ENGINEERING MECHANICS BAA1113

Chapter 4: Force System Resultants (Static)

by
Pn Rokiah Bt Othman
Faculty of Civil Engineering & Earth Resources
rokiah@ump.edu.my

Chapter Description

- Aims
  - To explain the Moment of Force (2D-scalar formulation & 3D-Vector formulation)
  - To explain the Principle Moment
  - To explain the Moment of a Couple
  - To explain the Simplification of a Force and Couple System
  - To explain the Reduction of Simple Distributed Loading

- Expected Outcomes
  - Able to solve the problems of MOF and COM in the mechanics applications by using principle of moments

- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14th Edition

Chapter Outline

4.1 Moment of Force (MOF) –Part I

4.2 Principle of Moment –Part II

4.3 Moment of Couple (MOC) Part III

4.4 Simplification of a Force and Couple System

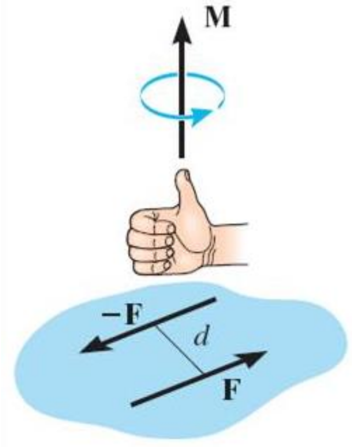
4.5 Reduction of Simple Distributed Loading- part IV

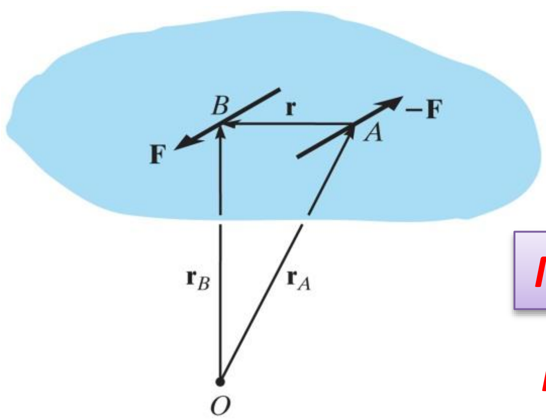
4.3 Moment of a Couple

A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance "d."

- Resultant force = 0
- Tendency to **rotate in specified direction**
- Couple moment is a **free vector**
- It can be compute d by **any point**
- Choose the line action of **one of the force** in the couple
- A resultant couple moment = sum of the couple moments of the system
- $\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$

- The moment of a couple is defined as



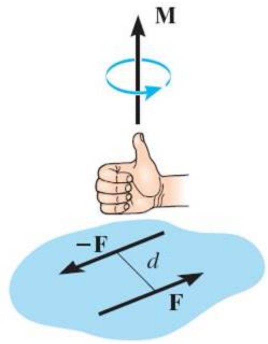


$M_O = \boldsymbol{r} \times \boldsymbol{F}$ (using a vector analysis)

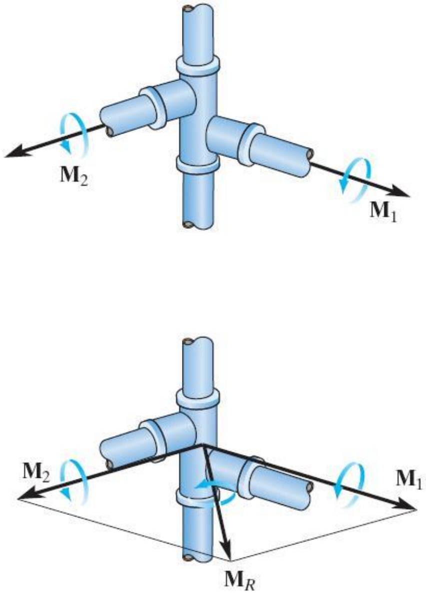
$\boldsymbol{r}$ is any position vector from the line of action of $\boldsymbol{F}$ to the line of action of $\boldsymbol{F}$

M_O = F d (using a scalar analysis)

