BFF1303: ELECTRICAL / ELECTRONICS ENGINEERING

Direct Current Circuits : Methods of Analysis

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Direct Current Circuit (DC)- Methods of Analysis

BFF1303 ELECTRICAL/ELECTRONICS ENGINEERING

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Contents:

- Outcomes
- Nodal Analysis
- Mesh Analysis

Outcomes

Compute the solution of circuits containing linear resistors and independent and dependent sources by using node analysis

Compute the solution of circuits containing linear resistors and independent and dependent sources by using mesh analysis

Nodal Analysis

- There is another method to solve for currents and voltages.
  - Easier
  - More methodical
  - Still based on Ohm's Law, KVL and KCL

- The methods are the nodal and mesh analysis.

- Nodal analysis is based on the KCL.

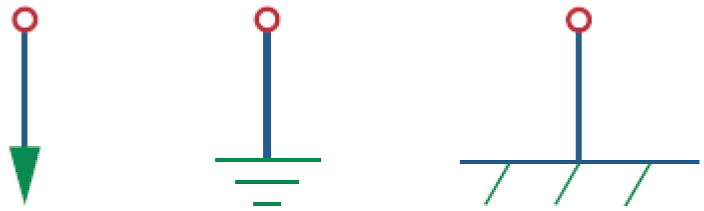
- Mesh analysis is based on the KVL.

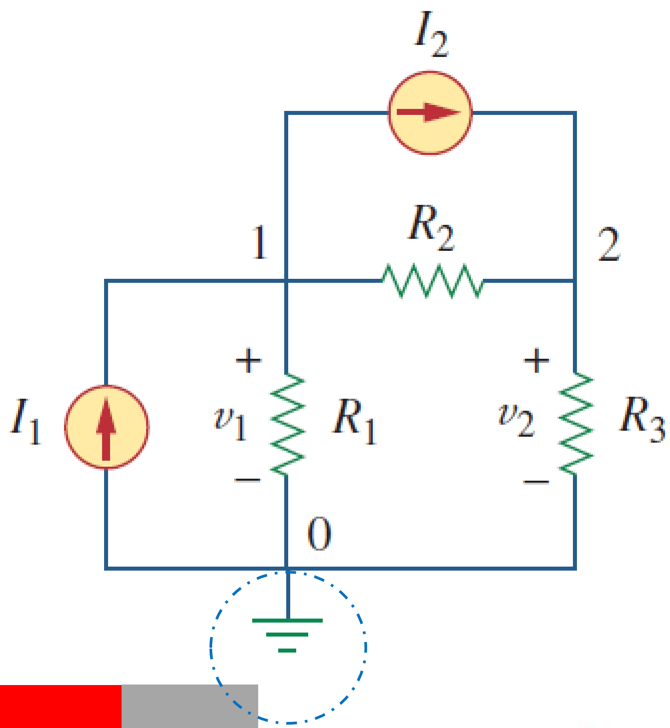
WITHOUT VOLTAGE SOURCE

- Analyze the circuit using node voltages as the circuit variables.
- The node voltages is chosen instead of the elements voltages.
- To simplify matters, it is first assumed that the circuits do not contain voltage sources.

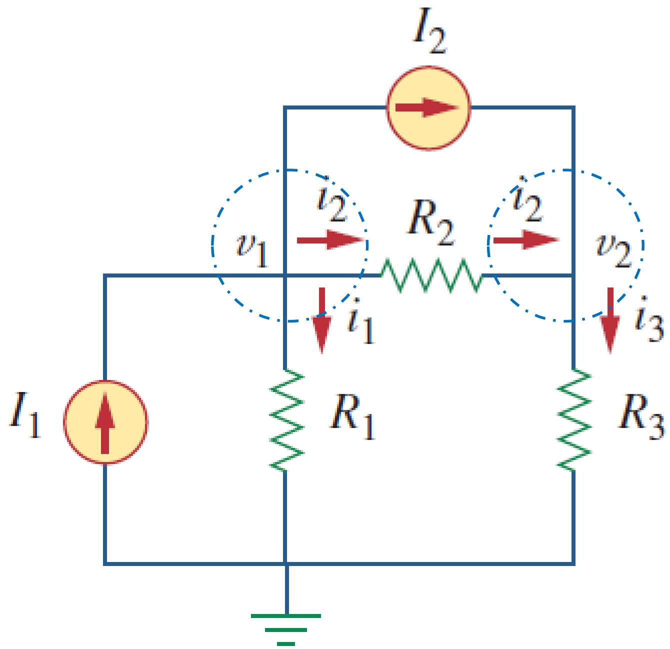
Steps to determine node voltages:

i. Select a node as a reference node – This reference node also called the ground, have zero potential.

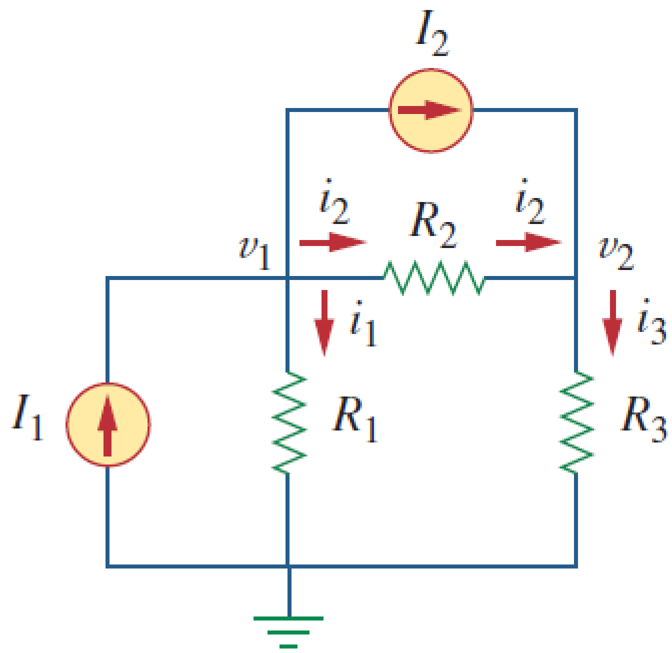




ii. The other node (nonreference nodes) will be assigned as $v_1, v_2, \cdots$



iii. Apply KCL to each of the nonreference node.



At node v_1:

$$I_1 = I_2 + i_1 + i_2$$

At node v_2:

$$i_3 = I_2 + i_2$$

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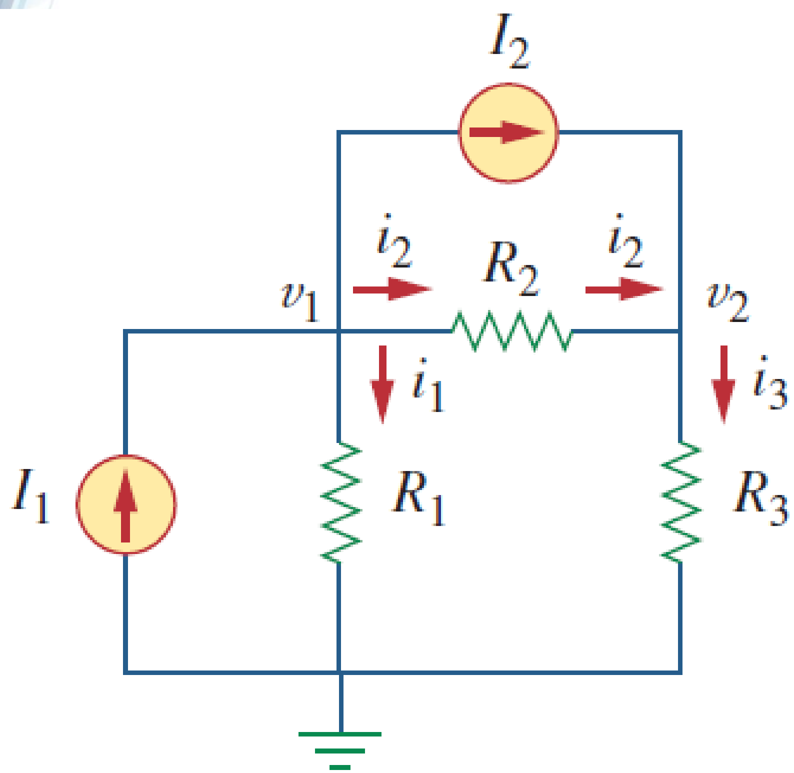
iv. Apply the Ohm's law to express the unknown current i_1, i_2 and i_3 in terms of node voltages.

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

Note: Current flows from a higher potential (positive terminal) to a lower potential (negative terminal) in resistor.

Nodal Analysis



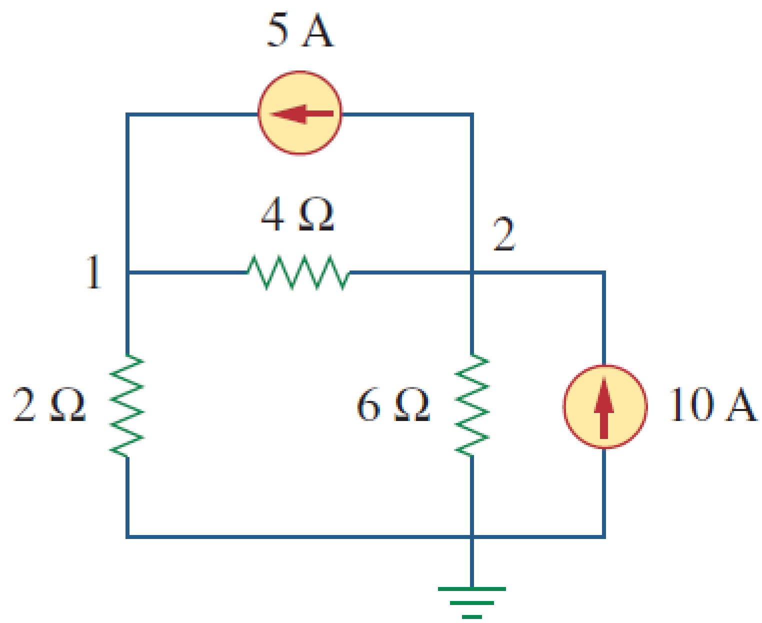
$$i_1 = \frac{v_1 - 0}{R_1} \qquad i_2 = \frac{v_1 - v_2}{R_2}$$

$$i_3 = \frac{v_2 - 0}{R_3}$$

$$I_1 = I_2 + \frac{v_1 - 0}{R_1} + \frac{v_1 - v_2}{R_2} \qquad \frac{v_2 - 0}{R_3} = I_2 + \frac{v_1 - v_2}{R_2}$$

