ENGINEERING MECHANICS
BAA1113

Chapter 4: Force System Resultants (Static)

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Chapter Description

- Aims
  - To explain the Moment of Force (2D-scalar formulation & 3D-Vector formulation)
  - To explain the Principle Moment
  - To explain the Moment of a Couple
  - To explain the Simplification of a Force and Couple System
  - To explain the Reduction of Simple Distributed Loading

- Expected Outcomes
  - Able to solve the problems of MOF and COM in the mechanics applications by using principle of moments

- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14th Edition

Chapter Outline

4.1 Moment of Force (MOF) –Part I

4.2 Principle of Moment –Part II

4.3 Moment of Couple (MOC) Part III

4.4 Simplification of a Force and Couple System

4.5 Reduction of Simple Distributed Loading- part IV

4.4 Simplification of force and couple systems

What is Equivalent system?

- A force has the effect of both translating and rotating a body
- The extent of the effect depends on how and where the force is applied
- It is possible to simplify a system of forces and moments into a **single resultant** and **moment** acting at a **specified point O**
- A **system of forces and moments** is then **equivalent** to the **single resultant force and moment acting at a specified point O**

Equivalent System

Point O is on the Line of Action

- Consider body subjected to force **F** applied to point A
- Apply force to point O without altering external effects on body

  - Apply equal but opposite forces **F** and –**F** at O

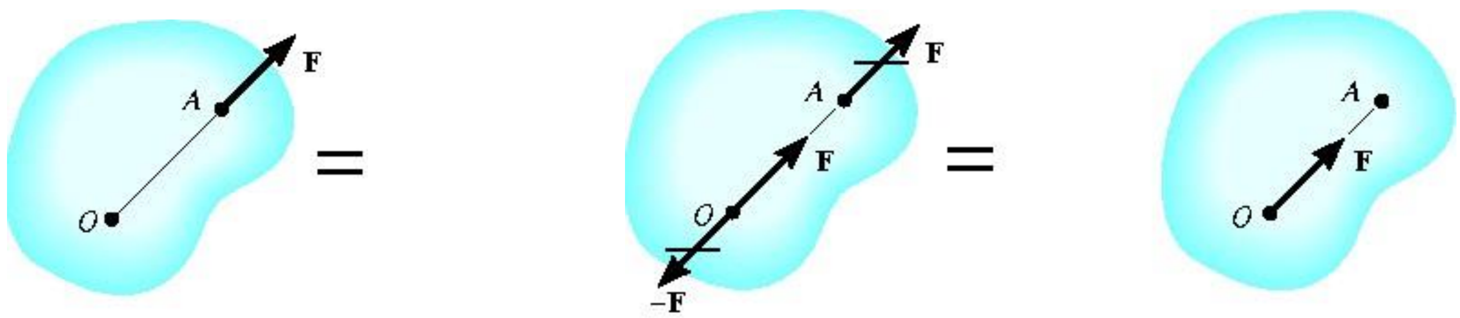
  - Two forces indicated by the slash across them can be cancelled, leaving force at point O

  - An **equivalent system** has be maintained between each of the diagrams, shown by the equal signs

  - Force has been simply transmitted along its line of action from point A to point O

  - External effects remain unchanged after force is moved

  - Internal effects depend on location of **F**



(a) (b) (c)