4.3 Moment of a Couple

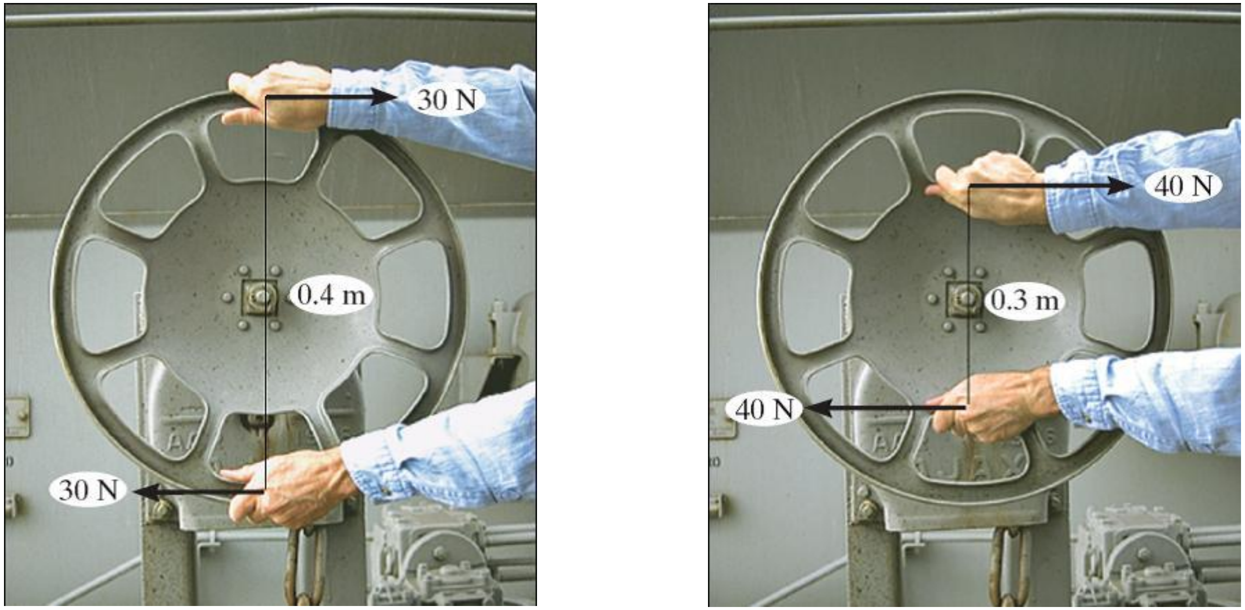


- The net external effect of a couple is that the net force equals zero and the magnitude of the net moment equals F *d.



- Since the moment of a couple depends only on the distance between the forces, the moment of a couple is a free vector.
- It can be moved anywhere on the body and have the same external effect on the body.

Application (Moment of a Couple)



A torque or moment of 12 N·m is required to rotate the wheel. Why does one of the two grips of the wheel above require less force to rotate the wheel?

Application (Moment of a Couple)

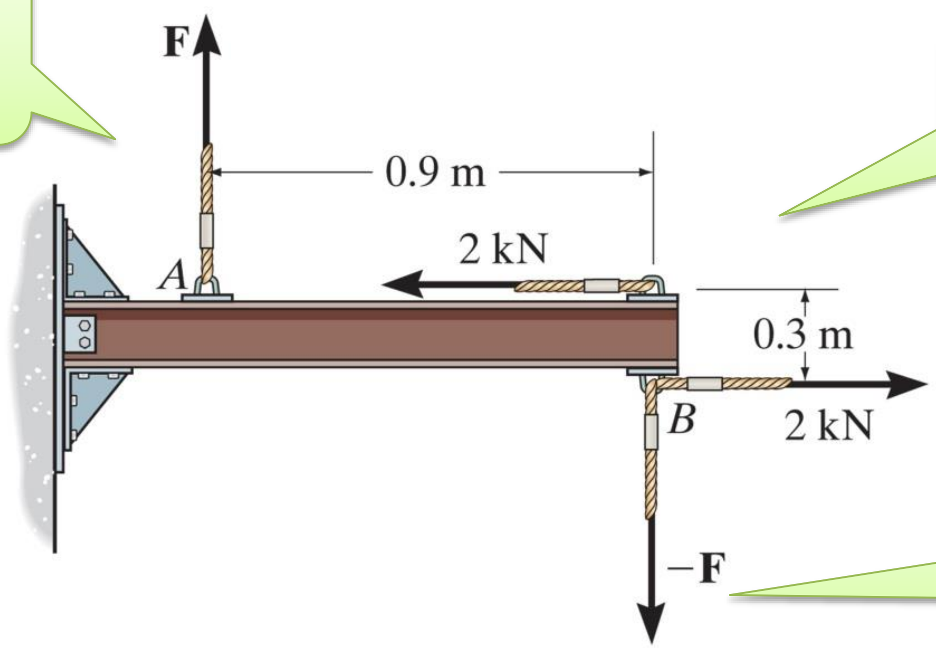
- When you grip a vehicle's steering wheel with both hands and turn, a couple moment is applied to the wheel