Example #1

Obtain the node voltages for the following circuit



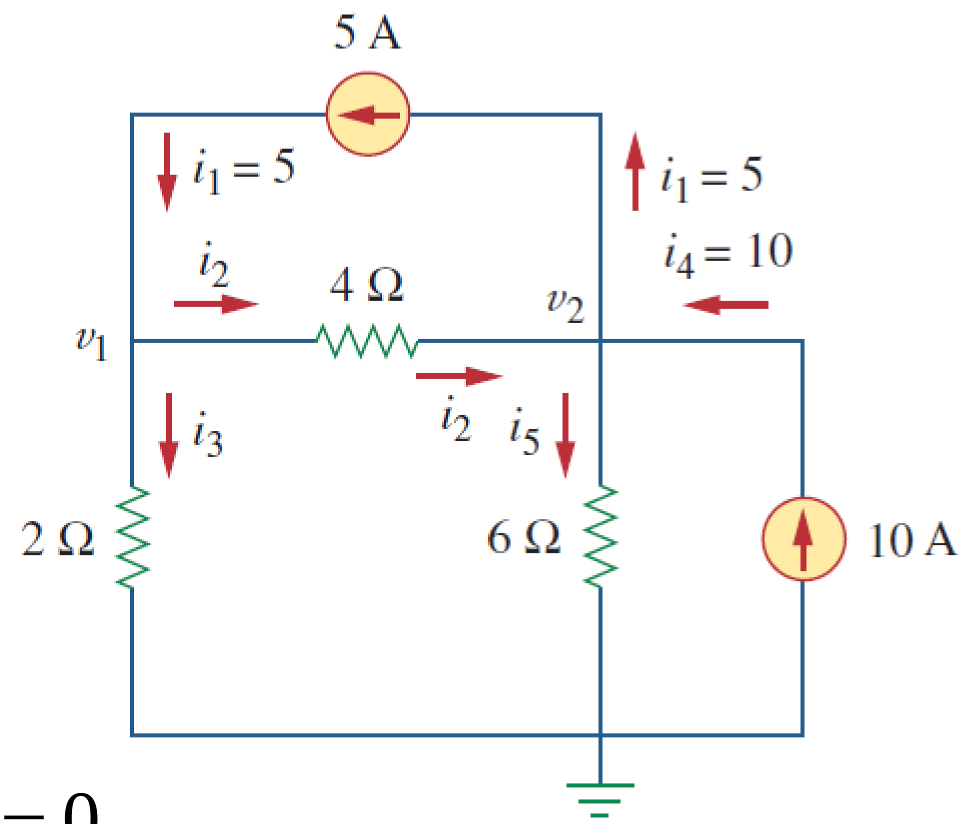
Example #1

Solution

i. Select a node as a reference node

ii. The other node (nonreference nodes) will be assigned as $v_1, v_2 \cdots$

iii. Apply KCL to each of the nonreference node.



At node v_1:

$$i_1 - i_2 - i_3 = 0$$

$$i_1 = i_2 + i_3$$

At node v_2:

$$i_2 + i_4 - i_1 - i_5 = 0$$

$$i_2 + i_4 = i_1 + i_5$$

Example #1

iv. Express the currents in term of node voltages.

$$i_2 = \frac{v_1 - v_2}{4} \qquad i_3 = \frac{v_1 - 0}{2} \qquad i_5 = \frac{v_2 - 0}{6}$$

$$5 = \frac{v_1 - v_2}{4} + \frac{v_1}{2} \qquad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2}{6}$$

Rearrange the equation

$$3v_1 - v_2 = 20$$

$$-3v_1 + 5v_2 = 60$$

Example #1

Then in matrix form

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

By using Cramer's Rule

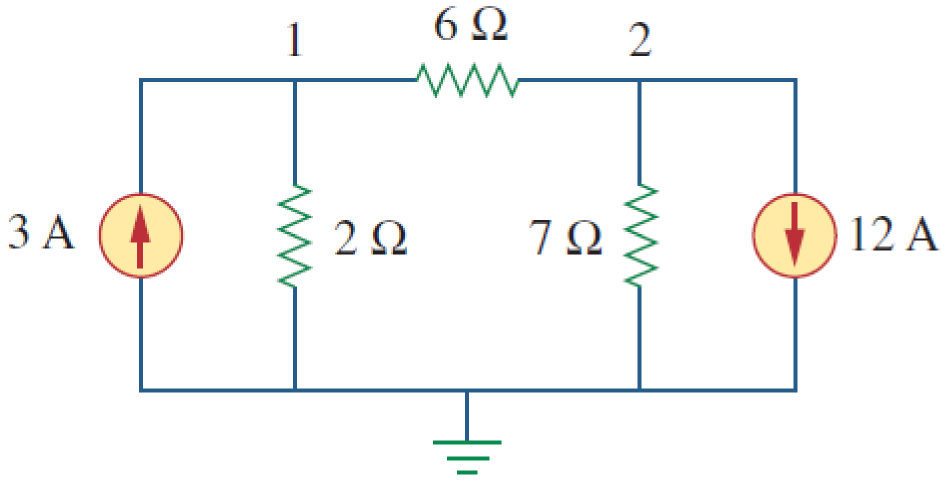
$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{bmatrix} 20 & -1 \\ 60 & 5 \end{bmatrix}}{\Delta} = \frac{100+60}{12} = 13.333\,\text{V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} 3 & 20 \\ -3 & 60 \end{bmatrix}}{\Delta} = \frac{180+60}{12} = 20\,\text{V}$$

Example #2

In the given circuit, obtain the node voltages

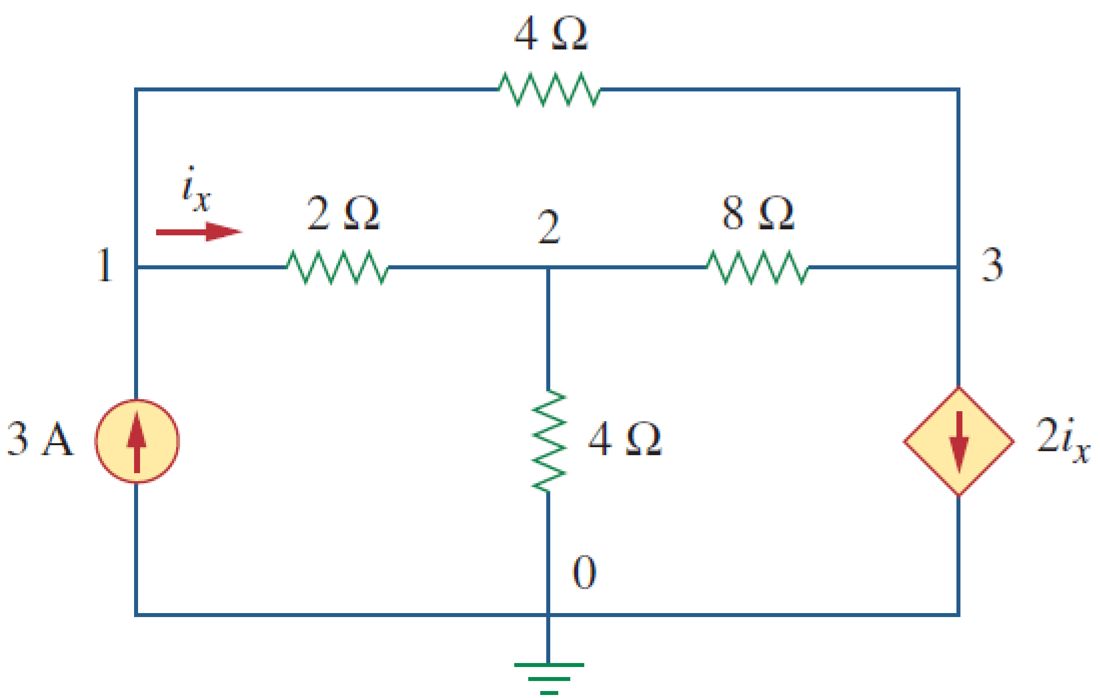


Answer

$$v_1 = -6\,\text{V} \qquad v_2 = -42\,\text{V}$$

Example #3

For the given circuit, determine the voltages at the nodes



Answer

$$v_1 = 4.8\,\text{V} \qquad v_2 = 2.4\,\text{V} \qquad v_3 = -2.4\,\text{V}$$

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Review of steps for nodal analysis without voltage sources:

i. Select a node as a reference node – This reference node also called the ground, have zero potential.

ii. The other node (nonreference nodes) will be assigned as $v_1, v_2, \cdots$

iii. Apply KCL to each of the nonreference node.

iv. Apply the Ohm's law to express the unknown current i_1, i_2 and i_3 in terms of node voltages.

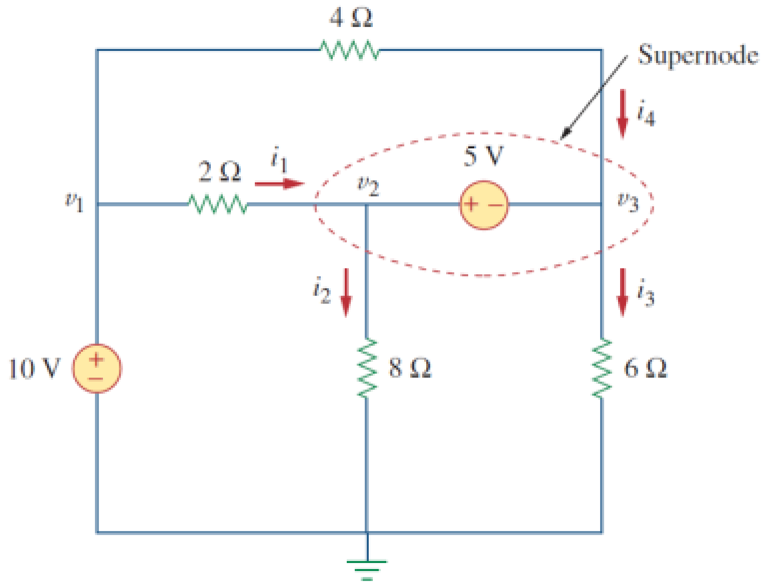
$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

WITH VOLTAGE SOURCE

- Case 1

If a voltage source is connected between the reference node and a nonreference node. The voltage at the nonreference node equal to the voltage source.