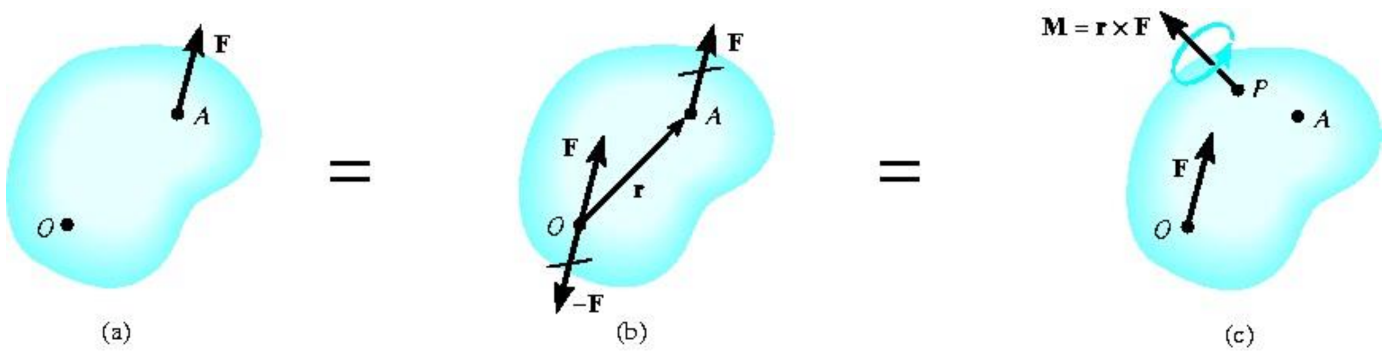
Equivalent System

Point O is Not on the Line of Action

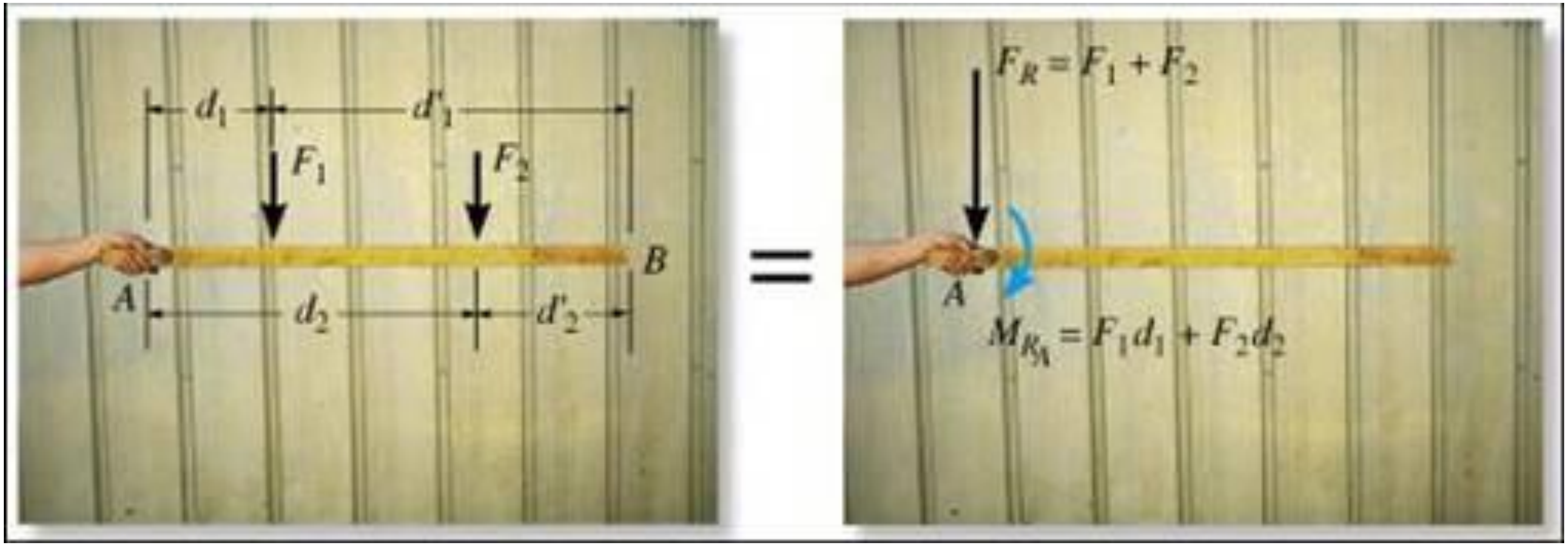
- **F** is to be moved to point 0 without altering the external effects on the body
- Apply equal and opposite forces at point O
- The two forces indicated by a slash across them, form a couple that has a moment perpendicular to **F**
- The moment is defined by cross product

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

- Couple moment is free vector and can be applied to any point P on the body



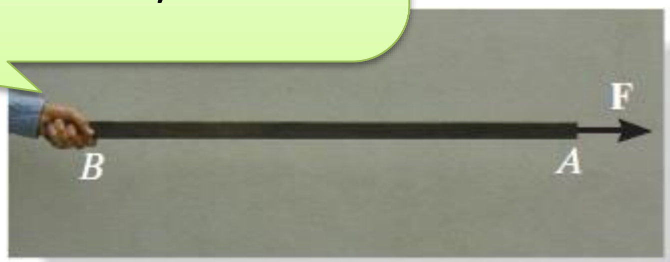
Application

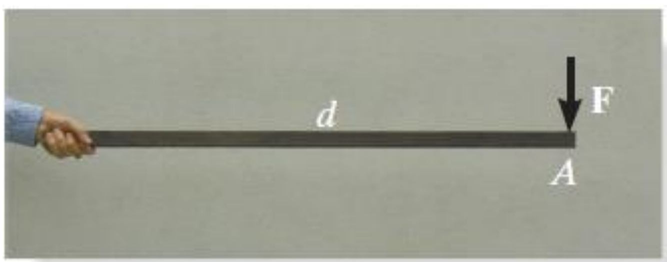


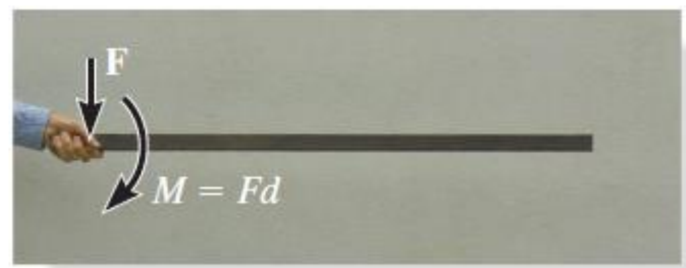
- Determine the effect of moving a force
- Determine an equivalent force-couple system for a system of forces and couples

Application

What are the resultant effects on the person’s hand when the force is applied in these four different ways?

