carefully handle the dependent variable.



Example #5

Use source transformation to find v_o, in the circuit given



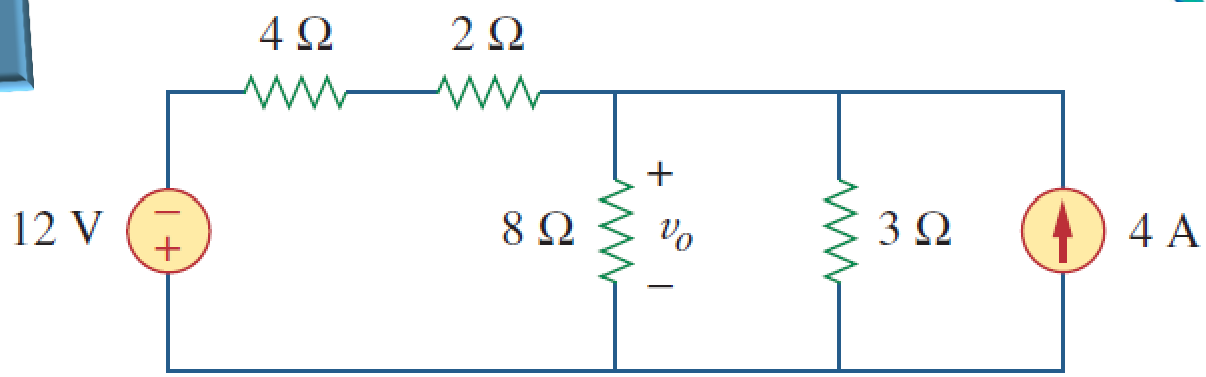
Solution

First transform the current and voltage sources to obtain the circuit

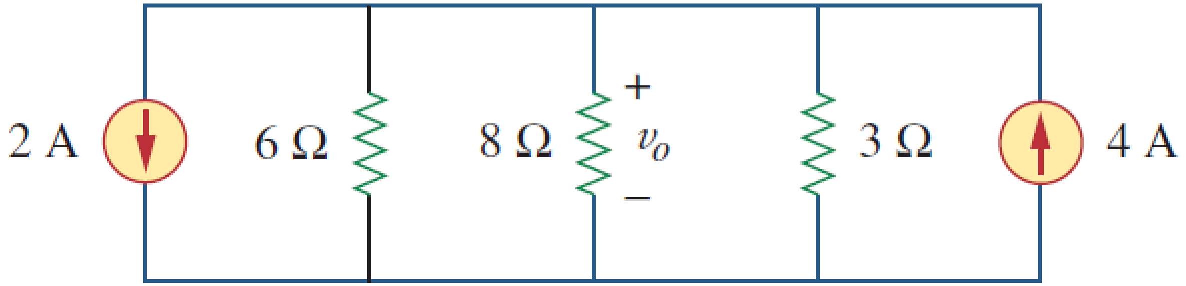
$$v_s = 4 \times 3 = 12\,\text{V}$$

$$i_s = \frac{12}{3} = 4\,\text{A}$$

Example #5

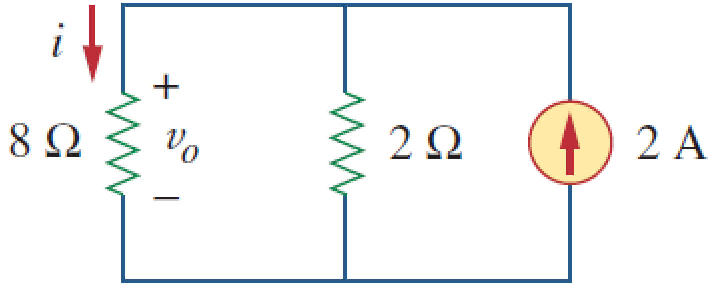


Combining the 4 Ω and 2 Ω in series and transforming 12 V voltage source



Combining the 2 A and 4 A and repeating the source transformation

Example #5



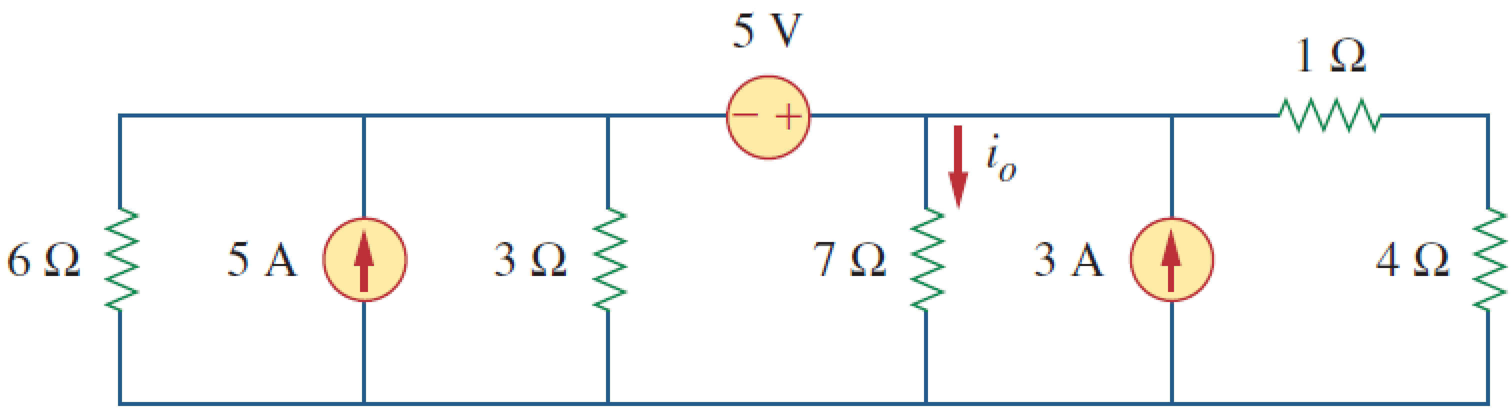
By using current divider

$$i = \left(\frac{2}{10}\right)2 = 0.4\,\text{A}$$