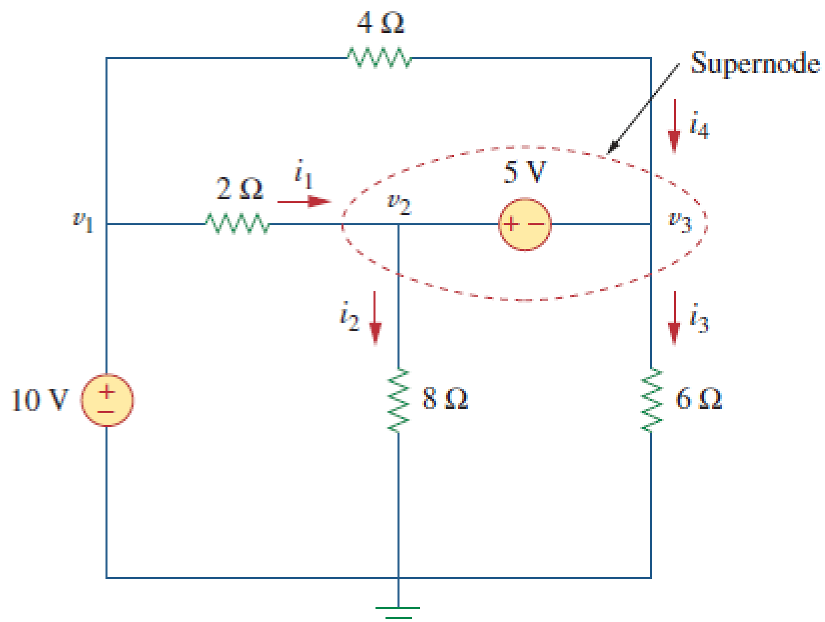
$$v_1 = 10\,\text{V}$$



Nodal Analysis

Case 2

If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or **supernode**.



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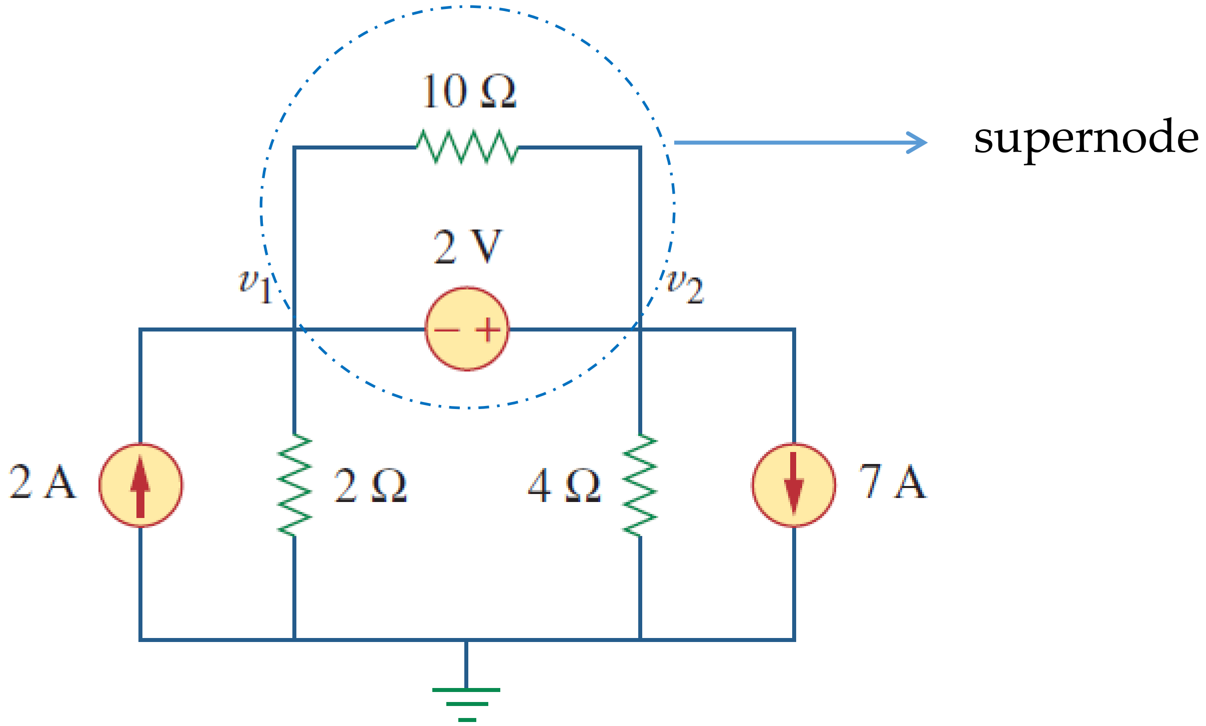
A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.



Nodal Analysis

- A supernode requires the applications of both KCL and KVL.
- A supernode has no voltage of its own.
- A supernode can be assumed as one big node.

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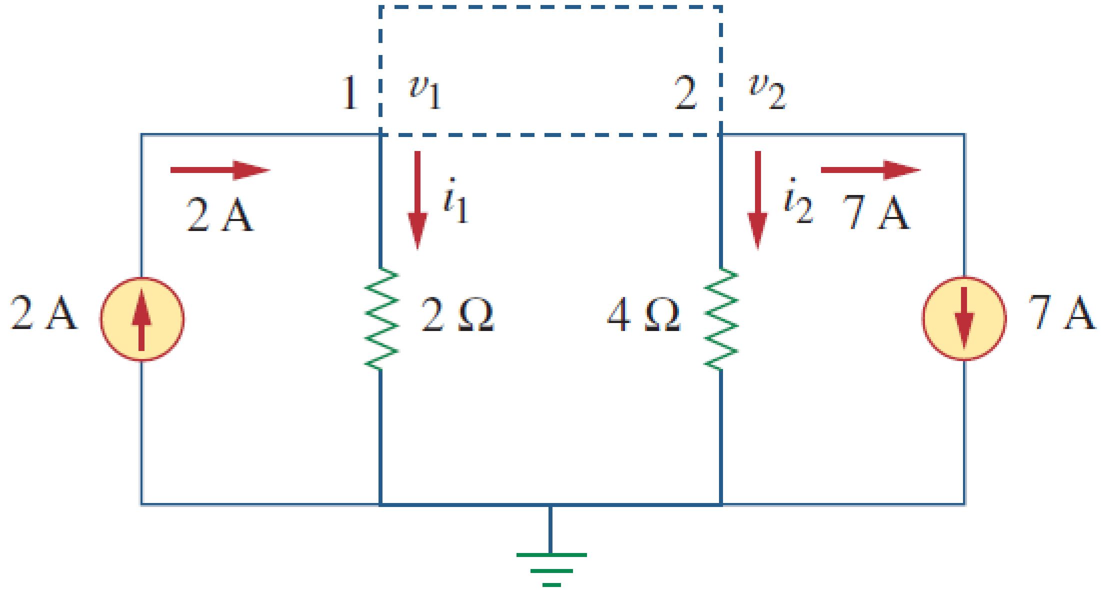
Steps for nodal analysis with voltage source:

i. Select a node as a reference node – This reference node also called the ground, have zero potential.

ii. Assign the other nodes (nonreference nodes) as $v_1, v_2, \cdots$ and identify the supernode.

iii. Apply KCL to each of the nonreference node and the supernode.

Note: For supernode, apply the KCL around the supernode. Assume the supernode as one big node

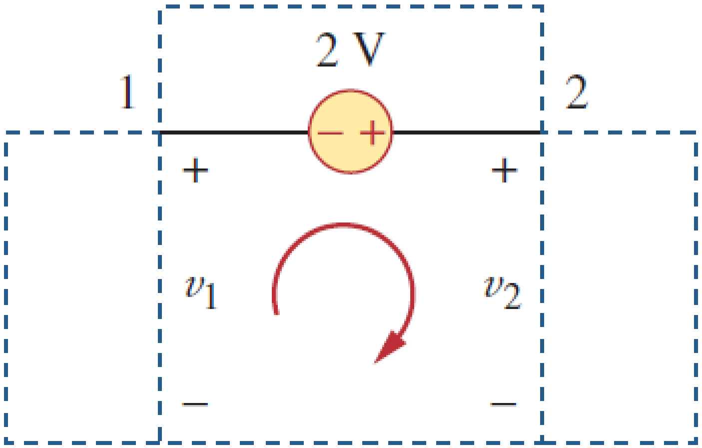


Nodal Analysis

iv. Express the currents in term of node voltages.