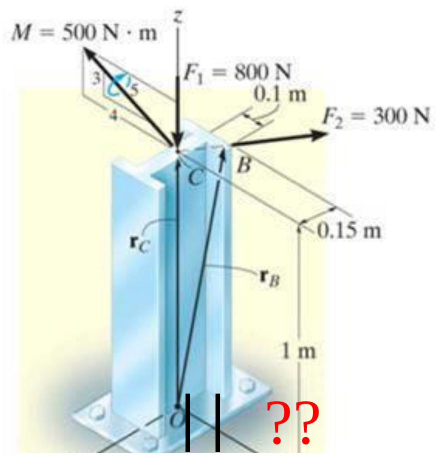


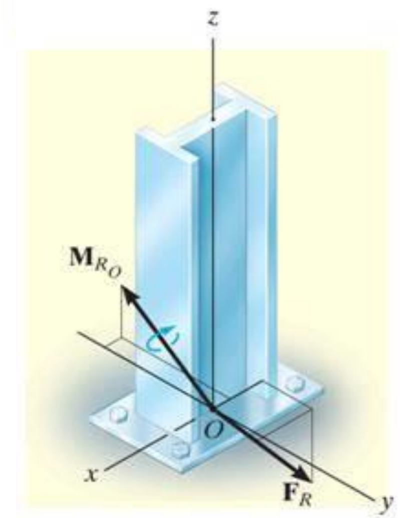




Why is understanding these differences important when designing various load-bearing structures?

Application

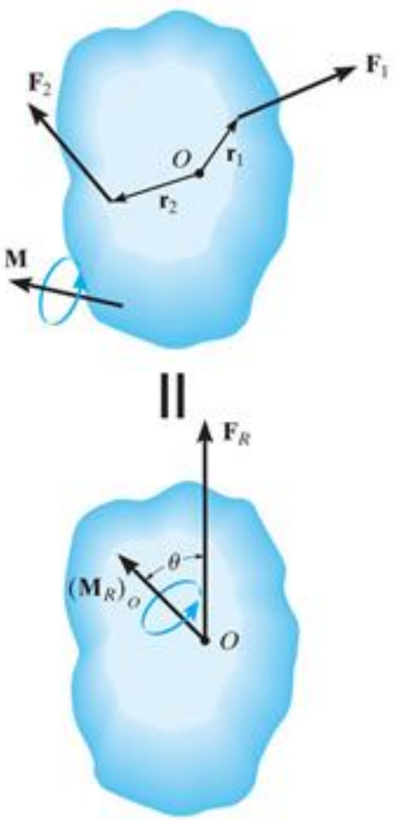




Several forces and a couple moment are acting on this vertical section of an I-beam.

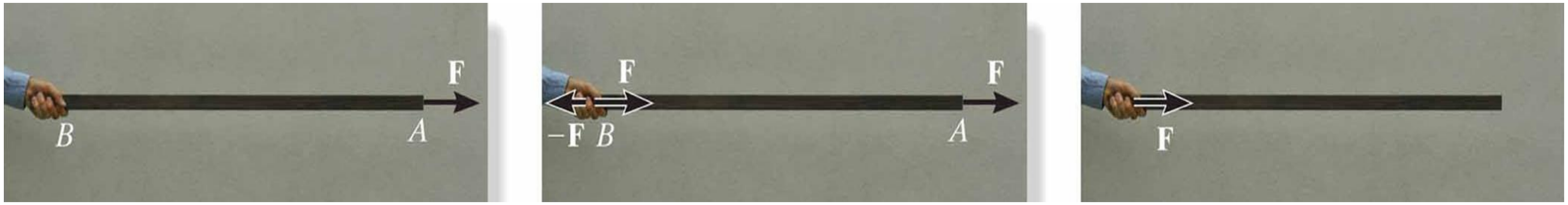
For the process of designing the I-beam, it would be very helpful if you could replace the various forces and moment just one force and one couple moment at point O with the same external effect? How will you do that?

Simplification force and couple system



- When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect.
- The two force and couple systems are called equivalent systems since they have the same external effect on the body.

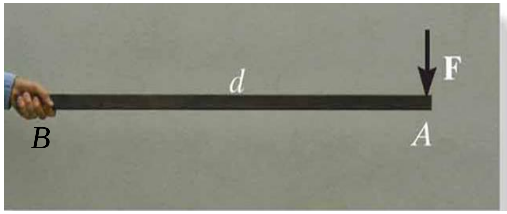
Moving a force on its line of action

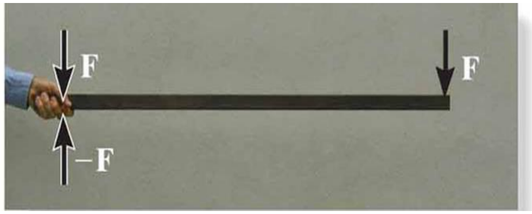


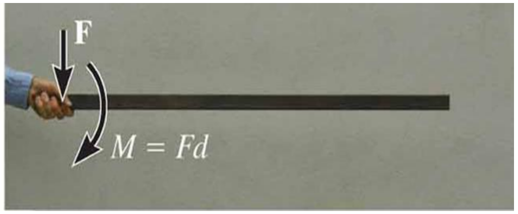
Moving a force from A to B, when both points are on the vector's line of action, does not change the external effect.

Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).

Moving a force on its line of action





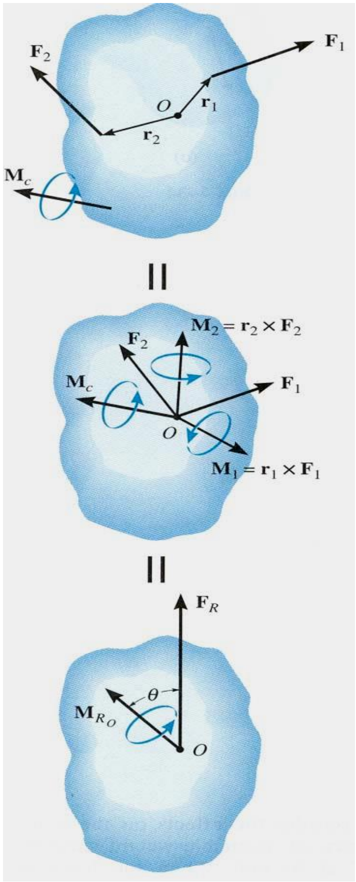


When a force is moved, but not along its line of action, there is a change in its external effect!

Essentially, moving a force from point A to B (as shown above) requires creating an additional couple moment. So moving a force means you have to "add" a new couple.

Since this new couple moment is a "free" vector, it can be applied at any point on the body.

Simplification force and couple system

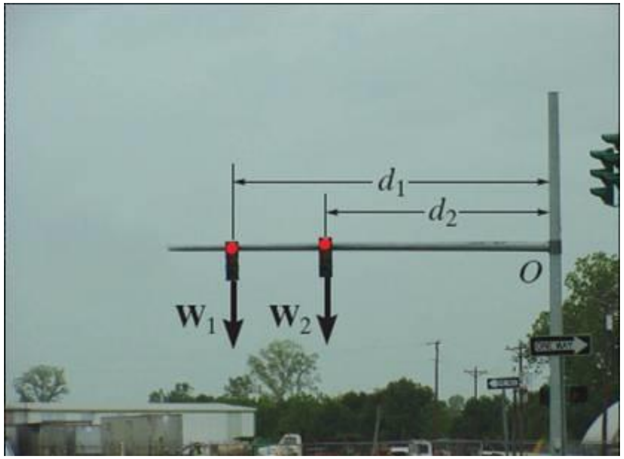


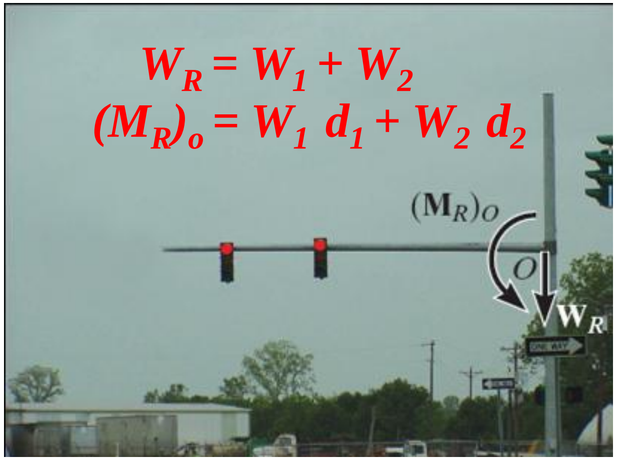
- To simplify any force and couple moment system to a resultant force acting at point O and a resultant couple moment, can use the following equations

$$\mathbf{F}_R = \sum \mathbf{F}$$
$$\mathbf{M}_R = \sum \mathbf{M}_C + \sum \mathbf{M}_O$$

The resultant couple moment is equivalent to the sum of all the **couple moments** plus the **moments about point O** of all the forces

Simplification force and couple system





If the force system lies in the x-y plane (a 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{Rx} = \sum F_x$$
$$F_{Ry} = \sum F_y$$
$$M_{Ro} = \sum M_C + \sum M_O$$