Would older vehicles without power steering have needed larger or smaller steering wheels?

Example 4.15

Two couples act on the beam with the geometry shown. Determine the magnitude of F so that the resultant couple moment is 1.5 kN.m clockwise

Step 1: FBD (Sketch outline shape)



Step 2: det d

use formula: scalar analysis

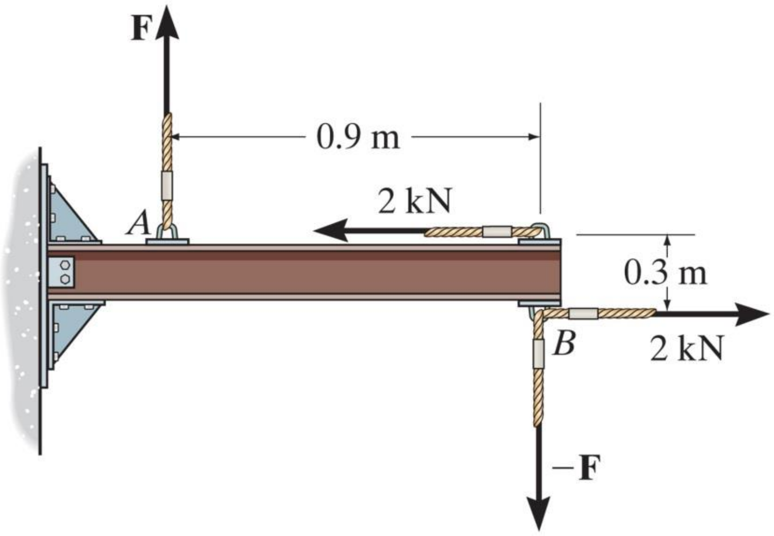
$$M_O = Fd$$

Det F?

Solution Example 4.15

Net moment = 1.5 kN.m

$M_O = Fd$



- The net moment is equal to:

$+ \Sigma M = - F\,(0.9) + (2)\,(0.3)$

$= - 0.9\,F + 0.6$

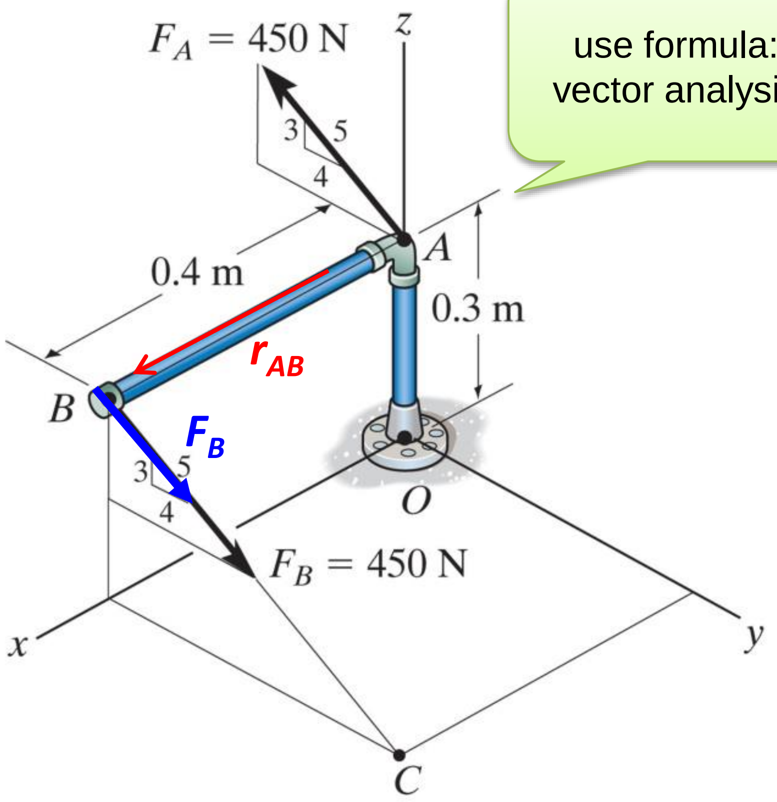
$- 1.5\ \text{kN·m} \quad = - 0.9\,F + 0.6$