and $v_o = 8i = 8(0.4) = 3.2\,\text{V}$

Example #6

Use source transformation to find i_o, in the circuit given

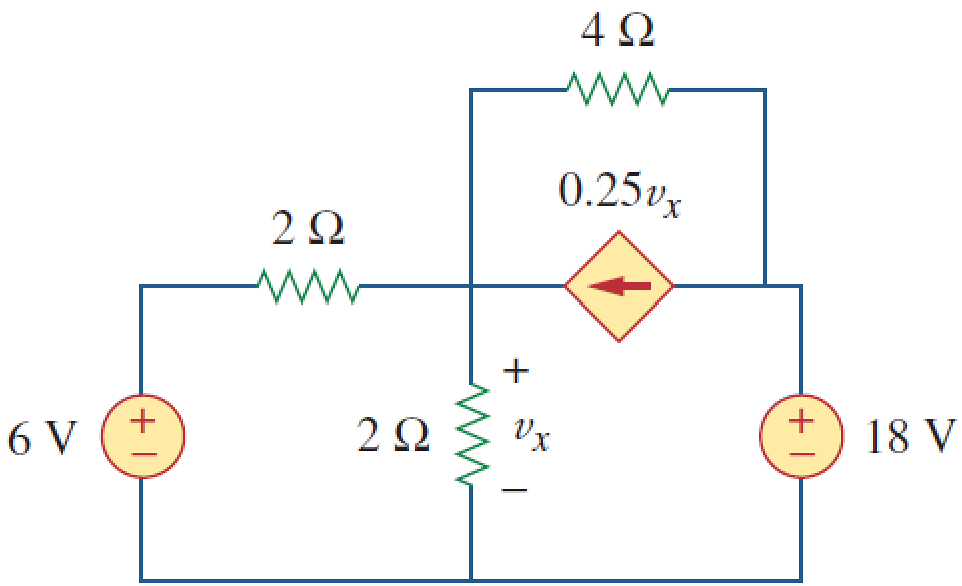


Answer

$$i_o = 1.78\,\text{A}$$

Example #7

Find v_x in the circuit given using source transformation



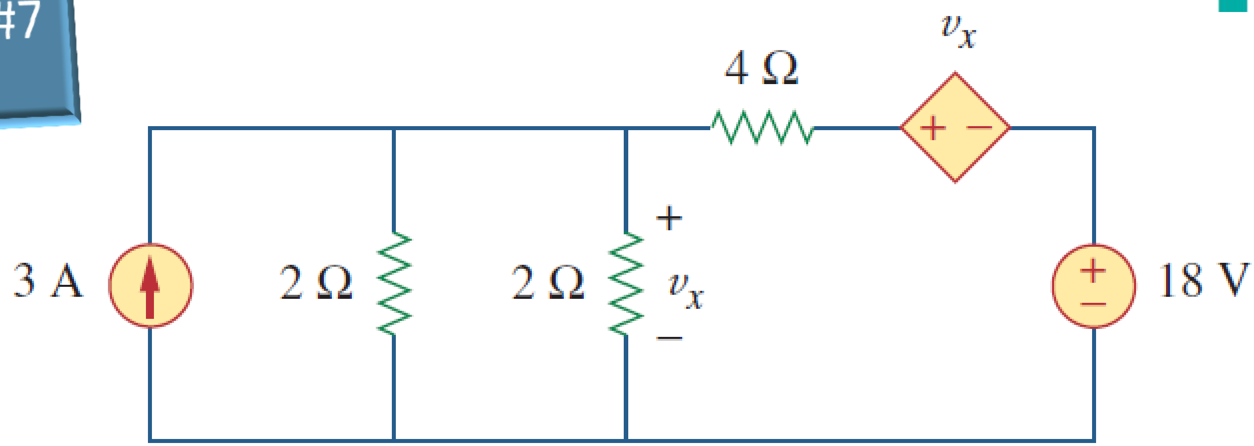
Solution

First transform the dependent current source and voltage sources

$$v_s = 4 \times 0.25 = 1\text{V}$$

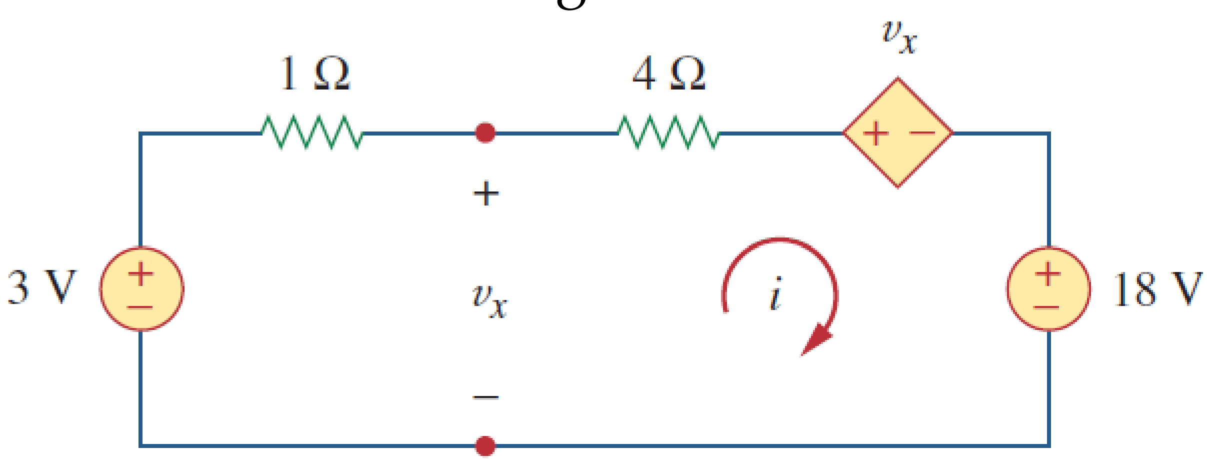
$$i_s = \frac{6}{2} = 3\,\text{A}$$

Example #7



since the 2 Ω and 2 Ω in parallel $2 \parallel 2 = \frac{2\times 2}{2+2} = 1\text{A}$

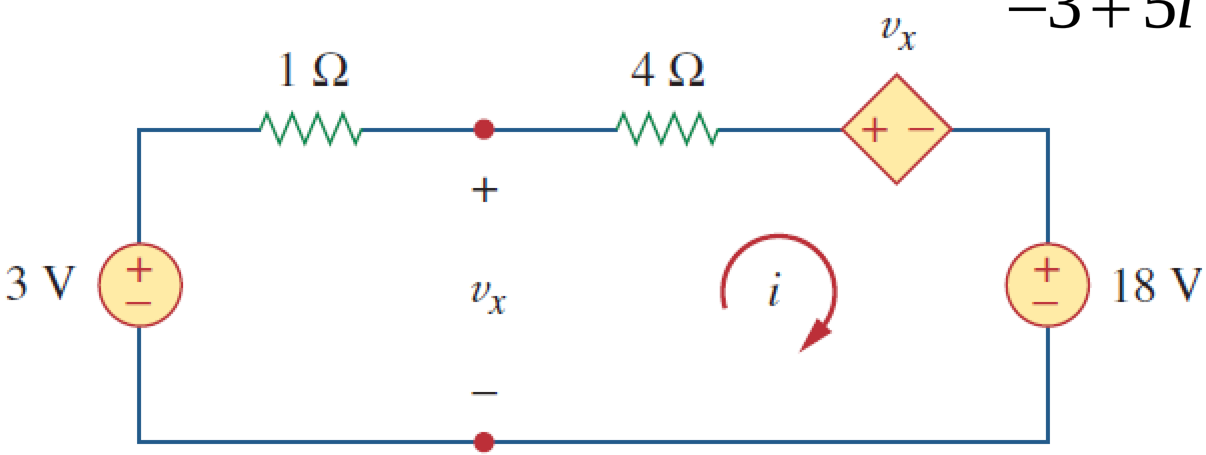
Transform the 2 Ω and 3A will give $v = 3\times 2 = 6\,\text{V}$