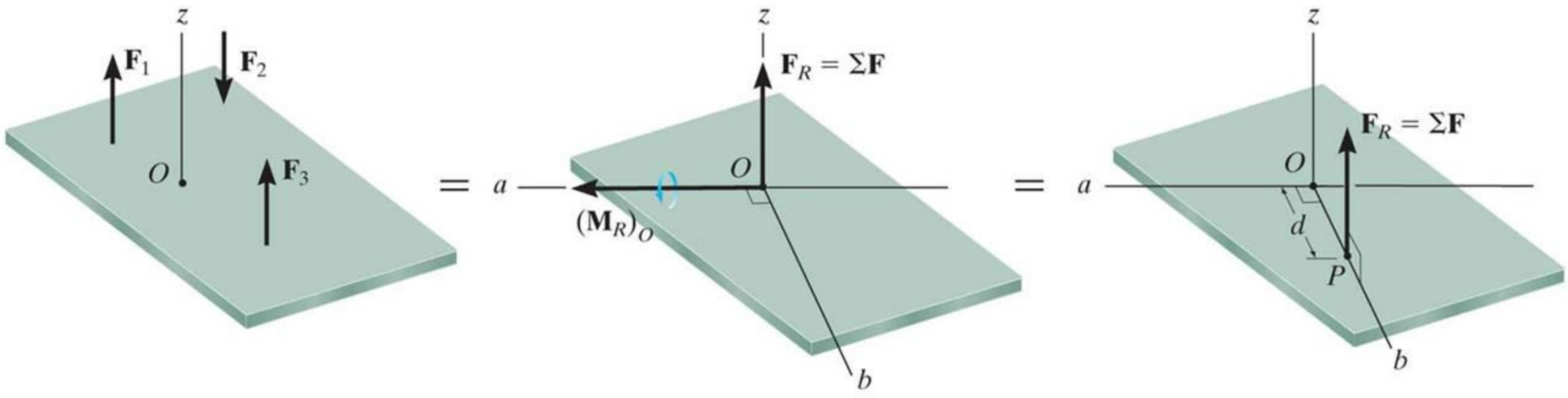
Procedure to use the following equation

$$\mathbf{F}_R = \sum \mathbf{F}$$
$$\mathbf{M}_R = \sum \mathbf{M}_C + \sum \mathbf{M}_O$$
$$F_{Rx} = \sum F_x$$
$$F_{Ry} = \sum F_y$$
$$M_{Ro} = \sum M_C + \sum M_O$$

- Establish the coordinate axes with the origin located at the point O and the axes having a selected orientation
- Force Summation
  - For coplanar force system, resolve each force into x and y components
  - If the component is directed along the positive x or y axis, it represent a positive scalar
  - If the component is directed along the negative x or y axis, it represent a negative scalar
  - In 3D problems, represent forces as Cartesian vector before force summation

- Moment Summation
  - For moment of coplanar force system about point O, use Principle of Moment
  - Determine the moments of each components rather than of the force itself
  - In 3D problems, use vector cross product to determine moment of each force
  - Position vectors extend from point O to any point on the line of action of each force

Further Simplification force and couple system

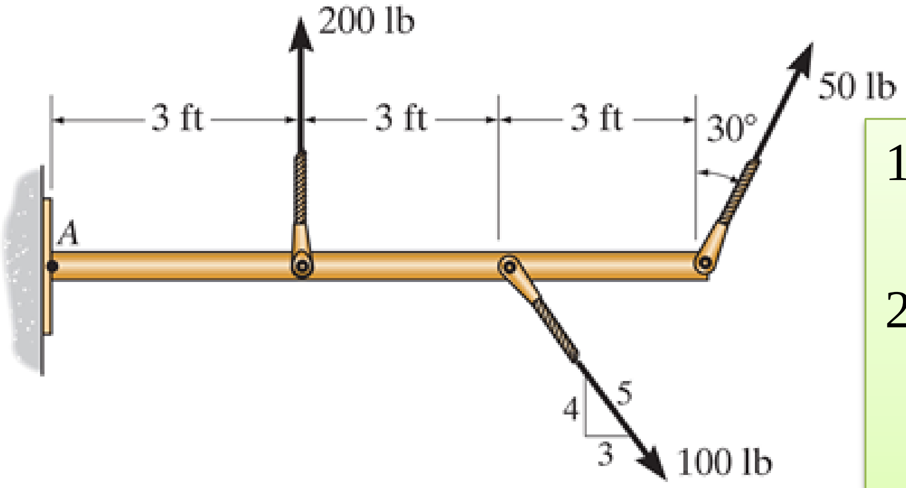


If F_R and M_{RO} are perpendicular to each other, then the system can be further reduced to a single force, F_R , by simply moving F_R from O to P.

In three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force.

Example 4.19

A 2-D force system with the geometry shown. Determine the equivalent resultant force and couple moment acting at A and then the equivalent single force location measured from A



1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force component to A.

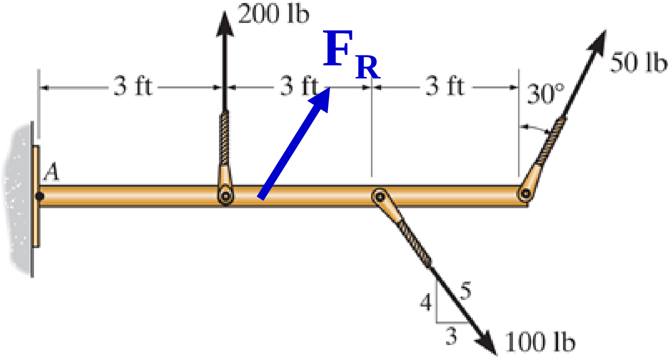
3) Shift F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

Solution Example 4.19

$+\rightarrow \Sigma F_{Rx} = 50(\sin 30) + 100(3/5)$
$= 85$ lb

$+\uparrow \Sigma F_{Ry} = 200 + 50(\cos 30) - 100(4/5)$
$= 163.3$ lb

$+ \quad M_{RA} = 200\,(3) + 50\,(\cos 30)\,(9)$
$- 100\,(4/5)\,6 = 509.7$ lb·ft CCW