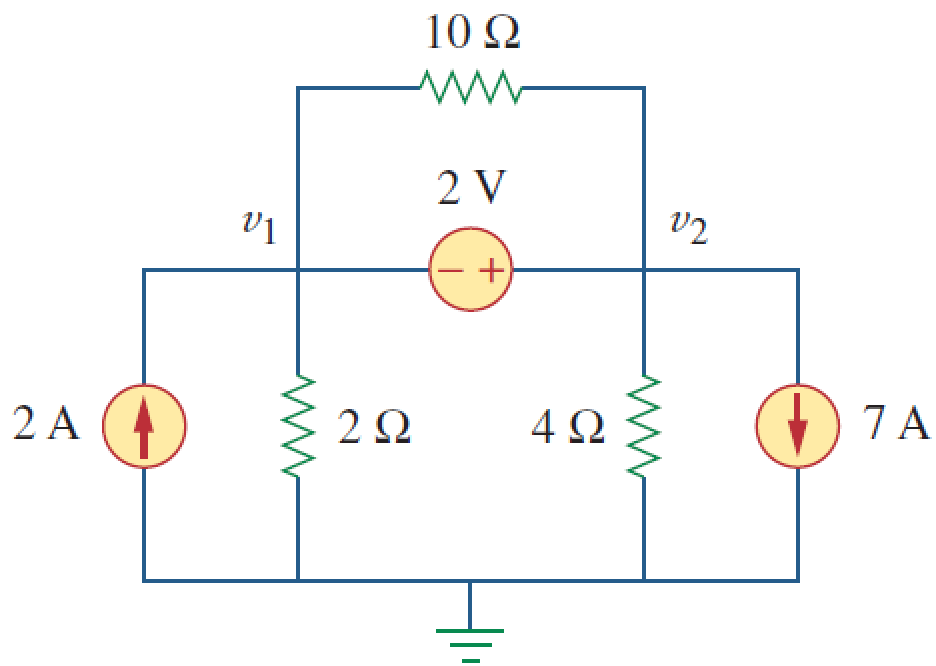
$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

v. Apply KVL around the supernode.



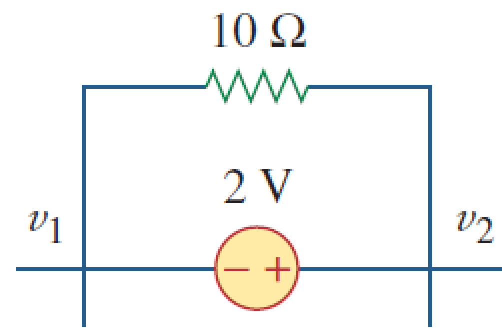
Example #4

Obtain the node voltages for the following circuit



Example #4

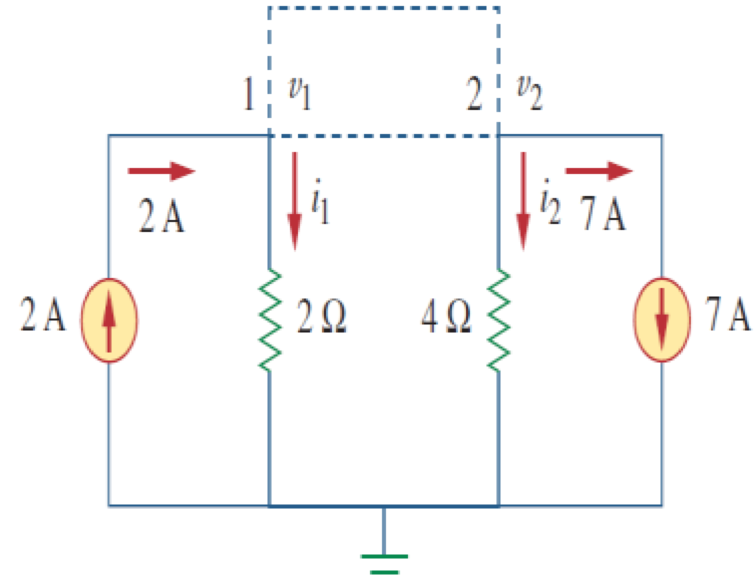
Solution



i. Select a node as a reference node

ii. Assign the other node (nonreference nodes) as $v_1, v_2 \cdots$ and identify the supernode

iii. Apply KCL to each of the nonreference node and the supernode.



At node v_1@1:

$$i_1 - 2 = 0$$

$$i_1 = 2\text{ A}$$

At node v_2@2:

$$i_2 + 7 = 0$$

$$i_2 = -7\text{ A}$$

At supernode

$$2 - i_1 - i_2 - 7 = 0$$

$$2 = i_1 + i_2 + 7$$

Example #4

iv. Express the currents in term of node voltages.

$$i_1 = \frac{v_1 - 0}{2} \qquad i_2 = \frac{v_2 - 0}{4} \qquad 2 = \frac{v_1}{2} + \frac{v_2}{4} + 7$$

Rearrange the equation

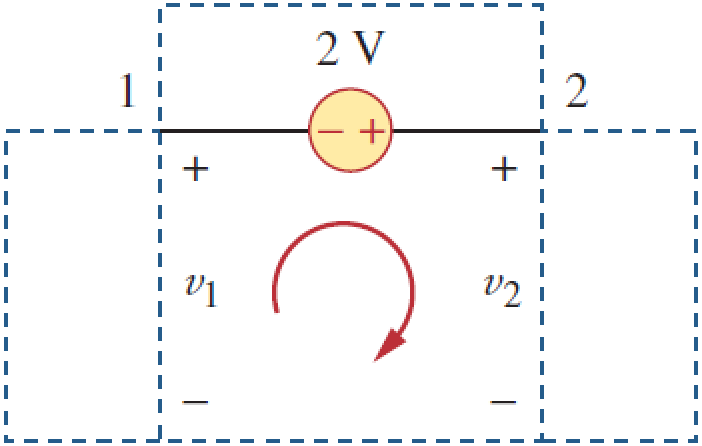
$$8 = 2v_1 + v_2 + 28$$

$$v_2 = -20 - 2v_1$$

v. Apply KVL around the supernode.

$$-v_1 - 2 + v_2 = 0$$

$$v_2 = v_1 + 2$$



Example #4

Then in matrix form

$$\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -20 \\ 2 \end{bmatrix}$$

By using Cramer's Rule

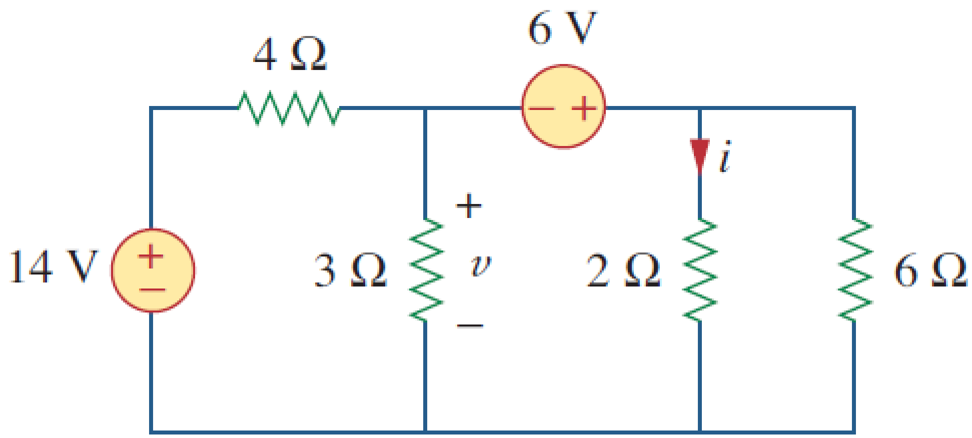
$$\Delta = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 2 + 1 = 3$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{bmatrix} -20 & 1 \\ 2 & 1 \end{bmatrix}}{\Delta} = \frac{-20 - 2}{3} = -7.333\,\text{V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} 2 & -20 \\ -1 & 2 \end{bmatrix}}{\Delta} = \frac{4 - 20}{3} = -5.333\,\text{V}$$

Example #5

Find *v* and *i* for the given circuit

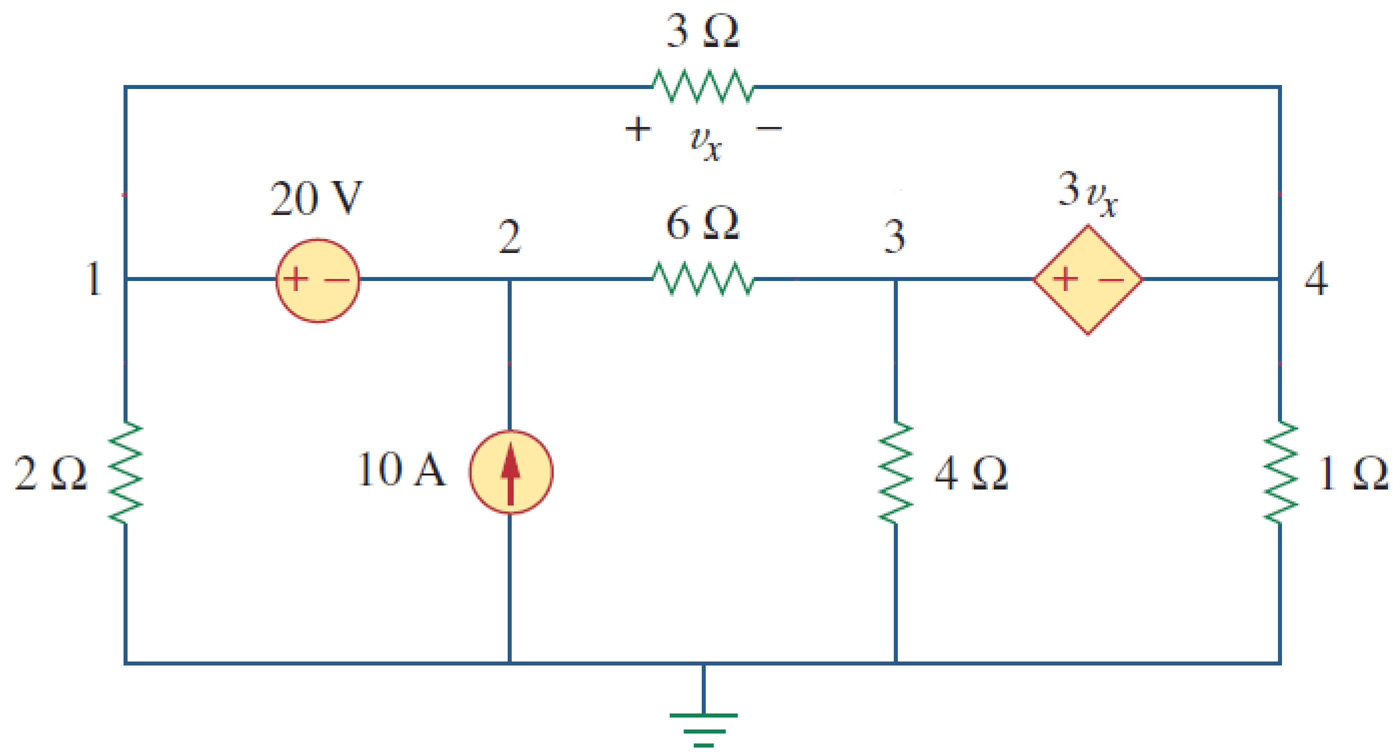


Answer

$$v = -400\,\text{mV} \qquad i = 2.8\,\text{A}$$

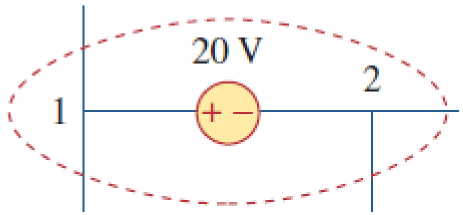
Example #6

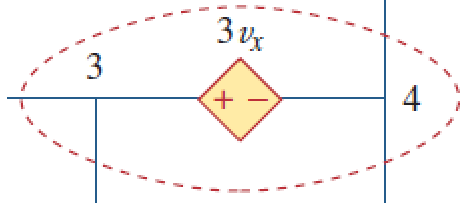
Obtain the node voltages for the following circuit