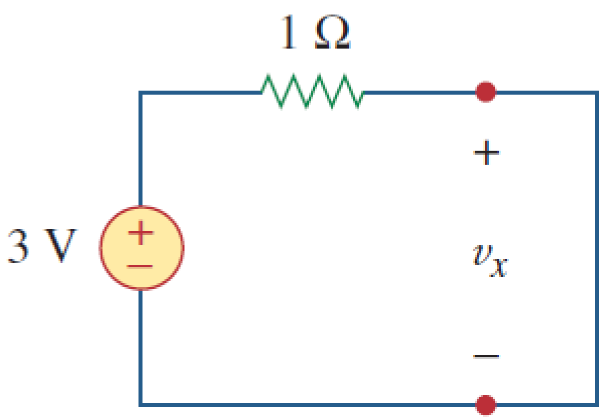


Example #7

Applying KVL around the loop

$$-3+5i+v_x+18=0$$





Applying KVL to the circuit

$$-3+i+v_x=0 \quad \Rightarrow v_x=3-i$$

$$15+5i+3-i=0 \quad \Rightarrow \quad i=4.5\,\text{A}$$

Thus $v_x=3+4.5=7.5\,\text{V}$

Thevenin's Theorem

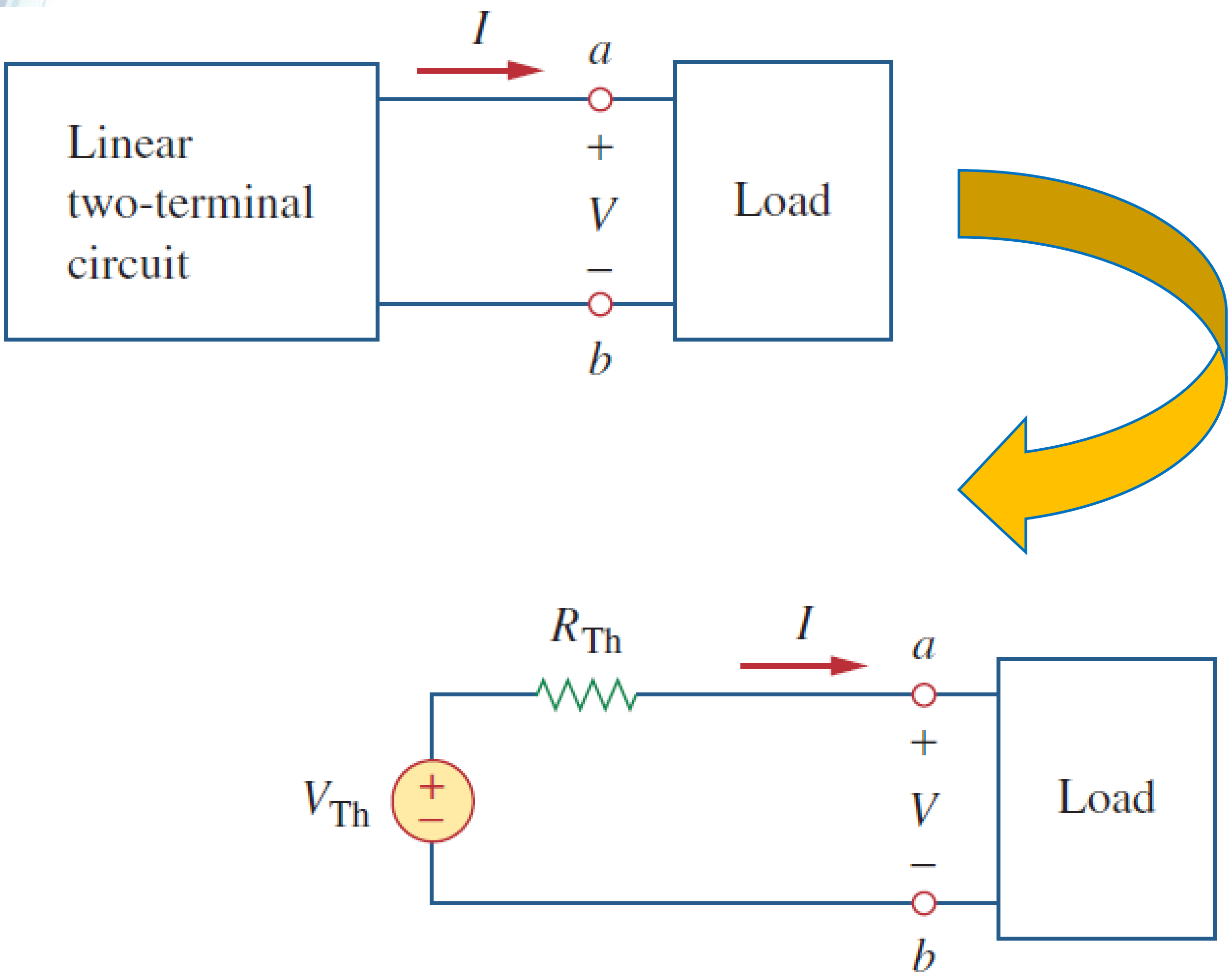
- Provides a technique by which the fixed part of the circuit replaced by the equivalent circuit.

- Thevenin's theorem: a linear 2-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{TH} in series with resistor R_{TH}.

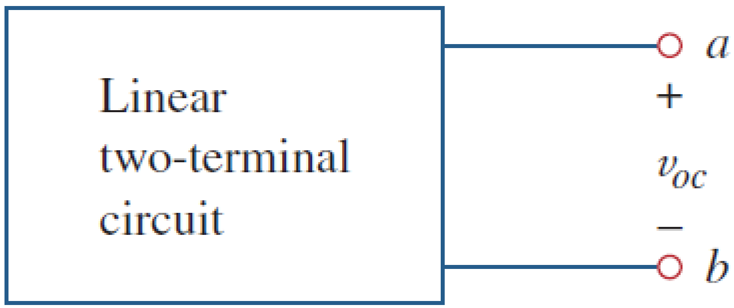
V_{TH} open-circuit voltage at terminal

R_{TH} input or equivalent resistance at the terminals when the independent source are turned off