$F_R = (85^2 + 163.3^2)^{1/2} = \underline{184 \text{ lb}}$

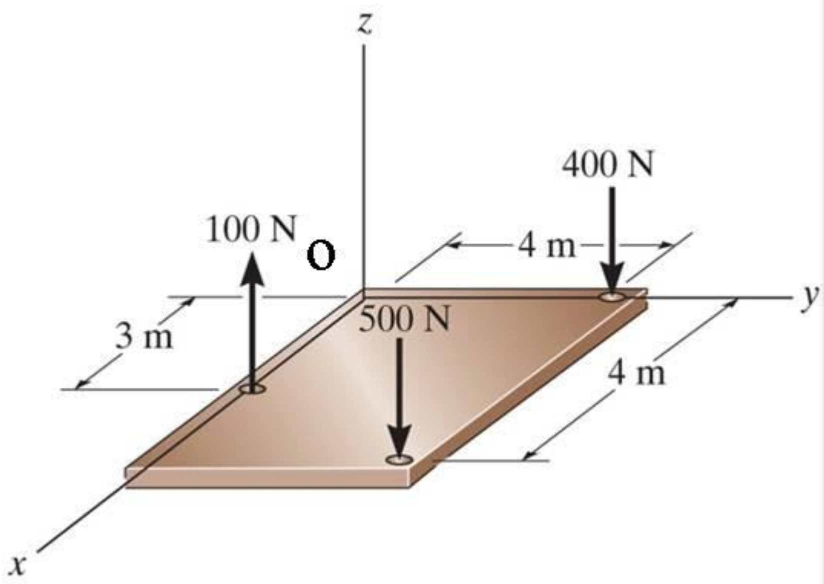
$\theta = \tan^{-1}(163.3/85) = \underline{62.5°}$

The equivalent single force F_R can be located at a distance d measured from A.

$d = M_{RA}/F_{Ry} = 509.7 / 163.3 = \underline{3.12 \text{ ft}}$

Example 4.20

The slab is subjected to three parallel forces. Determine the equivalent resultant force and couple moment at the origin O. Also find the location (x, y) of the single equivalent resultant force

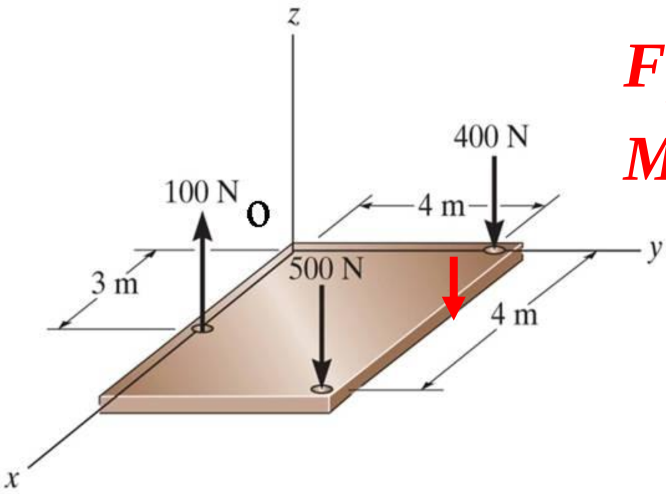


1) Find $\boldsymbol{F}_{RO} = \sum \boldsymbol{F}_i = F_{Rzo}\,\boldsymbol{k}$

2) Find $\boldsymbol{M}_{RO} = \sum (\boldsymbol{r}_i \times \boldsymbol{F}_i) = M_{RxO}\,\boldsymbol{i} + M_{RyO}\,\boldsymbol{j}$

3) The location of the single equivalent resultant force is given as $x = -M_{RyO} / F_{RzO}$ and $y = M_{RxO} / F_{RzO}$

Solution Example 4.20



$\boldsymbol{F_{RO}} = \{100\,\boldsymbol{k} - 500\,\boldsymbol{k} - 400\,\boldsymbol{k}\} = -800\,\boldsymbol{k}$ N

$\boldsymbol{M_{RO}} = (3\,\boldsymbol{i}) \times (100\,\boldsymbol{k}) + (4\,\boldsymbol{i} + 4\,\boldsymbol{j}) \times (-500\,\boldsymbol{k})$
$+ (4\,\boldsymbol{j}) \times (-400\,\boldsymbol{k})$
$= \{-300\,\boldsymbol{j} + 2000\,\boldsymbol{j} - 2000\,\boldsymbol{i} - 1600\,\boldsymbol{i}\}$
$= \{-3600\,\boldsymbol{i} + 1700\,\boldsymbol{j}\}$ N·m