Example #6

Solution

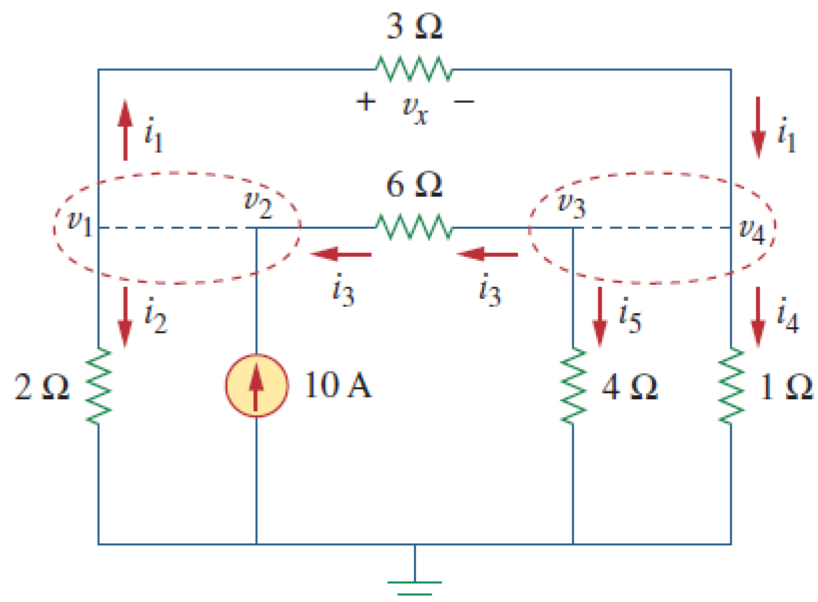




i. Select a node as a reference node

ii. Assign the other node (nonreference nodes) as $v_1, v_2 \cdots$ and identify the supernode

iii. Apply KCL to each of the nonreference node and the supernode.



At supernode 1-2:

$$i_3 + 10 - i_1 - i_2 = 0$$

$$i_3 + 10 = i_1 + i_2$$

At supernode 3-4:

$$i_1 - i_3 - i_4 - i_5 = 0$$

$$i_1 = i_3 + i_4 + i_5$$

Example #6

iv. Express the currents in term of node voltages.

$$i_1 = \frac{v_1 - v_4}{3} \qquad i_2 = \frac{v_1 - 0}{2} \qquad i_3 = \frac{v_3 - v_2}{6}$$

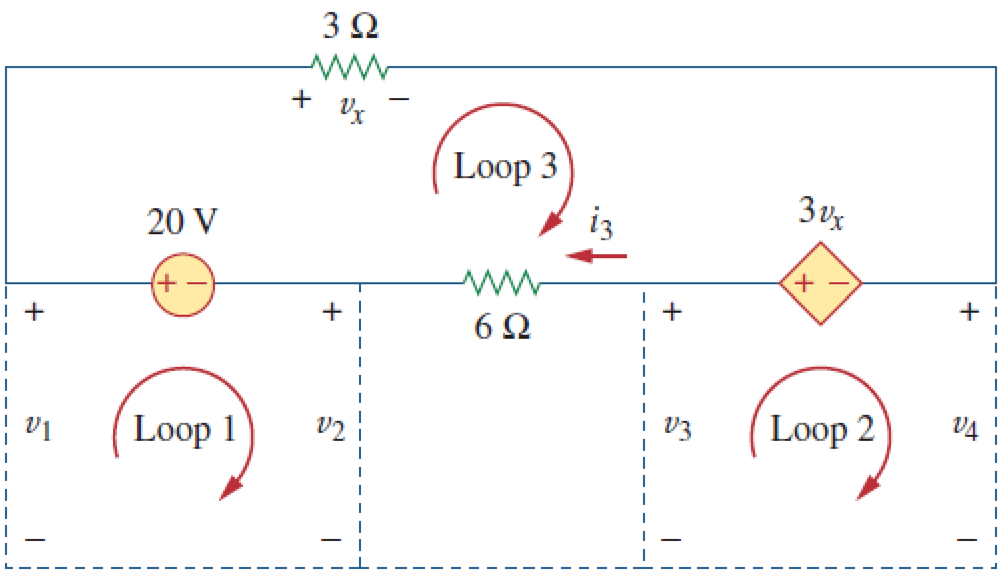
$$i_4 = \frac{v_4 - 0}{1} \qquad i_5 = \frac{v_3 - 0}{4}$$

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2} \qquad \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

$$5v_1 + v_2 - v_3 - 2v_4 = 60 \qquad 4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$$

Example #6

v. Apply KVL around the supernode.



At loop 1

$$-v_1 + 20 + v_2 = 0$$

$$v_1 - v_2 = 20$$

At loop 2

$$-v_3 + 3v_x + v_4 = 0$$

and $v_x = v_1 - v_4$

$$3v_1 - v_3 - 2v_4 = 0$$

At loop 3

$$v_x - 3v_x + 6i_3 - 20 = 0$$

$$-2v_1 - v_2 + v_3 + 2v_4 = 20$$

and

$$v_x = v_1 - v_4$$

$$6i_3 = v_3 - v_2$$

Example #6

From loop 1 $v_2 = v_1 - 20$

and subs this Eq. into

$$5v_1 + v_2 - v_3 - 2v_4 = 60$$

and

$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$$

$$6v_1 - v_3 - 2v_4 = 80$$

$$6v_1 - 5v_3 - 16v_4 = 40$$

Then in matrix form

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

Example #6

By using Cramer's Rule

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} = -18$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{bmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{bmatrix}}{\Delta} = \frac{-480}{-18} = 26.667\,\text{V}$$

Example #6

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{bmatrix} 3 & -0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{bmatrix}}{\Delta} = \frac{-3120}{-18} = 173.333\,\text{V}$$

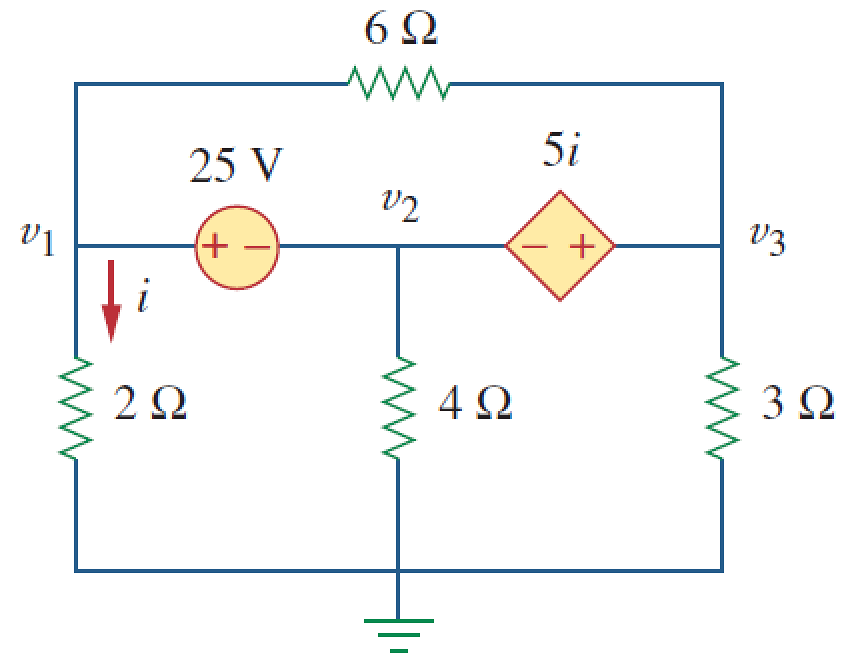
$$v_4 = \frac{\Delta_4}{\Delta} = \frac{\begin{bmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{bmatrix}}{\Delta} = \frac{840}{-18} = -46.667\,\text{V}$$

$$v_2 = v_1 - 20$$

$$v_2 = 26.667 - 20 = 6.667\,\text{V}$$

Example #7

By using nodal analysis obtain the node voltages for the following circuit



Answer

$v_1 = 7.608\,\text{V}$ $\quad$ $v_2 = -17.39\,\text{V}$ $\quad$ $v_3 = 1.6305\,\text{V}$

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WITHOUT CURRENT SOURCE

- Mesh analysis use the mesh currents as the circuit variables.
- Recall: nodal analysis use the node voltages as the circuit variables.
- Recall: a loop is a closed path with no node passed more than once.
- A mesh is a loop that does not contain any other loop within it.