- Solving for force F,

F = 2.33 kN

Example 4.16

A 450 N force couple acting on the pipe as shown. Determine the couple moment in cartesian vector notation.



use formula:
vector analysis

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \boldsymbol{F}_B$

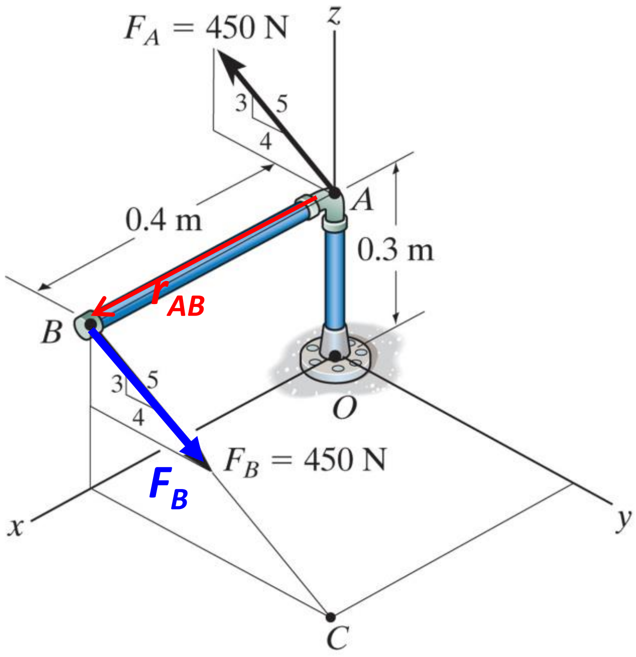
3) Write in the cross product to find $\boldsymbol{M}$

Solution Example 4.16

$\mathbf{r}_{AB} = \{ 0.4\ \mathbf{i} \}$ m

$\mathbf{F}_B = \{0\ \mathbf{i} + 450(4/5)\ \mathbf{j} - 450(3/5)\ \mathbf{k}\}$ N

$= \{0\ \mathbf{i} + 360\ \mathbf{j} - 270\ \mathbf{k}\}$ N



$\mathbf{M} = \mathbf{r}_{AB} \times \mathbf{F}_B$

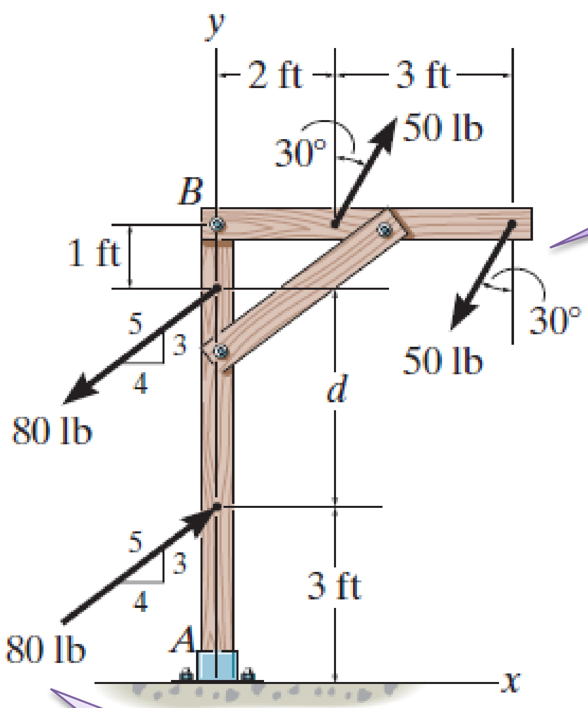
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.4 & 0 & 0 \\ 0 & 360 & -270 \end{vmatrix} \text{ N·m}$$

$= [\{0(-270) - 0(360)\}\ \mathbf{i} - \{4(-270) - 0(0)\}\ \mathbf{j} + \{0.4(360) - 0(0)\}\ \mathbf{k}]$ N·m

M= {0 $\mathbf{i}$ + 108 $\mathbf{j}$ + 144 $\mathbf{k}$} N·m

Example 4.17

Two couples act on the beam with the geometry shown and d = 4 ft. Determine the resultant couple.