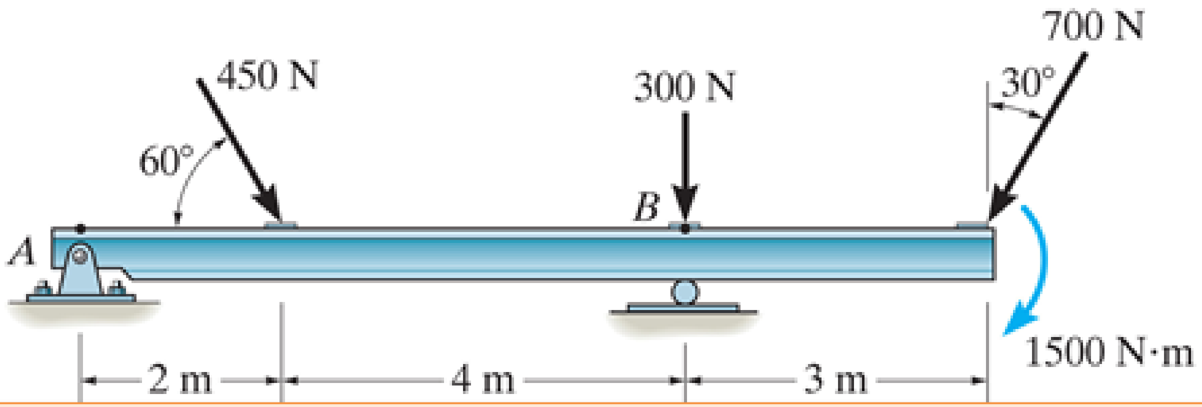
The location of the single equivalent resultant force is given as,

$x = -M_{Ryo} / F_{Rzo} = (-1700) / (-800) = 2.13$ m

$y = M_{Rxo} / F_{Rzo} = (-3600) / (-800) = 4.5$ m

Example 4.21

A 2-D force and couple system as shown. Determine the equivalent resultant force and couple moment acting at A.

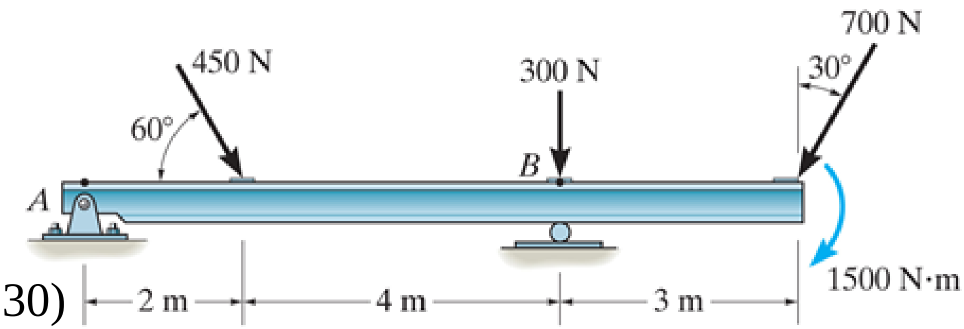


1) Sum all the x and y components of the two forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A and add them to the 1500 N·m free moment to find the resultant M_{RA} .

Solution Example 4.21

Summing the force components:



$\xrightarrow{+} \Sigma F_x = 450\,(\cos 60) - 700\,(\sin 30)$

$= -125\ \text{N}$

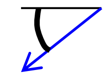
$+\uparrow \Sigma F_y = -450\,(\sin 60) - 300 - 700\,(\cos 30)$

$= -1296\ \text{N}$

Now find the magnitude and direction of the resultant.

$F_{RA} = (125^2 + 1296^2)^{1/2} = \underline{1302\ \text{N}}$ and $\theta = \tan^{-1}(1296/125)$

$= \underline{84.5°}$

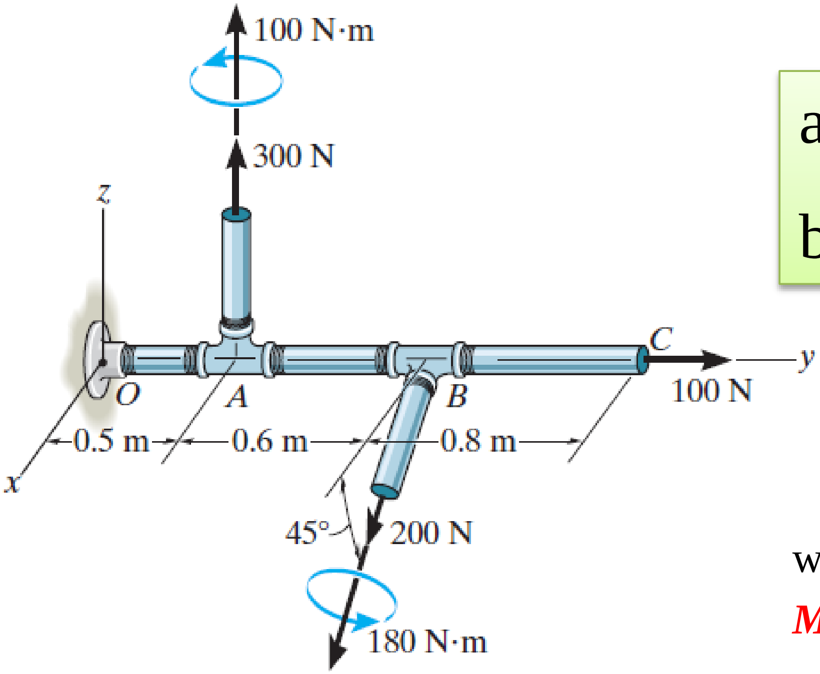


$+\circlearrowleft M_{RA} = 450\,(\sin 60)\,(2) + 300\,(6) + 700\,(\cos 30)\,(9) + 1500$