Thevenin's Theorem



Thevenin's Theorem



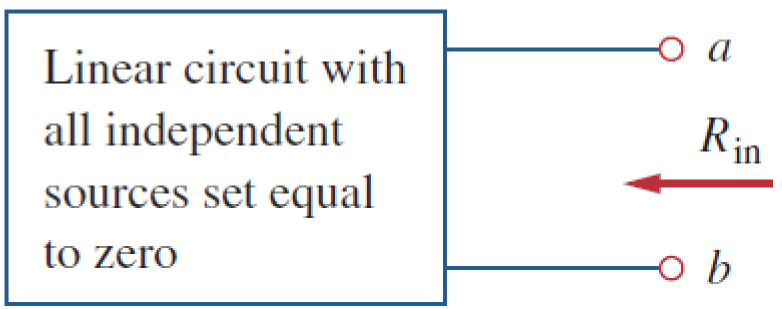
$V_{Th} = v_{oc}$

- By removing the load, there is no current flows, so that the open-circuit voltage across to terminal $a - b$
- The open-circuit voltage is equal to the V_{TH}

$$V_{TH} = v_{oc}$$

- With the load disconnected and we turn off all the independent sources the input/equivalent resistance must be equal to the R_{TH}

$$R_{TH} = R_{in}$$



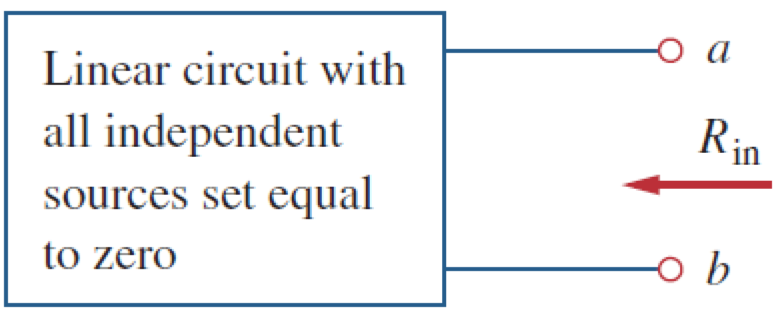
$R_{Th} = R_{in}$

Thevenin's Theorem

- To find the Thevenin resistance, R_{TH} we need to consider 2 cases.

- Case 1

If the network has no dependent sources, we turn off all independent sources. R_{TH} is the input resistance of the network looking between terminals a and b

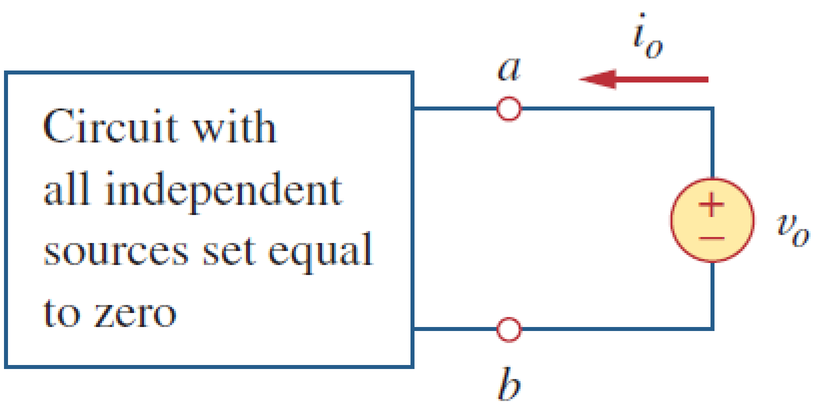


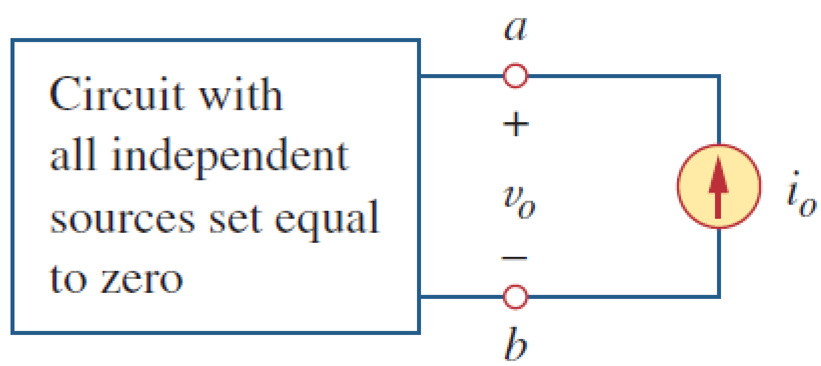
$R_{Th} = R_{in}$

$$R_{Th} = R_{in}$$

Case 2

If the network has dependent sources, we turn off all independent sources and need to apply v_o at terminal a and b and determine the resulting v_o

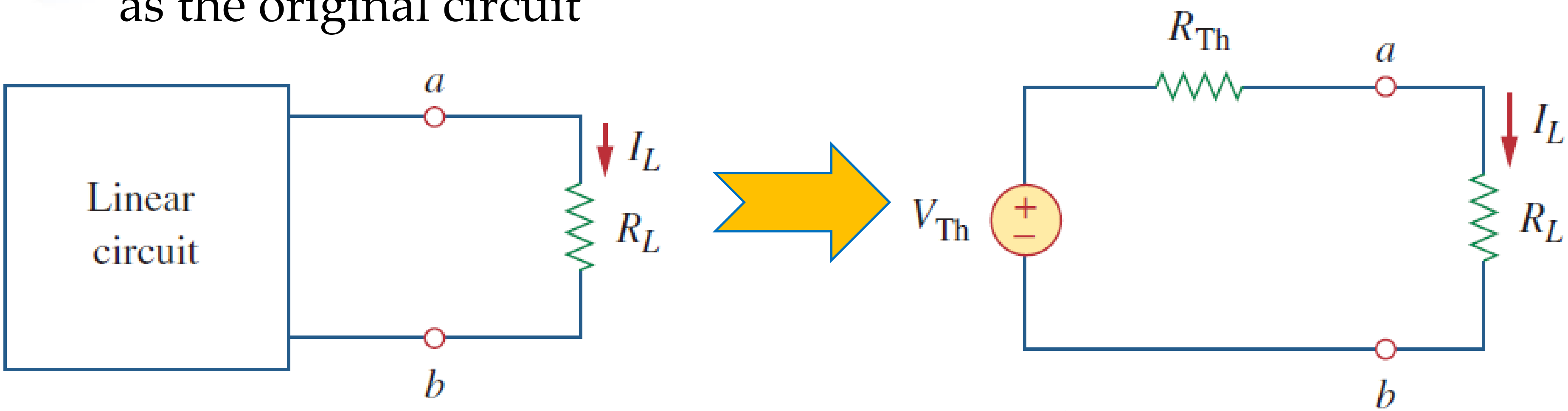




$$R_{Th} = \frac{v_o}{i_o}$$

Thevenin's Theorem

- A linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load.
- The equivalent network behaves the same way externally as the original circuit



$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \left(\frac{R_L}{R_{Th} + R_L} \right) V_{Th}$$

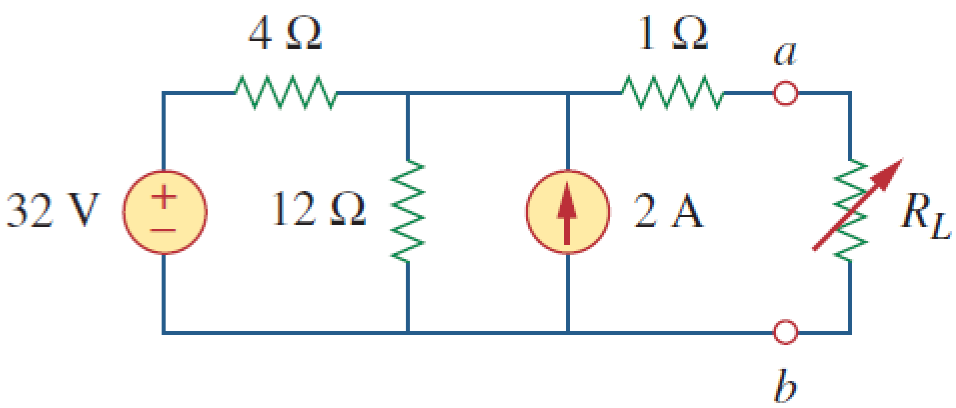
Thevenin's Theorem

- A linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load.