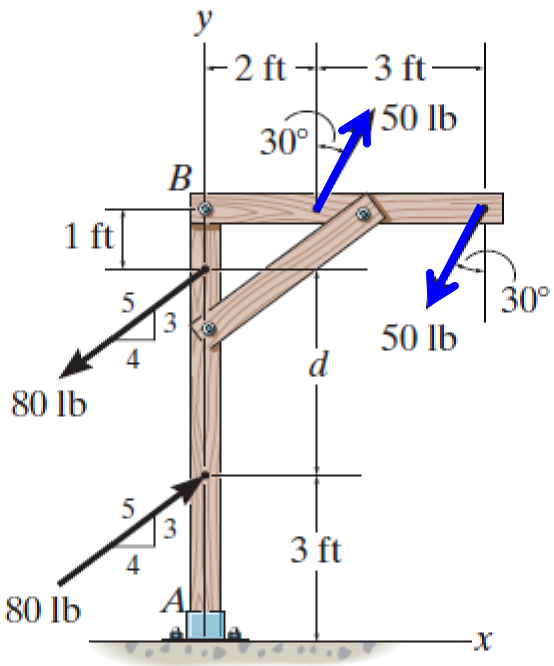
First , Resolve 50 lb into x y

2nd , Resolve 80 lb into x y

1) Resolve the forces in x and y-directions so they can be treated as couples.

2) Add these two couples to find the resultant couple.

Solution Example 4.17



The x and y components of the upper-left 50 lb force are:

50 lb (cos 30°) = 43.30 lb vertically up

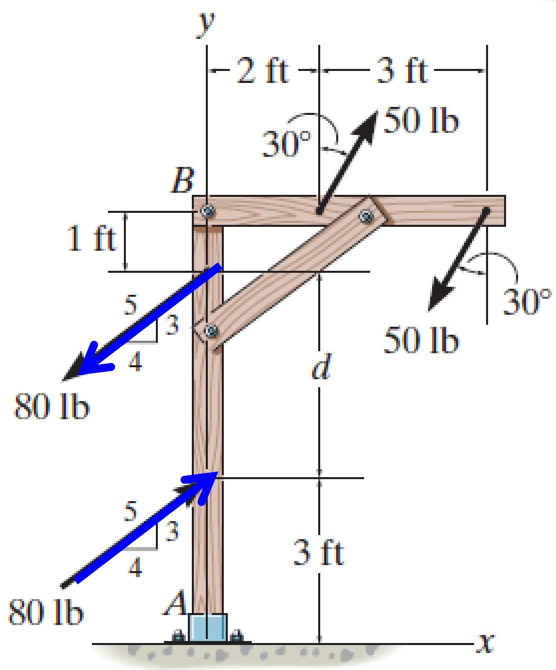
50 lb (sin 30°) = 25 lb to the right

Do both of these components form couples with their matching components of the other 50 force?

No! Only the 43.30 lb components create a couple. Why?

Solution Example 4.17

Do both of these components create a couple with components of the other 80 force?



Now resolve the lower 80 lb force:

(80 lb) (3/5), acting up

(80 lb) (4/5), acting to the right

The net moment is equal to:

+ ΣM = – (43.3 lb)(3 ft) + (64 lb)(4 ft)