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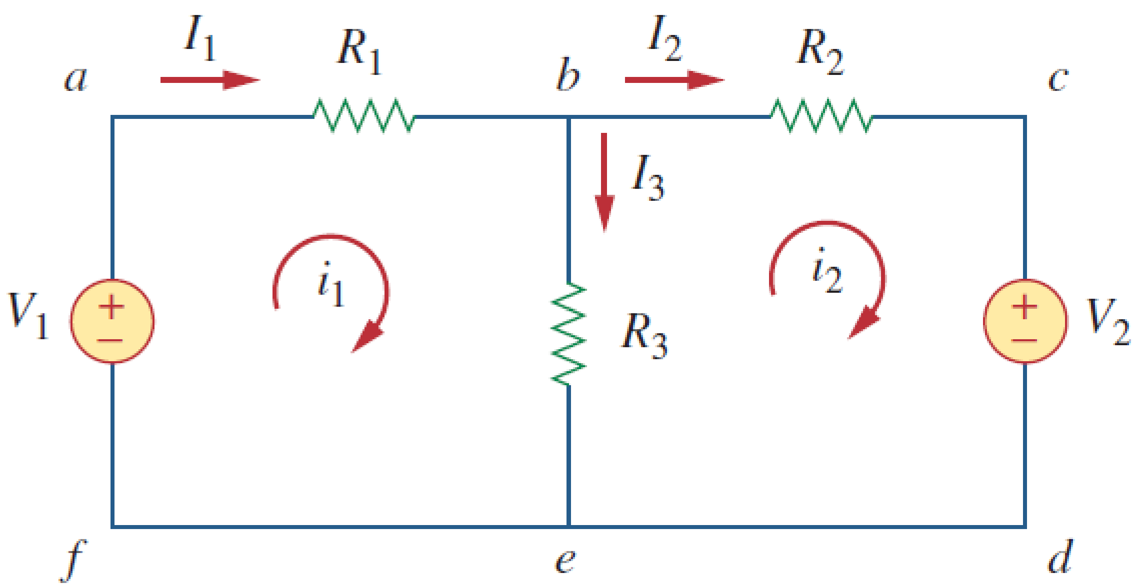
- Recall: nodal analysis applies **KCL** to find unknown voltages.
- Mesh analysis applies **KVL** to find unknown currents.



abefa & bcdeb are meshes.

abcdefa is not a mesh.

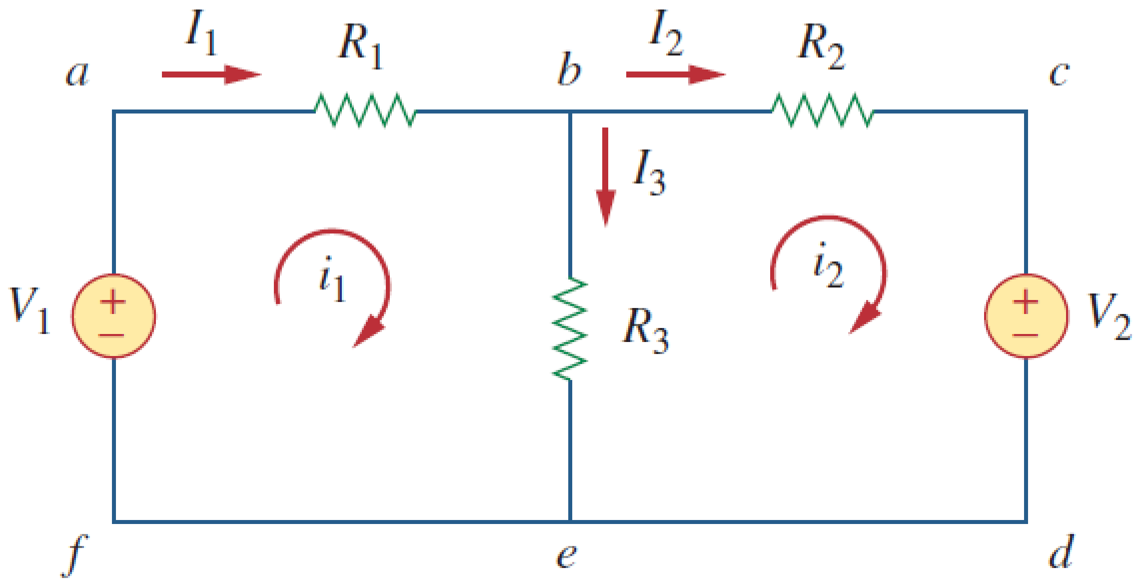
Mesh Analysis

- The current through a mesh is known as mesh current.

- First, the analysis will concentrate on circuit with no current source.

- Steps to determine mesh currents:
  i. Assign mesh currents $i_1, i_2, \cdots, i_n$ to the n meshes. The flow of the mesh currents is conventionally assume in clockwise direction.
  ii. Determine the polarity & the direction of current flow through each element.
  iii. Apply KCL for each node in term of mesh current.
  iv. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

Mesh Analysis



i. Assign the mesh currents: i_1 and i_2

ii. Determine the polarity & the direction of current.

Note: let the direction of current at the outside branch = direction of the mesh current

iii. Apply KCL.

$$I_3 = I_1 - I_2$$

since $I_1 = i_1$

$I_2 = i_2$

$$I_3 = i_1 - i_2$$

Note: I for branch current, i for mesh current

iv. Apply KVL to each mesh.

Mesh 1 $-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$

$$3i_1 - 2i_2 = 1 \qquad \rightarrow (1)$$

Mesh Analysis

Mesh 2

$$6i_2 + 4i_2 - 10(i_1 - i_2) - 10 = 0$$

$$i_1 - 2i_2 = -1 \qquad \rightarrow (2)$$

Then in matrix form

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

By using Cramer's Rule

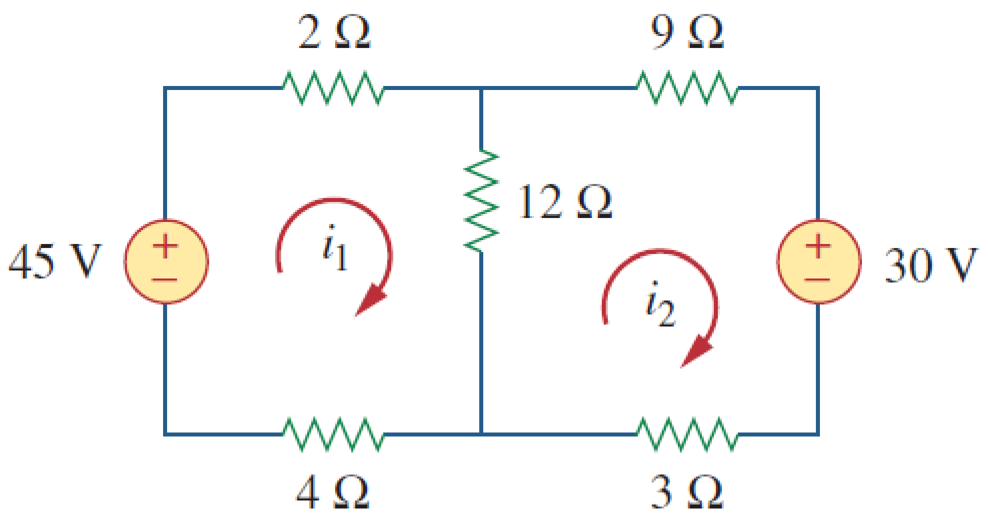
$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}}{\Delta} = \frac{2+2}{4} = 1\text{A}$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}}{\Delta} = \frac{3+1}{3} = 1\text{A}$$

Example #8

Calculate the mesh current i_1 and i_2 for the following circuit

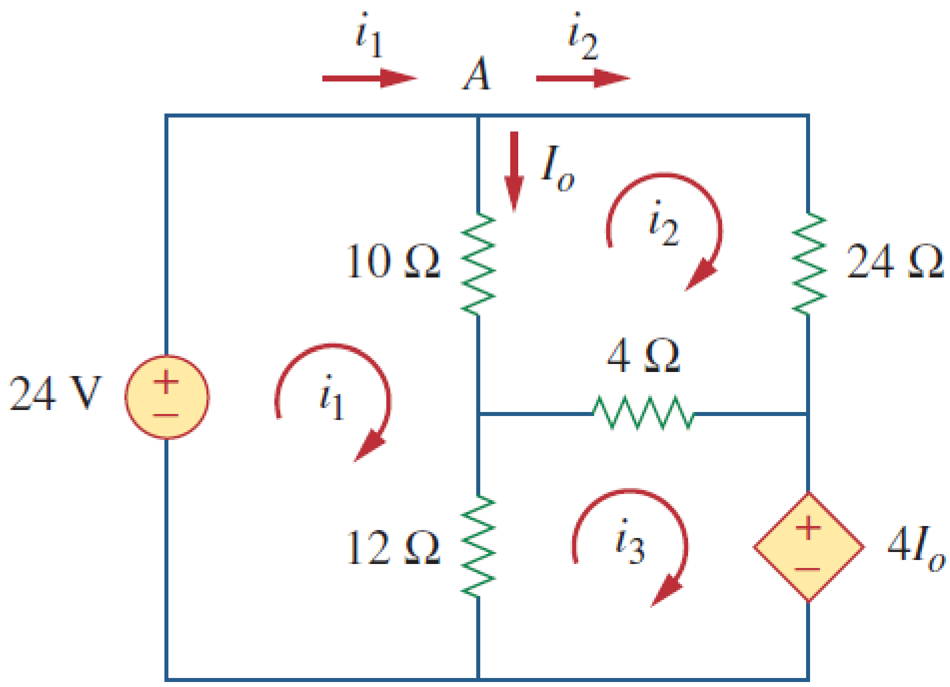


Answer

$$i_1 = 2.5\,\text{A} \qquad i_2 = 0\,\text{A}$$

Example #9

Use mesh analysis to find I_o for the following circuit



Mesh Analysis

i. Assign the mesh currents: i_1 and i_2.

ii. Determine the polarity & the direction of current.

iii. Apply KCL.

$$I_o = i_1 - i_2$$

iv. Apply KVL to each mesh.