- The equivalent network behaves the same way externally as the original circuit

Example #8

Find the Thevenin equivalent circuit of the circuit shown in following figure to the left of the terminal $a - b$. then find the current through $R_L = 6, 16$ and $36\ \Omega$



Solution

To find R_{Th} turn off the voltage source (short circuit) and current source (open circuit).

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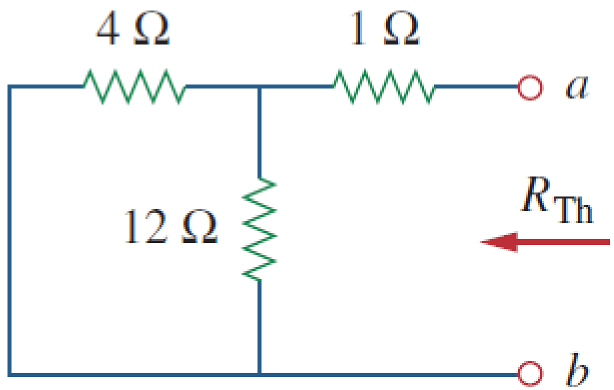
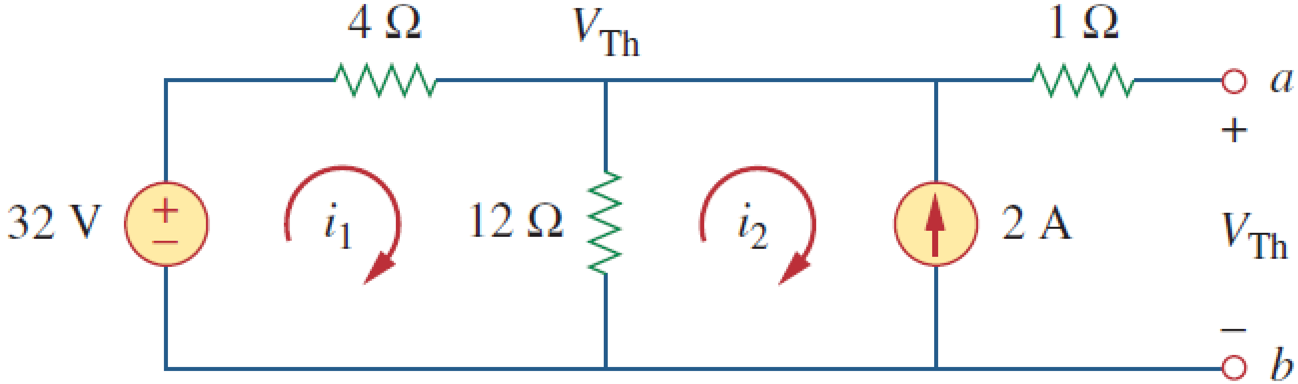
Example #8



$$R_{Th} = (4 \| 12) + 1 = \left(\frac{48}{16}\right) + 1 = 4\Omega$$



To find V_{Th} apply mesh analysis to the two loops

Example #8

Loop 1 $$-32+4i_1+12(i_1-i_2)=0$$

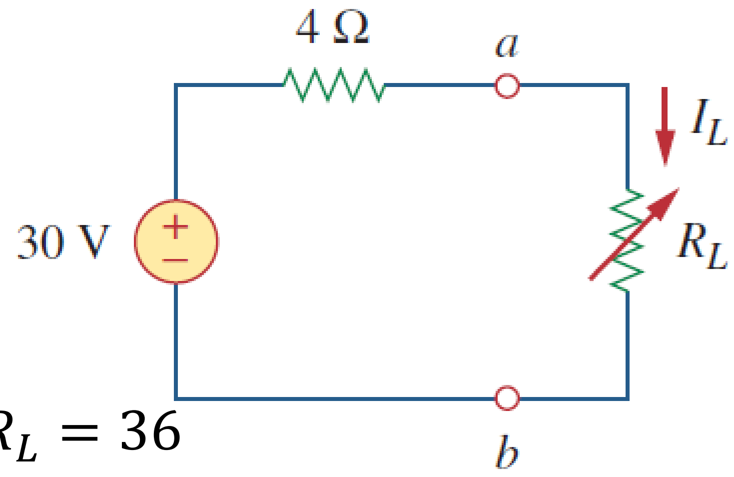
Loop 2 $$i_2=-2\text{ A}$$

Solving for i_1 $$i_1=0.5\text{ A}$$

Thus

$$V_{Th}=12(i_1-i_2)=12(0.5+2.0)=30\text{ V}$$

Then the current through R_L

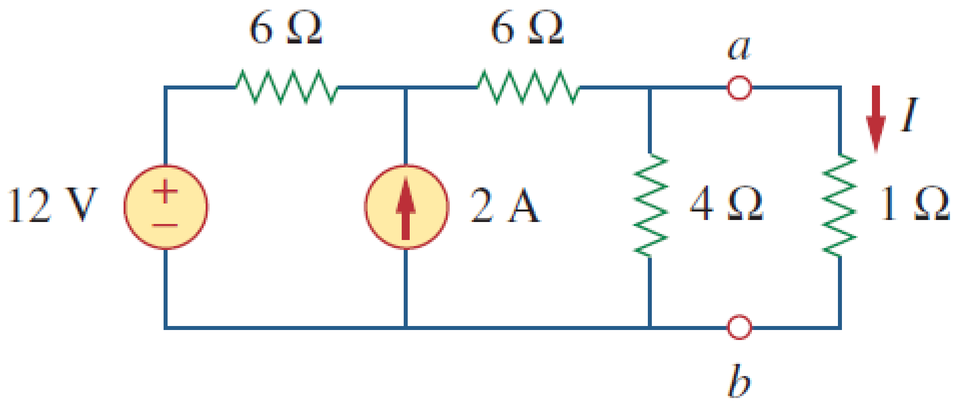


$$I_L=\frac{V_{Th}}{R_{Th}+R_L}=\frac{30}{4+R_L}$$

when $R_L = 6$	when $R_L = 16$	when $R_L = 36$
$I_L=3\text{ A}$	$I_L=1.5\text{ A}$	$I_L=0.75\text{ A}$