$= \underline{9535\ \text{N·m}}$ $\circlearrowleft$

Example 4.22

Forces and couple moments are applied to the pipe.Determine the equivalent resultant force and couple moment at point O



a) Find $\boldsymbol{F_{RO}} = \Sigma\, \boldsymbol{F_i} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$

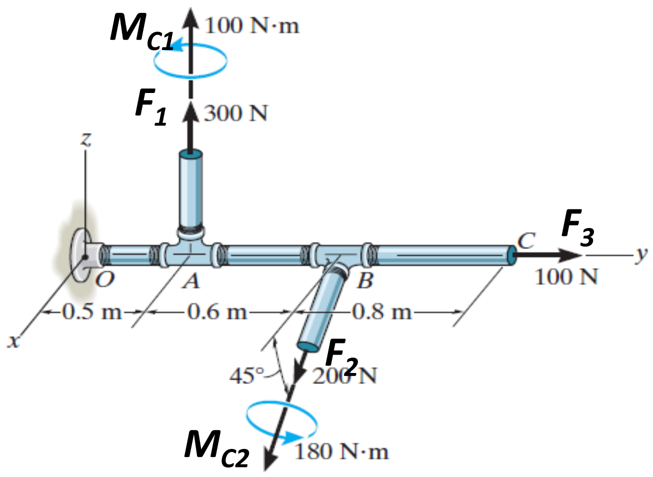
b) Find $\boldsymbol{M_{RO}} = \Sigma\, \boldsymbol{M_C} + \Sigma\, (\boldsymbol{r_i} \times \boldsymbol{F_i})$

where,

$\boldsymbol{M_C}$ are any free couple moments.

$\boldsymbol{r_i}$ are the position vectors from the point O to any point on the line of action of $\boldsymbol{F_i}$.

Solution Example 4.22



F_1 = {300 k} N

F_2 = 200{cos45° i – sin 45° k} N

= {141.4 i – 141.4 k} N

F_3 = {100 j} N

r_1 = {0.5 i } m, r_2 = {1.1 i } m,

r_3 = {1.9 i } m

Free couple moments are:

M_{C1} = {100 k} N·m

M_{C2} = 180{cos45° i – sin 45°k}N·m

= {127.3 i – 127.3k}N·m

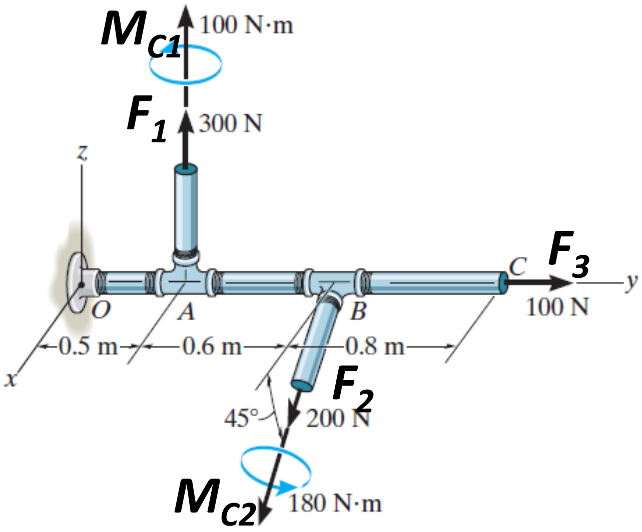
Solution Example 4.22

Resultant force and couple moment at point O:

$\mathbf{F}_{RO} = \Sigma \mathbf{F}_i = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$= \{300\,\mathbf{k}\} + \{141.4\,\mathbf{i} - 141.4\,\mathbf{k}\} + \{100\,\mathbf{j}\}$

$\mathbf{F}_{RO} = \{\underline{141}\,\mathbf{i} + \underline{100}\,\mathbf{j} + \underline{159}\,\mathbf{k}\}\ \underline{\text{N}}$



$\mathbf{M}_{RO} = \Sigma \mathbf{M}_C + \Sigma(\mathbf{r}_i \times \mathbf{F}_i)$

$\mathbf{M}_{RO} = \{100\,\mathbf{k}\} + \{127.3\,\mathbf{i} - 127.3\mathbf{k}\}$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.1 & 0 \\ 141.4 & 0 & -141.4 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.9 & 0 \\ 0 & 100 & 0 \end{vmatrix}$$

$\mathbf{M}_{RO} = \{122\,\mathbf{i} - 183\,\mathbf{k}\}\ \underline{\text{N·m}}$

Conclusion of The Chapter 4

- Conclusions

  - The simplification force and moment has been identified
  - The scalar and vector analysis have been implemented to solve Moment problems in specified axis

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