Mesh 1

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad \rightarrow (1)$$

Mesh Analysis

Mesh 2

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad \rightarrow (2)$$

Mesh 3

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

$$-i_1 - i_2 + 2i_3 = 0 \qquad \rightarrow (3)$$

Then in matrix form

Mesh Analysis

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

By using Cramer's Rule

$$\Delta = 192$$

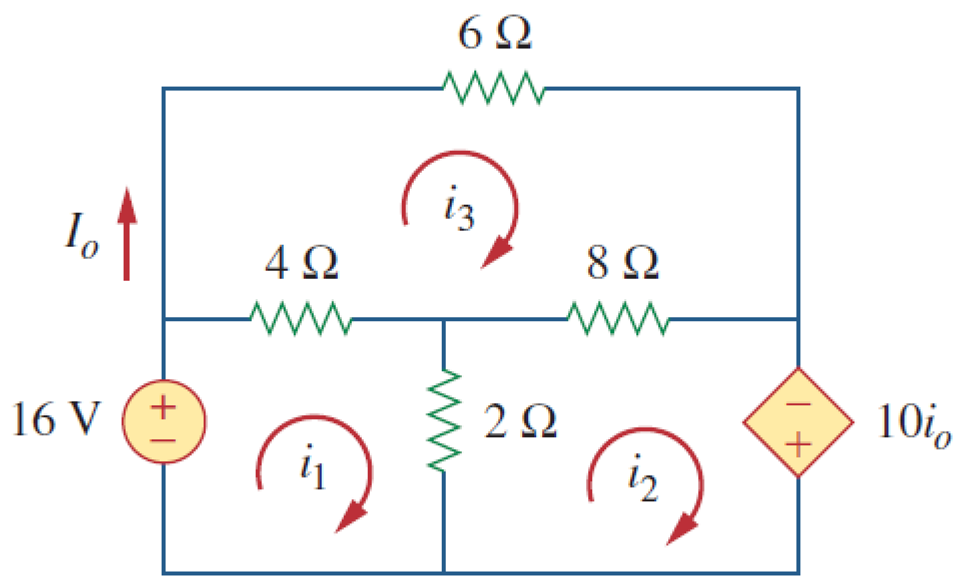
$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25\,\text{A} \quad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75\,\text{A}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5\,\text{A}$$

Thus, $I_o = i_1 - i_2 = 1.5\,\text{A}$

Example #10

By using mesh analysis find I_o in the following circuit



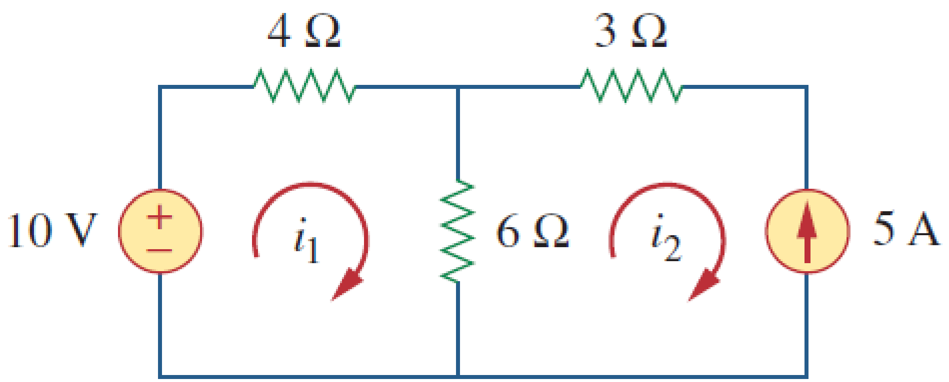
Answer

$$I_o = -4\,\text{A}$$

WITH CURRENT SOURCE

- Case 1

When the current source exist only in **one mesh**



Set the value of $i_2 = -5$ A and write a mesh equation for the other mesh in the usual way.

$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \quad \Rightarrow \quad i_1 = -2\text{ A}$$

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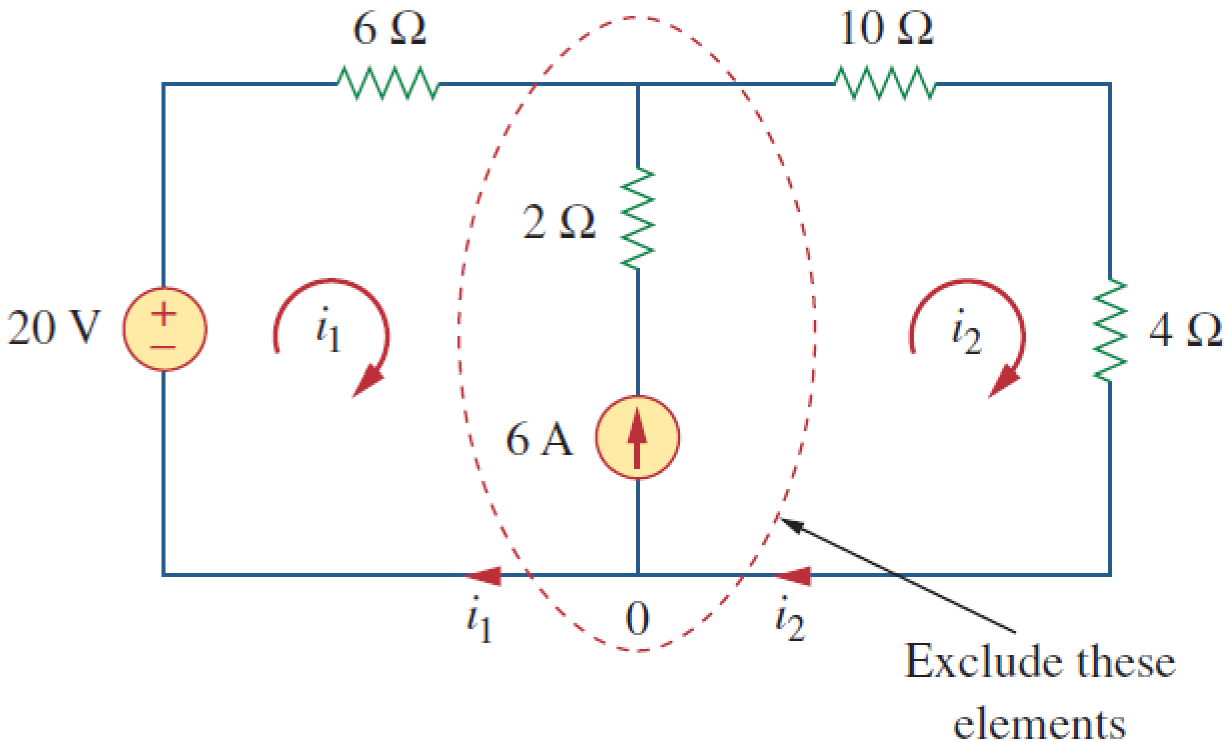
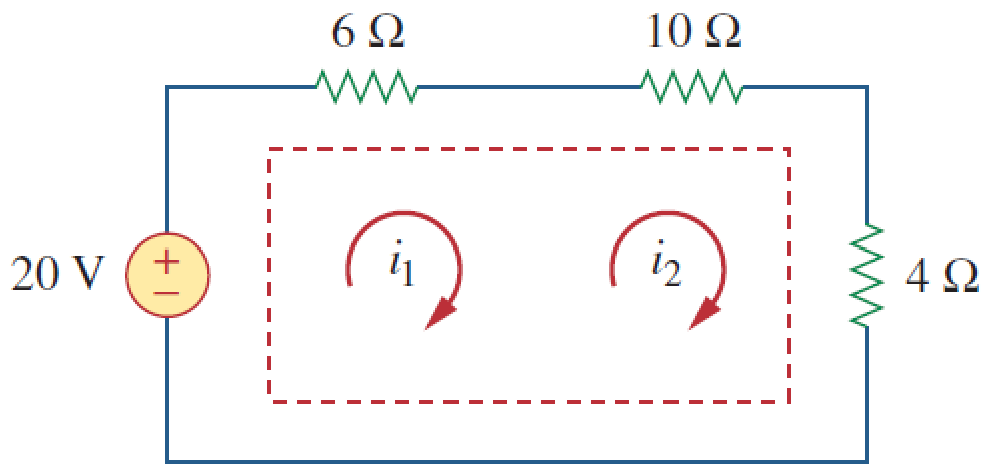
- Case 2

When the current source exist between **two meshes**.



need to create a **supermesh** by excluding the current source and any elements connected in series with it.

Mesh Analysis



NOTE: A supermesh results when two meshes have a (dependent or independent) current source in common.

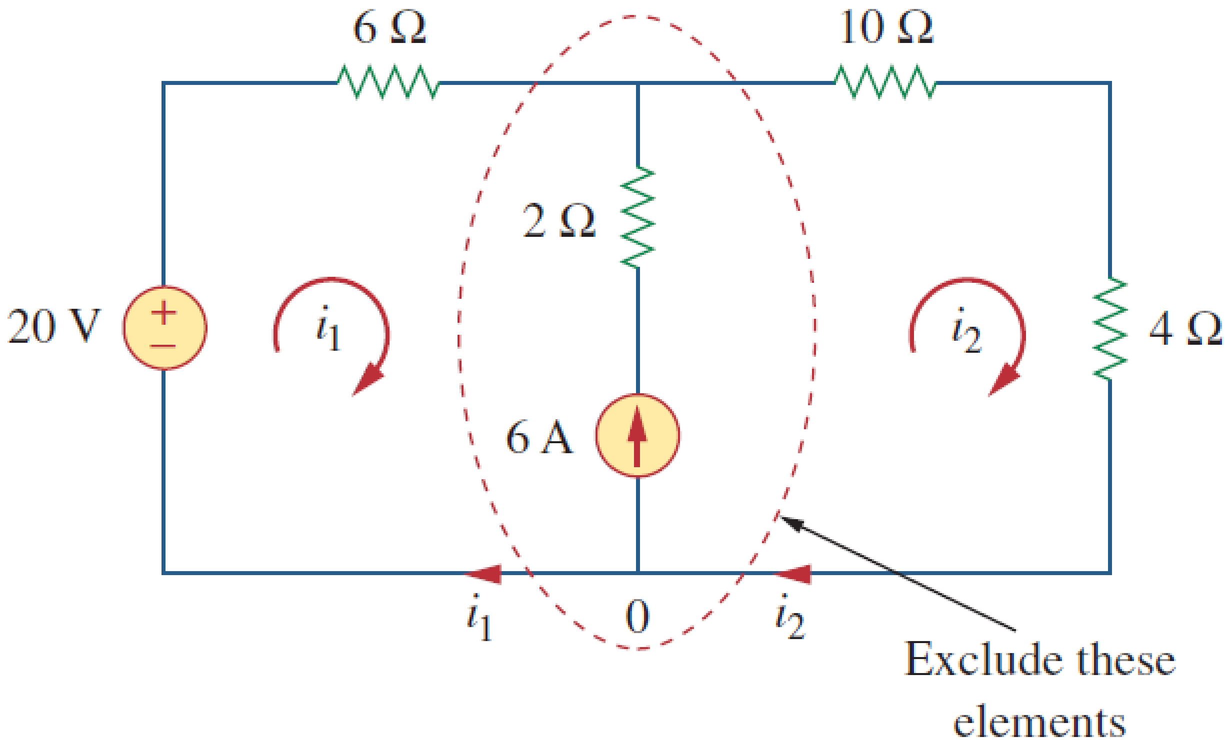
Applying KVL to the supermesh

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20 \qquad \rightarrow (1)$$

Applying KCL to the circuit

$$i_2 = i_1 + 6 \qquad \rightarrow (2)$$



Then solve the Eq. (1) and (2)