Example #9

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown. Then find *I*



Answer

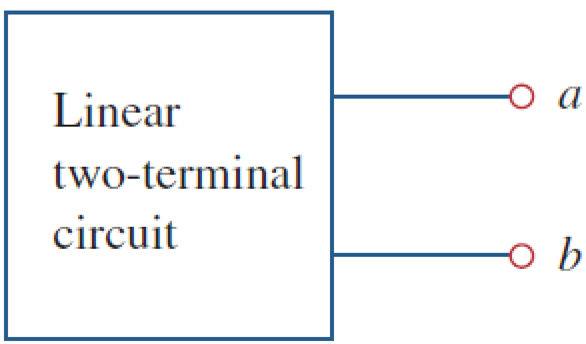
$$V_{Th} = 6\,\text{V}, \quad R_{Th} = 3\Omega, \quad I = 1.5\,\text{A}$$

Norton's Theorem

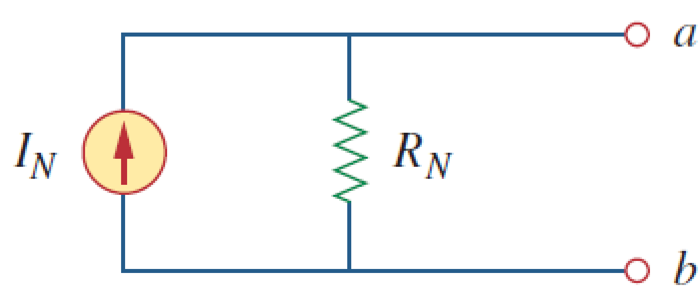
- **Norton's theorem**: a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in **parallel** with a resistor R_N.

I_N **short-circuit current** through the terminal

R_N input or **equivalent resistance** at the terminals when the **independent source are turned off**

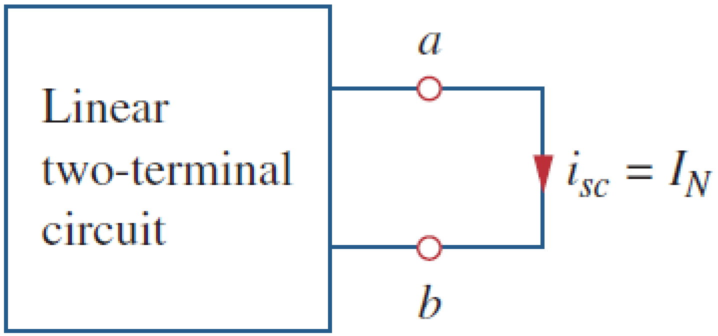


$$R_N = R_{Th}$$



Norton's Theorem

- To find the value of I_N we determine the short circuit current following from terminal *a* to *b*



$$I_N = i_{sc}$$

Since

$$R_N = R_{Th}$$

Then

$$I_N = \frac{V_{Th}}{R_{Th}}$$

Norton's Theorem

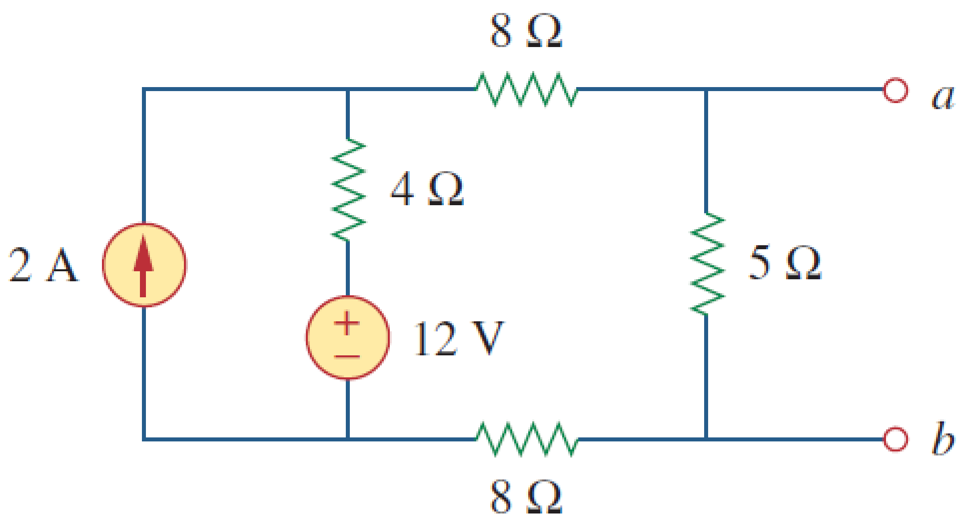
To determine the Thevenin or Norton equivalent circuit requires that we find:

- The open circuit voltage v_{oc} across terminal a and b.
- The short circuit current i_{sc} at terminal a and b.
- The equivalent or input resistance R_{in} at terminal a and b when all independent sources are turned off.

$$V_{Th} = v_{oc} \qquad I_N = i_{sc} \qquad R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N$$

Example #10

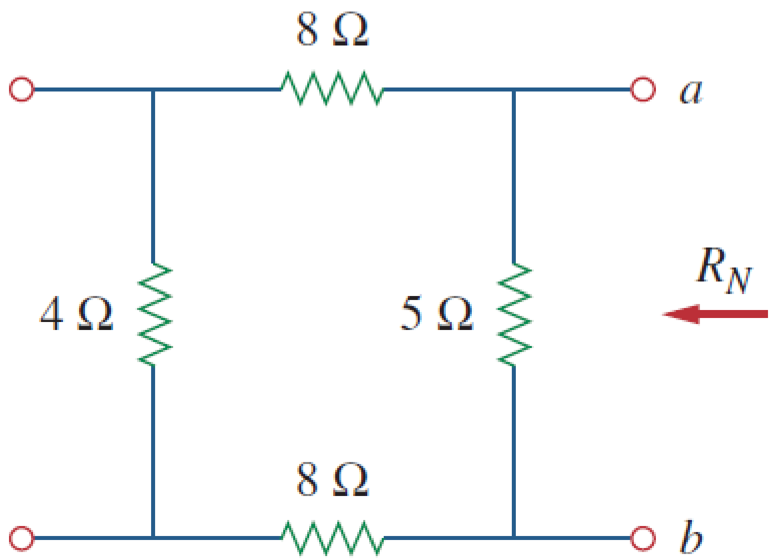
Find the Norton equivalent circuit of the circuit in figure shown at terminals $a - b$



Solution

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero.