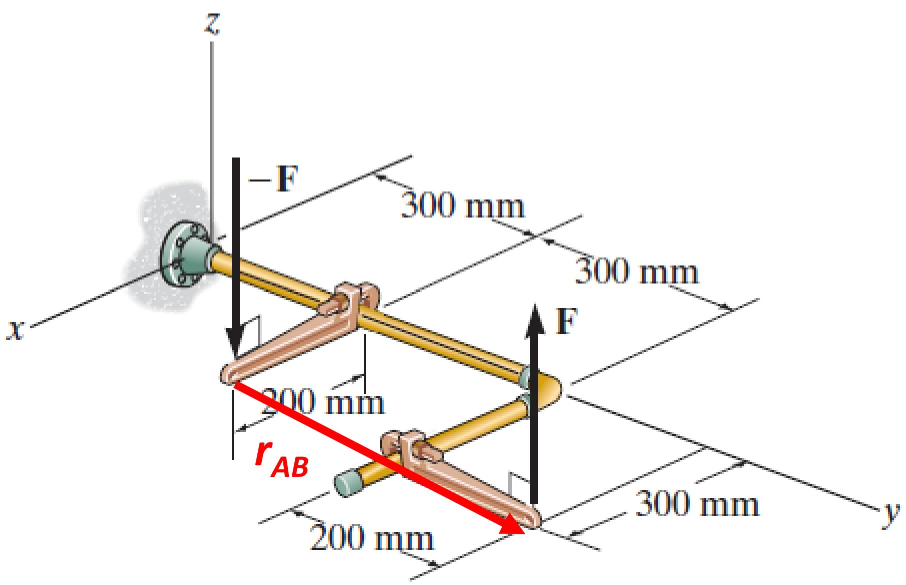
= – 129.9 + 256

= 126 ft·lb CCW

Example 4.18

Two couples act on the beam with the geometry shown. ***F*** = {80 ***k***} N and

– ***F*** = {– 80 ***k***} N . Determine The couple moment acting on the pipe assembly using Cartesian vector notation



1) Use $\mathbf{M} = \mathbf{r} \times \mathbf{F}$ to find the couple moment.

2) Set $\mathbf{r} = \mathbf{r}_{AB}$ and $\mathbf{F}$ = {80 $\mathbf{k}$} N.

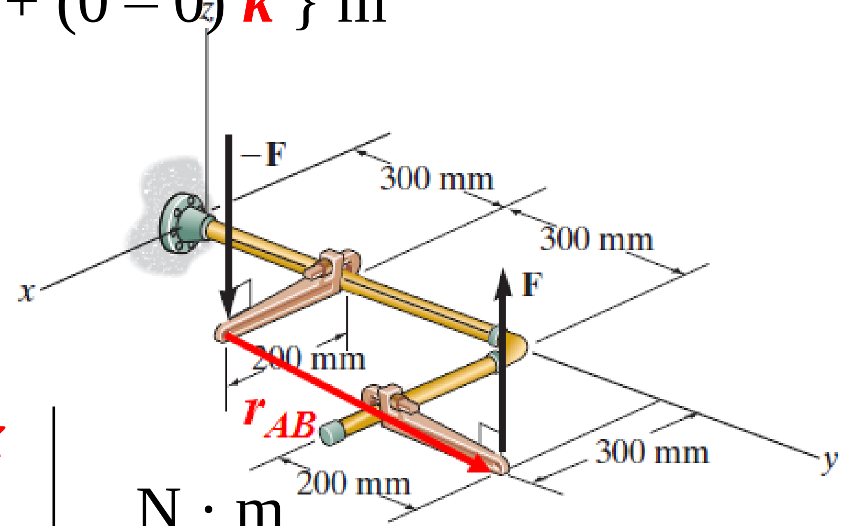
3) Write the cross product to determine ***M***.

Solution Example 4.18

$\boldsymbol{r}_{AB} = \{ (0.3 - 0.2)\,\boldsymbol{i} + (0.8 - 0.3)\,\boldsymbol{j} + (0 - 0)\,\boldsymbol{k} \}$ m

$= \{ 0.1\,\boldsymbol{i} + 0.5\,\boldsymbol{j} \}$ m

$\boldsymbol{F} = \{80\,\boldsymbol{k}\}$ N



$$\boldsymbol{M} = \boldsymbol{r}_{AB} \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 80 \end{vmatrix} \text{ N} \cdot \text{m}$$

$= \{(40 - 0)\,\boldsymbol{i} - (8 - 0)\,\boldsymbol{j} + (0)\,\boldsymbol{k}\}$ N · m

$= \{ 40\,\boldsymbol{i} - 8\,\boldsymbol{j} \}$ N · m

Conclusion of The Chapter 4

- Conclusions

  - The Moment of couple has been identified
  - The scalar and vector analysis have been implemented to solve Moment problems in specified axis

Credits to:

Dr Nurul Nadhrah Bt Tukimat
nadrah@ump.edu.my

En Khalimi Johan bin Abd Hamid
khalimi@ump.edu.my

Roslina binti Omar
rlina@ump.edu.my