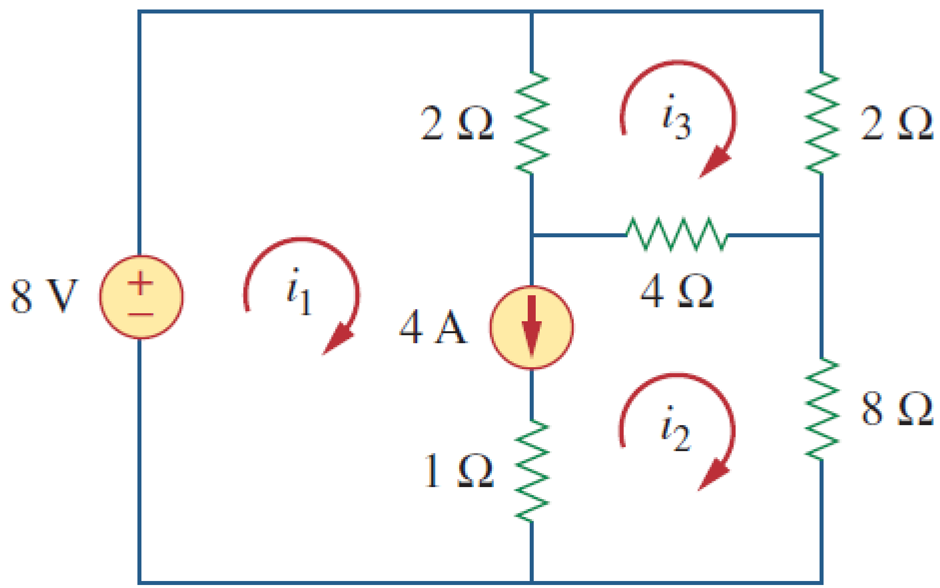
$$i_1 = -3.2\,\text{A} \qquad i_2 = 2.8\,\text{A}$$

Mesh Analysis

- The current source in the supermesh provides the constraint equation necessary to solve for the mesh currents.
- A supermesh has no current of its own.
- A supermesh requires the application of both KVL and KCL.

Example #11

By using mesh analysis find i_1, i_2 and i_3 in the following circuit



Example #11

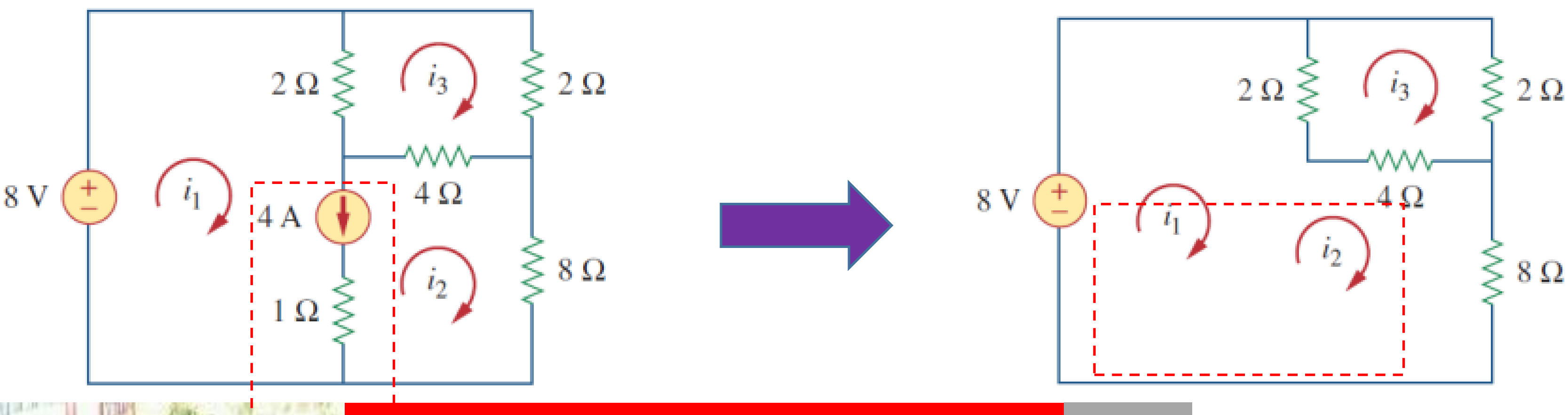
i. Assign the mesh currents: i_1, i_2 and i_3.

ii. Determine the polarity & the direction of current.

iii. Applying KCL to supermesh.

$$i_1 = 4 + i_2 \qquad \rightarrow (1)$$

Notice that meshes (1) and (2) form a supermesh since they have independent current source in common



Example #11

iv. Apply KVL to supermesh.

$$-8+2(i_1-i_3)+4(i_2-i_3)+8i_2=0$$

$$2i_1+12i_2-6i_3=8 \qquad \rightarrow (2)$$

Applying KVL in Mesh 3

$$2i_3+4(i_3-i_2)+2(i_3-i_1)=0$$

$$-2i_1-4i_2+8i_3=0 \qquad \rightarrow (3)$$

Then in matrix form

Example #11

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 12 & -6 \\ -2 & -4 & 8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$$

By using Cramer's Rule $\Delta = 76$

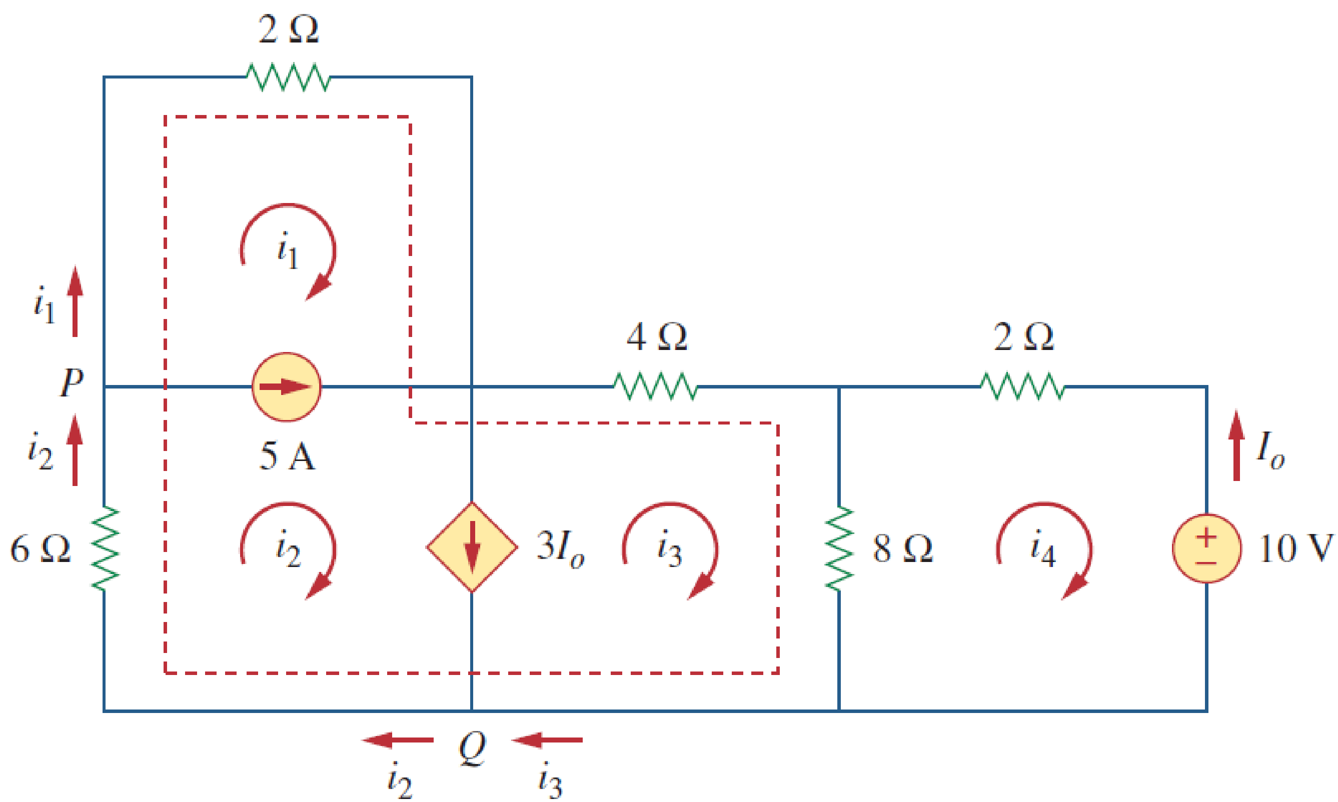
$$i_1 = \frac{\Delta_1}{\Delta} = \frac{352}{76} = 4.3516\,\text{A}$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{48}{76} = 0.6316\,\text{A}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{112}{76} = 1.4737\,\text{A}$$

Example #12

By using mesh analysis find i_1, to i_4 in the following circuit



Answer $i_1 = -7.5\,\text{A}$ $i_2 = -2.5\,\text{A}$ $i_3 = 3.93\,\text{A}$ $i_2 = 2.143\,\text{A}$