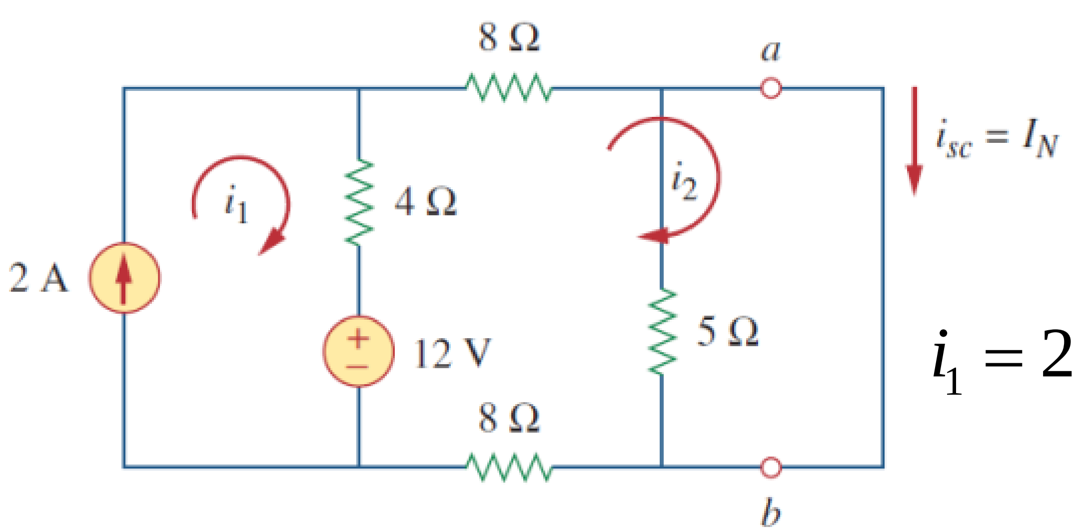
Example #10



$$R_N = 5 \parallel (8+4+8)$$

$$R_N = 5 \parallel 20 = 4\,\Omega$$

To find the value of I_N we short circuit terminals a to b



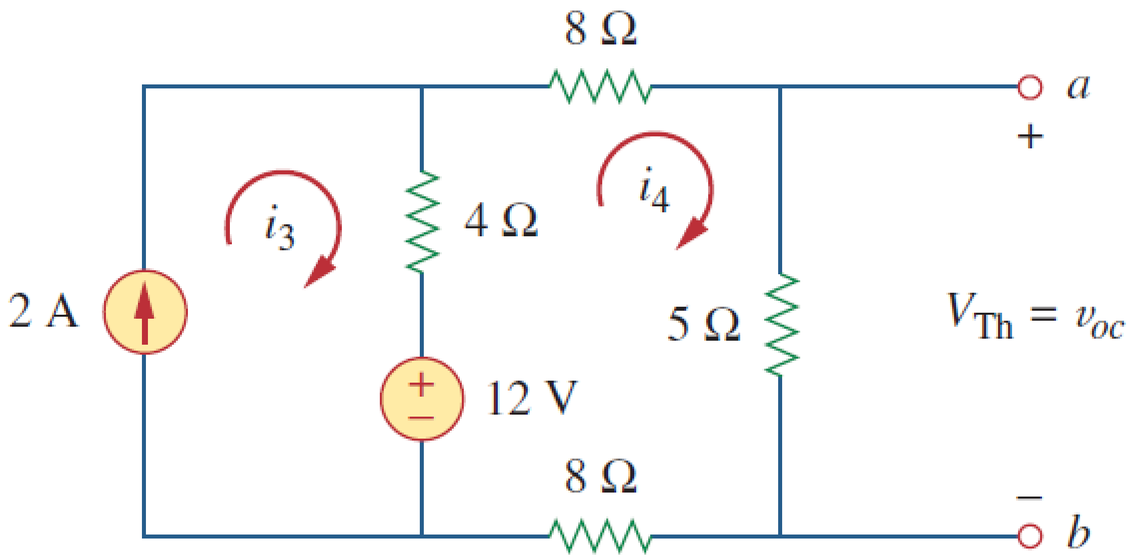
we ignored the 5 Ω resistor because it has been short circuit

$$i_1 = 2\text{ A} \qquad 20i_2 - 4i_1 - 12 = 0$$

$$i_2 = 1A = i_{sc} = I_N$$

Example #10

We obtain V_{Th} as the open circuit voltage across terminal $a - b$



By using mesh analysis

$$i_3 = 2\,\text{A}$$

$$25i_4 - 4i_3 - 12 = 0$$

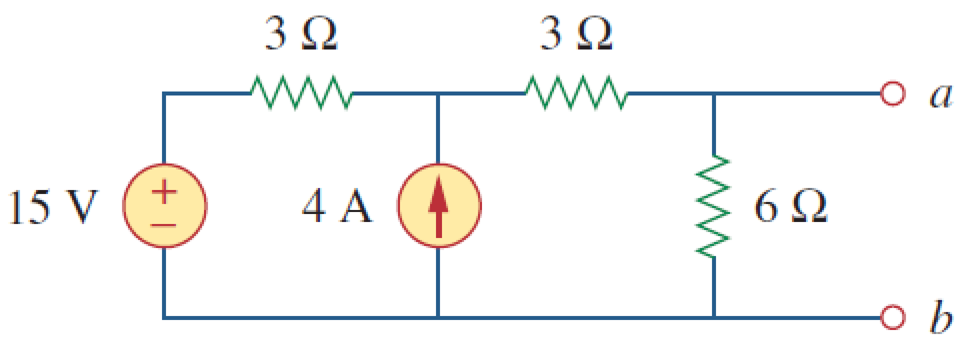
$$i_4 = 0.8\,\text{A}$$

$$v_{oc} = V_{Th} = 5i_4 = 4\,\text{V}$$

$$\therefore I_N = \frac{V_{Th}}{R_N} = 1\,\text{A}$$

Example #11

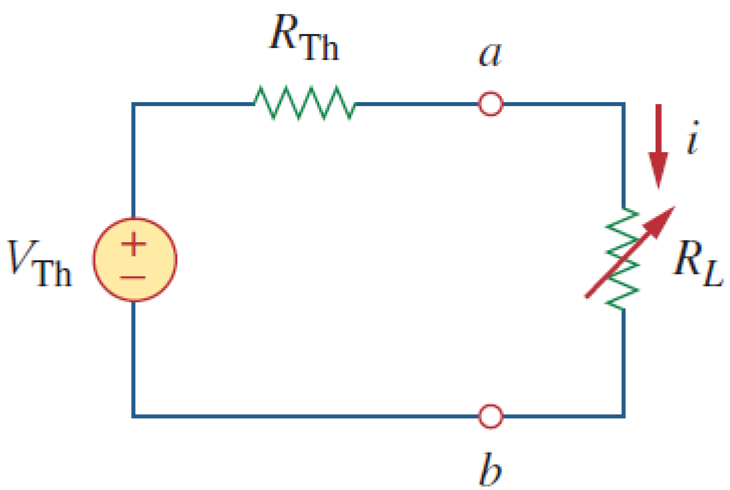
Find the Norton equivalent circuit of the circuit in figure shown at terminals $a - b$

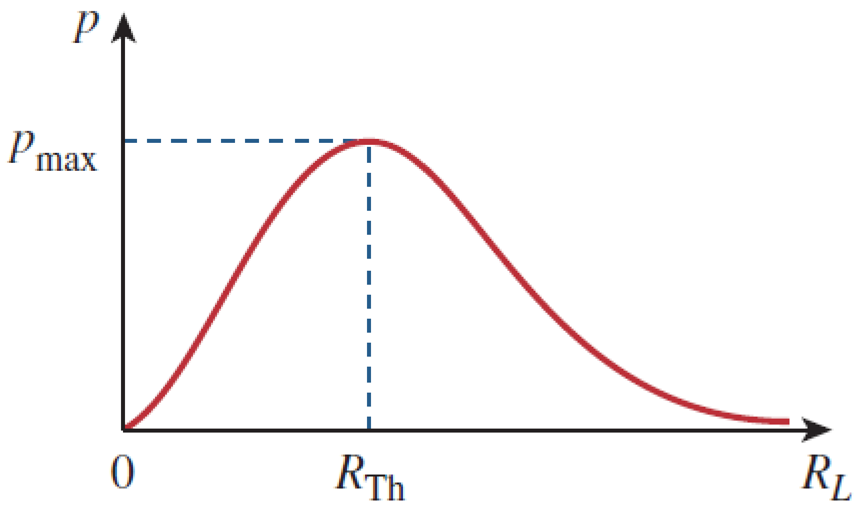


Answer

$$R_N = 3\Omega, \quad I_N = 4.5\,\text{A}$$

- In many practical situations, a circuit is designed to provide power to a load.
- There are applications in areas such as communications where it is desirable to maximize the power delivered to a load.
- The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load.





Maximum Power Transfer

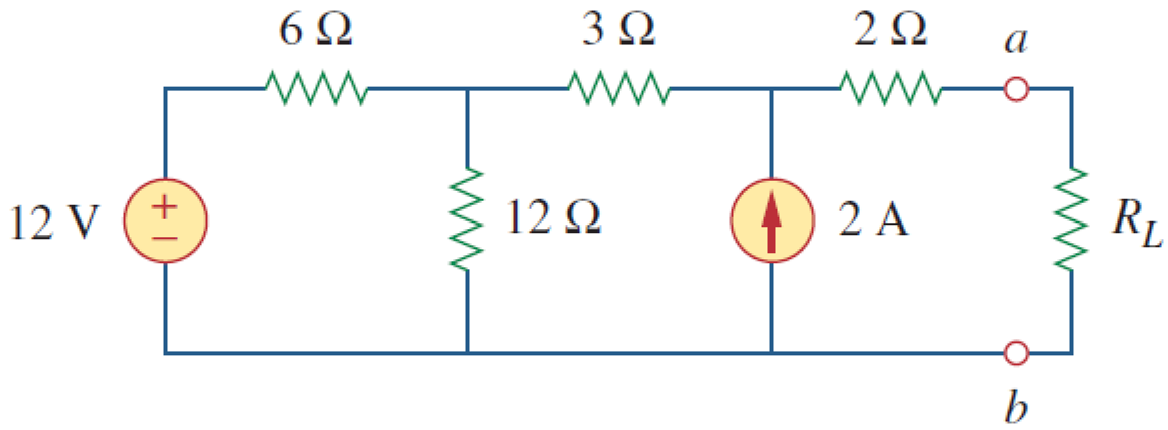
$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

Maximum Power is transferred to the load when the load resistance equal the Thevenin resistance as seen from the load

$$R_L = R_{Th} \qquad p_{\max} = \frac{V_{Th}^2}{4R_{Th}}$$

Example #12

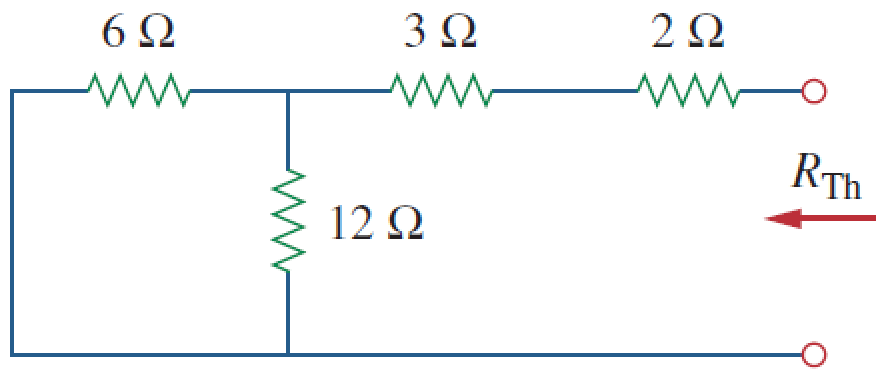
Find the value of R_L for the maximum power transfer for the circuit shown and give the maximum power transfer.



Solution

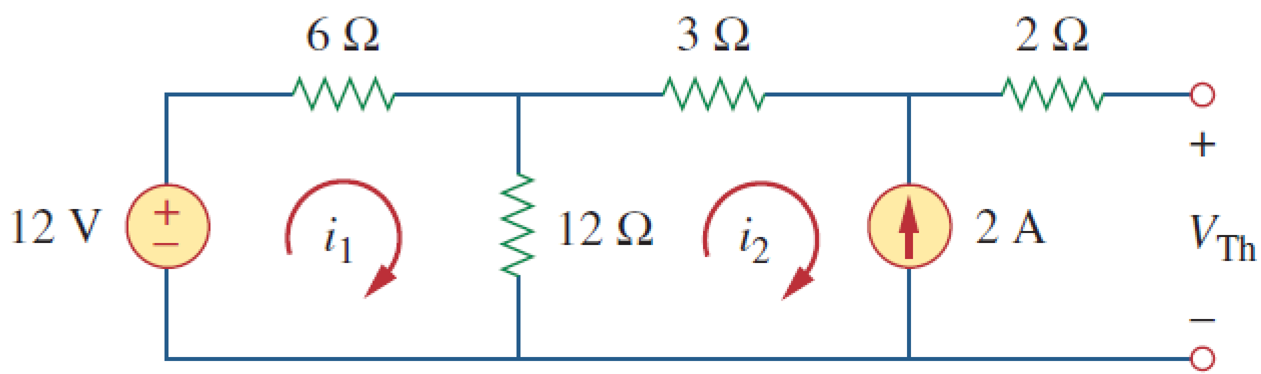
We find R_{Th} by set the independent sources equal to zero.

Example #12



$$R_{Th} = 2 + 3 + 6 \parallel 12 = 5 + \frac{72}{18}$$

$$R_{Th} = 9\,\Omega$$



Example #12

By using mesh analysis

$$-12+18i_1-12i_2=0 \qquad i_2=-2\,\text{A}$$

Then $i_1=-{}^{2}\!/_{3}\,\text{A}$

Applying KVL around the outer loop in order to find V_{Th}

$$-12+6i_1+3i_2+2(0)+V_{Th}=0$$

$$V_{Th}=22\,\text{V}$$

For maximum power transfer

$$R_L=R_{Th}=9\,\Omega$$

the maximum power transfer

$$p_{max}=\frac{V_{Th}^2}{4R_L}=\frac{(22)^2}{36}=13.44\,\text{W}$$

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